Sam F Perry

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ARITHMETIC,

Both in the

THEORY AND PRACTICE,

Made plain and easy in all the common and useful RULES, both in

Whole Numbers and Fractions,

VULGAR and DECIMAL:

Also INTEREST { *Simple* and / *Compound.* } and ANNUITIES.

LIKEWISE

Extraction of the Square and Cube Roots.

TOGETHER WITH

Arithmetical and Geometrical Progression, and the Combination and Election, Permutation and Composition of Numbers and Quantities.

With the Addition of

Several ALGEBRAICAL QUESTIONS.

By JOHN HILL, Gent.

With a PREFACE, *by* H. DITTON, *Gent.*

The THIRTEENTH EDITION, accurately Revised and Corrected.

EDINBURGH:
Printed by JOHN GRAY and GAVIN ALSTON.
Sold by them, and by JOHN WOOD at his Shop in the Luckenbooths.
MDCCLXV.

Prof. L. C. Karpinski
4-30-1924

TO THE

READER.

IT being a sort of disparagement to things which are evidently *very good, to say* much *in commendation of them; I shall therefore, in* justice *to this book, give but a very short account of it.*

It appears to me, upon the perusal of it, to be a curious piece: it is clean, methodical, *and handsomely dressed: so* plain, *that the dullest person may learn by it, and so* complete, *that he need learn no more.*

The Author (whoever he was) has in this treatise gone much beyond the bounds which the common writers of this Science *use to advance to.*

And though many of the practices *he delivers,* ex. gr. *the making of* LOGARITHMS, INTEREST, *and* COMBINATION *of* QUANTITIES, *are to be done with greater advantage and exactness, by the help of* superior methods, *as* ALGEBRA, *&c.; yet take him purely* as an Arithmetician, *and he has not only done more, and much better, than* Wingate, Cocker, Leyburn, *or any other of the writers in our tongue, but indeed all that*

can

can be done by Arithmetic. And therefore, if no other book on this subject comes out till this performance is really mended, *I am satisfied we shall have no new book of Arithmetic very soon.*

CHRIST's-Hospital,
Nov. 12. 1712.

H. DITTON.

I Have perused this book; and finding it very well done, recommend it to such as desire a good knowledge in Arithmetic.

From my school in Foster-Lane,
March 7. 17$\frac{20}{21}$

C. SNELL.

ARITH-

ARITHMETIC,

BOTH IN

THEORY and PRACTICE.

The INTRODUCTION.

SECTION I.

Containing the general Præcognita.

1. ARITHMETIC is an art or ſcience that teacheth us the dexterous handling of numbers, and contains three branches,

viz. { VULGAR, LOGARITHMICAL, ANALYTICAL.

2. For the well managing of which, the *Arabians*, as may be ſuppoſed by their way of reading, invented the following ſymbols or characters, commonly called *digits*, (as may be reaſonably gueſſed, from the fingers of the hand), which, though few in number, are ſufficient for managing the vaſteſt calculations.

See here their names and characters.

1	2	3	4	5	6	7	8	9	0
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Null

3. The cipher, null, ſtanding by itſelf, ſignifieth nothing; but being joined with other numbers, increaſeth or decreaſeth their value; and is indeed the beginning of all number, as we ſhall elſewhere demon-

ſtrate, contrary to what *Tacquet*, and ſome other modern artiſts affirm.

4. Number is compoſed of one or a multitude of units, and is that by which we ſay any thing is numbered; as 1 acre, 4 crowns, 7 days in a week.

5. Of numbers are ſeveral ſorts; as digits, articles, compounds, whole, broken, mixed, *&c.*

6. Digits are ſuch numbers as are under ten, as 2, 3, 4, 5, *&c.*

7. Articles are ſuch numbers as are compoſed of a digit and a cipher, as 10, 20, 30, *&c.* and are diviſible by ten without a remainder.

8. Compound numbers are ſuch as are compounded of many numbers, as 144, 282, 1702, 1728, *&c.*

9. A whole number either contains unity, or ſome number thereof; as 7, 21, 512, 2056, *&c.*

10. A fraction, or broken number, is always leſs than unity; as $\frac{3}{4}$ repreſents three quarters of any thing, or unity; and $\frac{6}{10}$, or .6, is ſix tenth parts of unity, *&c.*

11. A mixed number is always greater than unity, as $2\frac{1}{2}$ repreſents 2 integers, and one half of an integer or unity, and $7\frac{75}{100}$, or 7.75, ſignifies 7 integers, and 75 hundred parts of an integer or unity.

12. According to the diviſion of unity, a fraction comes to be ſtyled vulgar or decimal.

13. A vulgar fraction is divided into two parts, one above another, with a ſmall line drawn betwixt, of which the lower is called the *denominator*, and the higher the *numerator*, ſhewing how many of thoſe parts are ſignified by the fraction. So if we divide unity into 12 parts, 5 of thoſe parts will be expreſſed thus,

$\frac{5}{12}$ Numerator, Denominator, and 7 parts thus $\frac{7}{12}$, and ſo others.

14. A decimal fraction (which is the moſt genuine and natural way of dividing unity, and perhaps the moſt ancient) always ſuppoſes the integer to be divided into 10, 100, 1000 parts, *&c.* as you covet preciſeneſs in your operation. Hence the denominator being known, needs not to be expreſſed, but you

may

may place your fraction as an integer, by taking care to prefix its distinguishing point, or comma; so $\frac{5}{10}$ will be expressed thus, .5, and $\frac{5}{100}$ thus, .05: $\frac{75}{100}$ thus, .75, *&c.*

15. Numbers are said again to be equal, unequal, even, odd, evenly even, evenly odd, oddly odd, composite, prime, plain, solid, perfect, harmonic, square, cube, *&c.*

16. Equal numbers are such as contain an equal number of units.

17. Unequal numbers are such whose number of units differ.

18. An even number is such as may be divided into two equal parts.

19. An odd number is such as cannot be divided into two equal parts.

20. A number evenly even, is such as is composed of two even numbers. Such an one is 8; for $2\times4=8$.

21. A number evenly odd, is such as is composed of an even and an odd number: such is 18, composed of 6 and 3, for $6\times3=18$; of 9 and 2, for $9\times2=18$.

22. A number oddly odd, is such as is composed of two odd numbers: such is 21, composed of 7 and 3; for $7\times3=21$.

23. Some numbers are both evenly even, and evenly odd, as 24 composed of 6 and 4, for $6\times4=24$, and so is evenly even; and it may be composed of 8 and 3, for $8\times3=24$, and so is evenly odd.

24. Composite numbers are such as are measured by some other number than unity; such are 8, 12, 15, 25, *&c.*

25. Prime or incomposite numbers are such as unity only measureth; such are 3, 5, 7, 11, 17, 19, *&c.*

26. Plain numbers are such as are made by the multiplication of two numbers, as 12, 18, 36; the first made up of 6 and 2, and the second of 6 and 3, and the third of 9 and 4.

27. Solid numbers are such as are made by the multiplication of three numbers into one another: such

are 24, made by the multiplication of 2 into 3, into 4; and 60 made of 3 into 4, into 5; whence you may infer, that all plain and solid numbers are composite.

28. Perfect numbers are such, whose aliquot parts added, are equal to themselves; the first of which is 6, whose aliquot parts are 3, 2, 1=6; the second is 28, whose aliquot parts are 14, 7, 4, 2, 1=28; of these numbers are but few, only nine in a hundred thousand millions.

29. Harmonic numbers are such, that the aliquot parts of the one collected, make a sum equal to the other number.

30. Square numbers are such as are made by the multiplication of some number into itself; so 4 is the square of 2, 9 of 3, 16 of 4, and so on *ad infinitum.*

31. Cube numbers are made by the multiplication of some number twice into itself: such a one is 8, made by the multiplication of 2 into 2 into 2; such another is 27, and infinite more.

32. Numbers to one another may be termed aliquot parts, aliquant parts, prime, composite.

33. One number is said to be an aliquot part to another, when the first precisely measures the second: so 6 is an aliquot part of 18, and 7 of 28; for 6 measures 18 by 3, and 7 measures 28 by 4, *&c.*

34. One number is said to be an aliquant part of another number, when the first measures not the second without a remainder; so 5 is an aliquant part of 18, and 9 of 25, *&c.*

35. One number is said to be prime to another, when no number can be found to measure both precisely, excepting unity; so 11 and 15 are prime to one another; so are 13 and 36, and many more.

36. One number is composite to another, when a number can be found that measures both exactly besides unity: such are 12 and 36, 15 and 75; since 3 measures the first pair, and five the second, and so in many more.

37. Numbers

37. Numbers to one another may be said to have reason, ratio, or habitude, and may be twofold, either in respect of quantity or quality.

38. In respect of quantity, it is considered only betwixt two numbers, of which the first is called the *antecedent*, the second the *consequent*, and is either equal, as 3 to 3, or 7 to 7; or unequal, which may be of the greater to the less, as 6 to 4, or of the less to the greater, as 4 to 6.

39. Reason, as well of the greater to the less, as of the less to the greater, is fivefold, *viz.* first, *Multiple*; secondly, *Superparticular*; thirdly, *Superpartiens*; fourthly, *Multiplesuperparticular*; fifthly, and lastly, *Multiplesuperpartiens*. The three first of which are called *Simple*; the two last, *Mixed* reason or habitude. To give a name to their opposites or contraries, we join the preposition *sub*; then they are called *Submultiple*, *Subsuperparticular*, &c.

40. First, *Multiple* reason, is when the antecedent, or greater number, contains the consequent or less number, some certain number of times, without a remainder; as 6 to 3, commonly called *Duple*; 21 to 7, commonly called *Triple* reason. Their opposites are of the less to the greater, as 3 to 6, 7 to 21, that is, *subduple*, *subtriple* reason.

41. Secondly, *Superparticular* reason, is when the antecedent, or greater number, contains the consequent, or less number, but once with a fraction, whose numerator is always unity; such are 3 to 2, 4 to 3, 5 to 4, *&c.* commonly called *Sesquialtera*, *Sesquitertia*, *Sesquiquarta*, reason or proportion: its opposite is *Subsuperparticular*, as of 2 to 3, 3 to 4, 4 to 5, *&c.* commonly called *Subsesquialtera*, *Subsesquitertia*, *Subsesquiquarta*, &c.

42. Thirdly, *Superpartient* reason, is when the antecedent, or greater number, contains the consequent, or less number once with a fraction, whose numerator is always more than unity; such as 5 to 3, 7 to 4, *&c.* commonly called *Superdupartienstres*, and

Supertripartiens quartas, &c. Its opposite is *Subsuperpartiens*, as of 3 to 5, 4 to 7; or *Subsuperdupartiens tres*, *Subsupertripartiens quartas*, &c.

43. Fourthly, *Multiplesuperparticular* reason, is when the antecedent, or greater number, contains the consequent, or less number, divers times with a fraction, whose numerator is always unity; such as 9 to 4, or *Duplasesquiquarta*, 9 to 2, or *Quadruplasesquialtera*, 26 to 5, or *Quintaplasesquiquinta*, &c. Its opposite is, *Submultiplesuperparticular*, as 4 to 9, 2 to 9, 5 to 26, *&c.*

44. Fifthly, *Multiplesuperpartiens* reason, is when the antecedent, or greater number, contains the consequent, or less number, divers times with a fraction, whose numerator is always greater than unity; as 8 to 3, commonly called *Duplasuperdupartiens tertia*; 19 to 5, termed *Triplasuperquadripartiens quinta*, &c. Its opposite is, *Submultiplesuperpartiens*, as 3 to 8, 5 to 19, *&c.* Under some of these five species are comprehended all the variety that can happen betwixt two numbers, in respect of quantity; the same holds also in fractions, as well as mixed numbers.

45. In respect of quality, which is only a similitude of reasons commonly called *proportion*, it is considered betwixt more than two numbers: for though the reason of two numbers may be had, as before, yet a similitude of reasons cannot be found, unless the numbers be more than two, and is threefold: First, In respect of their *difference*. Secondly, In respect of their *Quote*. Thirdly, In respect of both. Of the first springeth Arithmetical; of the second, Geometrical; of the third, Harmonical proportion.

46. Arithmetical proportion, is an equality of differences; that is to say, When a rank of numbers have one and the same difference; and this is twofold, continued, or discontinued.

47. First, Continued; when of several the 2d exceeds or is less than the first, by the same number of

units

units as the 3d exceeds, or is leſs than the 2d, or as the 4th exceeds, or is leſs than the 3d, *&c*. So 1, 3, 5, 7, 9, 11, *&c*. are numbers in arithmetical proportion, increaſing by two. And 16, 13, 10, are numbers in arithmetical proportion, decreaſing by 3. And 1, 2, 3, 4, 5, 6, 7, are numbers in arithmetical proportion, continued, increaſing by unity; and theſe are what is commonly called *arithmetical progreſſion*.

48. Secondly, Diſcontinued, that is, when there is the ſame difference betwixt the 1ſt and the 2d, as there is betwixt the 3d and 4th, but not as between the 2d and 3d. So 1, 3 . . 7, 9, are four numbers in arithmetical proportion. The difference of 1 and 3, and of 7 and 9, being 2; which is not the difference of 3 and 7, which is 4.

49. Geometrical proportion is an equality of *ratios*; that is to ſay, when ſeveral numbers, being divided by one another, have ſeveral quotients; and is either continued, or diſcontinued.

50. Continued, when of ſeveral numbers the 1ſt bears the ſame *ratio*, or proportion, to the 2d, as the 2d doth to the 3d, and as the 3d doth to the 4th, *&c*. Thus 2, 3, 4, 6, are geometrical proportionals continued, ſince there is the ſame reaſon of 2 to 3, as of 4 to 6, each being *Subſeſquialtera*; 1, 2, 4, 8, 16, 32, *&c*. are numbers in geometrical proportion, for the ſame reaſon; and this is what is commonly called *geometrical progreſſion*.

51. Second, Diſcontinued or interrupted, when the proportion of the 1ſt to the 2d is the ſame as that of the 3d to the 4th, but not of the 2d to the 3d. Thus 3 : 6 :: 16 : 32, are geometrical proportions diſcontinued; 3 being contained in 6, as often as 16 in 32, that is, twice, which is not the proportion of 6 to 16; and this is what is commonly called *The Golden Rule*.

52. Harmonic, or muſical proportion, is when the 1ſt term is to the laſt, as the difference of the 1ſt and

2d to the difference of the two laſt. So theſe three numbers, 2, 3, 6, are in muſical proportion, ſince 2 is to 6, as 1, the difference of the two firſt, to 3, the difference of the two laſt. Thus alſo theſe 4 numbers are in harmonical proportion, *viz.* 2, 3, 6, 12: ſince the firſt is to the laſt, as the difference of the two firſt, to the difference of the two laſt.

SECT. II.

The diviſion of a pound Sterling.

4 Farthings	Make	1 Penny
12 Pence, or 3 Groats	Make	1 Shilling
5 Shillings	Make	1 Crown
4 Crowns, or 20 Shillings	Make	1 Pound

20 Groats, or 6 *s.* and 8 *d.*	Make	1 Noble
2 Nobles	Make	1 Merk
2 Angels	Make	1 Pound
3 Merks	Make	2 Pounds
1 Merk 1 Noble	Make	1 Pound
240 Pence	Make	1 Pound
960 Farthings	Make	1 Pound

The diviſion of a pound Troy.

24 Grains	Make	1 Penny Wt
20 Penny Weight	Make	1 Ounce
12 Ounces	Make	1 Pound *Tr.*
14 Oun. 12 Penny Wt	Make	1 Pound *Aver.*

240 Penny wt. make a Pound. So will 5760 grains.

Averdupois.

Averdupois Weight.

16 Drams	Make	1 Ounce
16 Ounces		1 Pound
14 Pound		1 Stone
2 Stone, or 28 pound		$\frac{1}{4}$ Hundred
4 Stone, or 56 pound		$\frac{1}{2}$ Hundred
8 Stone, or 112 pound		1 Hundred
5 Hundred		1 Hogshead
10 Hundred		1 Pipe or butt
20 Hundred		1 Tun or load

256 drams make a lb. 1792 oz. make a C. wt. 28672 dr. make a C. wt. 3584 dr. make a stone, so will 224 ounces.

Apothecaries Weight.

20 Grains	Make	1 Scruple
3 Scruples		1 Dram
8 Drams		1 Ounce
12 Ounces		1 Pound

96 drams in a pound, 288 scruples in a pound, 5760 grains in a pound.

N. B. Apothecaries compound their medicines by the weights expressed in the above table; but they buy and sell their drugs by *Averdupois* weight.

Long Measure.

3 Barley-corns	Make	1 Inch
4 Inches		1 Palm
12 Inches, or 3 palms		1 Foot
3 Feet		1 Yard
3 Feet 9 inches		1 Ell *English*
5 Feet		1 Geom. pace
6 Feet, or 2 yards		1 Fathom
5 $\frac{1}{2}$ yards		1 Perch
40 Perch, or 132 paces		1 Furlong
8 Furl. or 320 perches		1 Mile
3 Miles		1 League

190080 Bar. cor. make a mile, 63360 inch. make a mile, 5280 feet make a mile, 1760 yards make a mile, 1056 paces make a mile, 320 perches make a mile.

Cloth

Cloth Measure.

4 Nails	Make	1 Quarter	
4 Quarters		1 Yard	16 nails one yard.
5 Quarters		1 Ell *English*	20 nails 1 ell *English*.
3 Quarters		1 Ell *Flemish*	12 nails 1 ell *Flemish*.

Dry Measure.

2 Pints	Make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon
2 Gallons		1 Peck
4 Pecks		1 Bushel land measure
5 Pecks		1 Bushel water measure
4 Bushels		1 Coomb
2 Coombs		1 Quarter
4 Quarters		1 Chalder
5 Quarters		1 Tun or Wey.

Liquid Measure.

2 Pints	Make	1 Quart
2 Quarts		1 Pottle
2 Pottles		1 Gallon (or herrings
8 Gallons		1 Firkin of ale, soap,
9 Gallons		1 Firkin of beer
2 Firkins		1 Kilderkin
2 Kilderkins		1 Barrel, or 36 gall.
42 Gallons		1 Tierce
63 Gallons		1 Hogshead
2 Hogsheads (lons		1 Pipe or butt
2 Butts, or 252 gal-		1 Tun

TIME.

60 Seconds	Make	1 Minute
60 Minutes		1 Hour
24 Hours		1 Day natural
7 Days		1 Week
4 Weeks (day		1 Month
13 Months and one		1 Year, or 365 days

Sometimes

Sometimes a fraction is expressed decimally; and in this case an unit is supposed to be divided into 10 parts; and every one of those 10 parts, into other 10 parts; whereby unity is divided into 100 parts. Again, every of those parts are supposed to be divided into other 10 parts, and then unity is divided into 1000 parts; and so as far as you please.

In any decimal fraction, the denominator is not expressed, but understood; and the numerator hath a point or comma prefixed, to distinguish it from an integer.

So if a pound be divided into 10 parts, 10 shillings, or $\frac{1}{2}$, will be thus expressed, $\frac{5}{10}$, or thus, .5.

Again, if a pound *Sterling* be divided into 100 parts, 5 shillings, or $\frac{1}{4}$ of a pound, will be expressed thus, $\frac{25}{100}$, or .25.

Thus you see the denominator of a decimal fraction may very well be omitted, because easily known, being always an unit with as many ciphers annexed as there are places in the numerator.

Note also, That ciphers placed to the left hand of an integral number, or to the right hand of a decimal, neither increase nor decrease the value; but placed contrary work contrary effects: for as ciphers placed to the right hand of an integer, increase the value in a tenfold proportion; so ciphers placed to the left hand of a decimal fraction, decrease the value in the same proportion.

So 5 pounds, by annexing a cipher to the right hand, becomes 50 pounds, ten times more than before. So 5 *l.* or 10 *s.* by annexing a cipher to the left hand, become .05 or 1 shilling, ten times less than before. But more of this in decimal arithmetic.

An explanation of certain CHARACTERS *or* SIGNS *made use of in this Work.*

+PLUS, or more, the sign of *Addition*; and, when placed between two numbers, signifies that they are to be added together: as 5+6 denotes, that 5 and 6 are to be added into one sum.

— *Minus*, or less, the sign of *Subtraction*; and, when it stands between two numbers, denotes that the latter is to be subtracted from the former; as 7—3, is 7 less 3, and signifies that 3 is to be subtracted from 7.

× Is the sign of *Multiplication*; and signifies, when placed between two numbers, that they are to be multiplied together: thus 2×3, is 2 multiplied by 3.

÷ Is the mark of *Division*; and signifies, when two numbers are placed one above another, that the number above is to be divided by that below: as $\frac{6}{2}$, or 6÷2, denote that 6 is to be divided by 2.

= The sign of *Equality*; and, when placed between two numerical expressions, denotes that they are equal between themselves: as 7+3=8+2, that is, the sum of 7 and 3 is equal to the sum of 8 and 2.

: Implies to, and : : so is. They are marks of proportionality, signifying that the numbers between which they are placed are proportional; as 2 : 4 : : 8 : 16, signifies, as 2 to 4 so is 8 to 16.

NUMERATION.

NUMERATION.

BY *Numeration* we know how to place, or give a just value to any number propounded; which that you may do, obſerve the nature of the following table.

The TABLE.

Hundreds of Millions	Ten of Millions	Millions	Hundreds of Thouſ.	Ten of Thouſ.	Thouſands	Hundreds	Tens	Units
								1
							2	1
						3	2	1
					4	3	2	1
				5	4	3	[illegible]	1
			6	5	4	3	2	1
		7	6	5	4	3	2	1
	8	7	6	5	4	3	2	1
9	3	7	6	5	4	3	2	1

In the foregoing table you may ſee how each place exceeds the former ten times, increaſing in value towards the left hand.

The firſt place is the place of units, the ſecond tens, the third hundreds, the fourth thouſands; and ſo on as far as you pleaſe.

The denomination of the firſt period, or of the firſt 3 places, is hundreds; the denomination of the ſecond period, or of the ſecond 3 places, is thouſands; of the third, is millions.

The fourth period, if it had been annexed, would have been thouſands of millions; but in practice we have ſeldom uſe for ſo large a number.

In reading the numbers, it is convenient that the young learner ſhould exerciſe himſelf in the ſmalleſt firſt, and ſo proceed to the greater, until he be perfect.

The value of 7654321, being the 7th number in the table, will be found to be in words at length, ſeven millions, ſix hundred fifty-four thouſand, three hundred, twenty-one. Of the fourth, to wit of 4321, the value in words at length will be, four thouſand, three hundred, twenty-one; and ſo of any other.

And though the former table go but to 9 places, yet it is ſufficient to find the value of any number, though it conſiſt of 90 thouſand places.

A ready way in long numbers, is pointing the places of millions, as in the number underneath.

M. of M. of M. of M. — Mill. of Mill. of Mill. — Millions of Millions — Millions

765432345678987654323456789142

ADDITION.

ADdition is the gathering together of ſeveral numbers into one total ſum.

Addition of INTEGERS.

Take care to place units under units, tens under tens, *&c.* And for every 10, carry one to the next place.

	Example.
To work this example, I begin in the units place, and say, 7 and 4 is 11, and 6 is 17; place 7 under the place of units, and for the ten carry 1 to the next place: then going to the place of tens, I say, 1 that I carried, and 6 is 7, and 7 is 14, and 1 is 15; set down 5, and carry 1:	4216 2974 2967 —— 10157 Tot. ——

then 1 I carried, and 9 is 10, and 9 is 19, and 2 is 21; set down 1, and carry 2: then 2 I carried, and 2 is 4, and 2 is 6, and 4 is 10; which being the last, set it all down, and the total sum will be 10157, as in the example may be seen.

Other examples for practice.

		41262
		12346
	46725632	71621
	12982624	32428
	37890167	4216
	34256782	2194
	42167142	2651
	46300001	3986
	29067892	7894
	——	6724
		167
	——	729
		814
		672
32142	72900	500
12162	4678	27
42164	290	42
59786	46	61
21214	7	7
——	——	——
——	——	——

In addition of numbers of divers denominations, as Money, Weight, Meaſure, *&c.*

We ſhall firſt begin with money, that being the moſt common.

Having placed the numbers given to be added, in their order, *viz.* Pounds under pounds, ſhillings under ſhillings, pence under pence, *&c.*

Then,

For every 4 farthings carry one penny, for 12 pence carry one ſhilling, and for 20 ſhillings carry one pound.

Example.

l	*s.*	*d.*	*q.*
21	12	7	1
36	15	8	3
14	12	7	2
18	15	2	2
91	16	2	0

Begin with the farthings, and ſay, 2 and 2 is 4, and 3 is 7, and 1 is 8 farthings; ſet down a cipher, and carry 2 pence to the place of pence; then 2 I carried, and 2 is 4, and 7 is 11, and 8 is 19, and 7 is 26 pence; ſet down 2, and carry 2 ſhillings; then 2 I carried, and 5 is 7, and 2 is 9, and 5 is 14, and 2 is 16; ſet down 6 ſhillings, and carry 1 angel, which with the other 4 angels make 5 angels; ſet down 1 angel, and carry 2 *l.* Then in pounds work as in integers, and the ſum will be 91 *l.* 16 *s.* 2 *d.* 0 *q.*

Other

Other examples for practice.

l.	*s.*	*d.*	*q.*
42	16	9	1
86	18	2	1
34	17	9	2
16	16	8	1
74	17	7	3
18	12	5	1
224	19	6	1

s.	*d.*
17	9 $\frac{1}{2}$
12	7 $\frac{1}{2}$
13	9 $\frac{1}{4}$
7	2 $\frac{3}{4}$
4 11	5 0

l.	*s.*	*d.*	*q.*
365	16	8	1
321	12	5	2
178	18	8	2
421	12	7	1
624	13	8	3
424	12	7	1
100	00	0	0
724	16	0	3
146	17	10	1
741	18	8	2
178	12	8	2
246	16	8	2
146	17	11	3
424	12	5	1
129	18	8	0
5177	17	2	0

If your sum be long, you may point it, or divide it into parts; and the parts added together will be equal to the whole, which proves the work.

Addition of TROY-WEIGHT.

Having placed your numbers in order, that is, each under its own denomination; then, for every 24 grains carry one penny-weight, for 20 penny-weights carry one ounce, for 12 ounces carry one pound.

Example.

lb.	*oz.*	*pw.*	*gr.*
24	7	11	15
36	5	15	13
64	2	14	12
125	4	01	16

Begin with the grains, and say, 12 gr. and 13 is 25, and 15 is 40, which is one penny-weight, and 16 grains; set down 16 grains, and carry one penny-weight to the place of penny-weights. In penny-weights work as in shillings; in the ounces work as in pence; and in the pounds as in integers, and the total will be 125 *lb.* 4 *oz.* 1 *pw.* 16 *gr.*

Other examples.

lb.	oz.	pw.	gr.	lb.	oz.	pw.	gr.
364	7	17	11	4216	7	10	19
142	8	18	10	1216	5	7	05
219	6	10	14	7146	8	11	16
216	7	12	10	2162	5	13	10
943	6	18	21	14742	3	03	02

Addition of AVERDUPOISE-WEIGHT.

Having placed your numbers in their true places; for 16 drams carry one ounce, for 16 ounces carry one pound, for 28 pounds carry one quarter, for 4 quarters carry one hundred weight.

Example.

C.	q.	lb.	oz.
36	2	11	8
14	1	17	5
64	2	13	10
115	2	14	7

Begin with the ounces, and ſay, 10 ounces and 5 is 15, and 8 is 23; set down 7 ounces, and carry 1 pound to the pounds; then, 1 pound I carried and 13 is 14, and 17 is 31, and 11 is 42 pounds, which is 1 quarter and 14 pounds; ſet down 14 and carry 1 quarter. In the quarters work as in the farthings, and in the hundreds as in integers, and the ſum will be found to be, 115 *C.* 2 *q.* 14 *lb.* 7 *oz.*

Other examples.

lb.	oz.	dr.	C.	q.	lb.	oz.
71	11	10	142	2	11	7
36	8	12	678	1	14	10
14	5	10	241	2	19	8
36	5	6	362	3	10	5
14	7	5	176	1	15	6
173	6	11	1601	3	15	4

There

There are other weights and measures: but he that understands these, cannot be ignorant of any other. If he but take notice of the tables of weights and measures, in the introduction, where he may see how much of one denomination make one of another; then the work will be easy enough.

We shall therefore shew the learner the proof of Addition, and so conclude this rule.

Proof of ADDITION.

In a *proof of Addition*, add your numbers downward, contrary to the common way, carry as usually; so will you avoid making a mistake in the same place: if the total sum be the same both ways, you are right, else not.

Example.

In Money.

	l.	s.	d.
	146	7	9
	362	14	2
	174	11	5
Sum	683	13	4
Proof	683	13	4

In *Averdupoise*-weight.

	C.	q.	lb.	oz.
	142	2	11	6
	178	1	19	10
	242	2	18	5
	426	3	11	5
Sum	990	2	04	10
Proof	990	2	04	10

Questions in ADDITION.

A man at *Manchester* demands how far to *London?* and was answered, From hence to *Derby* is 38 miles, thence to *Harborough* 32 miles, thence to *St Albans* 46 miles, and so to *London* 20 miles.

What is the distance from *Manchester* to *London?*

	38
	32
Facit 136, as in the work.	46
	20
	136

An

An old man's age was required; and he answered, I have 5 sons and 3 daughters; betwixt the birth of each of my sons was 2 years; betwixt my last son and first daughter 4 years; and 3 years apiece betwixt the rest of my daughters; in my 20th year was my first son born, and that is the age of my youngest daughter.	20 2 2 2 2 4 4 4 20
What is the father's age?	
Answer, Sixty years.	60

SUBTRACTION.

BY *Subtraction* we find the difference of any two numbers, by taking or drawing the lesser from the greater, whereby the difference will appear.

Subtraction in INTEGERS.

Take care to place units under units, tens under tens; and in case of want, in subtracting, borrow 10; and for every ten so borrowed, pay one in the next place.

EXAMPLE.

Bought 7126 bundles of yarn, of which I have sold 1693 out again. What remains to sell?

Bought	7126
Sold	1693
Rest	5433

Place your numbers as in the margin, and beginning at the right hand, say, 3 from 6 and there remains 3; 9 from 2 I cannot, but 9 from 12 (for borrowing 10 makes the 2 12), rest 3; then go on, saying, 1 I borrowed and 6 is 7, from 1 I cannot; but 7 from 11, rest 4; lastly, 1 I borrowed and 1 is 2 from 7, rest 5: so will the remainder be found 5433, as in the work.

Other

Other examples for practice.

Lent	467256	From	6725426
Paid	414063	Subt.	6109826

Lent at one time	4246462 *l.*
at another	124216
at another	62142
at another	4215
Lent in all	4437035
Paid at one time	1263125
at another	642162
at another	82425
Paid in all	1987712
Rest to pay	2449323

In this last example I add the sums lent into one sum, and likewise what was paid; then subtracting as before, the remainder will be found to be 2449323.

Subtraction in MONEY.

In subtraction of numbers of divers denominations' we shall, as in *Addition*, begin with money in the first place, and of the rest in their order.

Subtraction in Money is not much different from *Integers*: only note, that having placed your numbers right, the less under the greater, and pounds under pounds, shillings under shillings, &c. you must in case of want in the farthings, borrow 4, or one penny; and in the pence borrow 12, or one shilling; and in the shillings, borrow 20 shillings, or one pound, remembering always to pay what you borrowed to the next place, by calling the lower figure one more than it is.

EX-

EXAMPLE.

Begin with the farthings and say, 2 farthings from 1 I cannot, but 2 from 4, rest 2, and 1 is 3, which set down; then go to the pence, saying, 1 borrowed and 9 is 10, from 7 I cannot, but 10 from 12 rest 2, and 7 is nine pence, which set down; then proceed to the shillings, and say, 1 shilling I borrowed and 14 is 15, from 12 I cannot, but from 20, rest 5, and 12 is 17, which set down; and going to the pounds, work as in integers, and the remain will be 48 *l.* 17 *s.* 9 *d.* 3 *q.*

	l.	*s.*	*d.*	*q.*
Lent	67	12	07	01
Paid	18	14	09	02
Rest	48	17	09	03

Other examples for practice.

	l.	*s.*	*d.*	*q.*
Lent	142	16	9	1
Paid	79	13	8	2

	l.	*s.*	*d.*
Lent	416	16	7
Paid	198	14	2

	l.	*s.*	*d.*
Lent at several times,	21	14	9
	36	18	2
	14	17	3
	91	14	6
	71	12	8
	83	16	7
	14	14	2
Paid	333	12	$5\frac{1}{2}$
Rest to pay			

Subtraction of TROY-WEIGHT.

In *Subtraction of Troy-weight*, in case of want, in the grains, borrow 24, in the penny-weights 20, in the ounces 12, and in the pounds as in integers; remembering still to pay what you borrow to the next place.

Begin with the grains, and say, 16 grains from 14 grains I cannot, but 16 from 24, rest 8, and 14 is 22, which set down; then proceeding to the pennyweights, there you may work as in shillings, and in the ounces as in pence, and in the pounds as in integers.

	lb.	*oz.*	*pw.*	*gr.*
Bought	672	11	12	14
Sold	149	08	13	16
Rest.	523	02	18	22

Other examples.

	l.	*oz.*	*pw.*	*gr.*
Bought	674	7	04	10
Sold	194	8	11	06
Rest				

	l.	*oz.*	*pw.*	*gr.*
Bought	4216	5	07	11
Sold	1982	8	10	14
Rest				

Subtraction of AVERDUPOISE-WEIGHT.

Having placed your numbers in order, as was intimated before, subtract as usually; but in case of want in the drams or ounces, borrow 16, in the pounds 28, in the quarters 4, and in the hundreds as in integers.

Begin with the ounces, and say, 8 ounces from 5 ounces I cannot, but 8 from 16 rest 8, and 5 is 13, which set down; then 1 I borrowed and 10 is 11, which subtracted from 11 rest 0, which set down; then proceed to the quarters, where work as in farthings, and in the *C's* work as in integers.

	C.	*q.*	*lb.*	*oz.*
Bought	142	2	11	5
Sold	79	3	10	8
Rest	62	3	0	13

Other

Other examples.

	C.	q.	lb.
Bought	436	2	19
Sold	198	3	25
Rest			

	C.	q.	lb.	oz.
Bought	144	2	14	05
Sold	79	3	19	10
Rest				

Proof of SUBTRACTION.

To prove Subtraction, do thus; add the sum to be subtracted to the remainder, the total will be equal to the number from which you were to subtract, if your work be right.

Example in money.

	l.	s.	d.	
Lent	42	16	09	
Paid	18	16	11	Add
Rest	23	19	10	
Proof	42	16	09	

In Troy Weight.

	lb.	oz.	pw.	gr.	
Bought	142	12	11	14	
Sold	79	08	15	17	Add
Rest	63	03	15	21	
Proof	142	12	11	14	

A bond dated in the year 1685; How many years are spent this present 1765?

From	1765
Subt.	1685
Rest	80 years, the answer.

The author hereof was born in the year of our Lord 1660; How old was he in the year 1753?

From	1753
Subt.	1660
Rest	93 years, the answer.

What number of pounds, shillings and pence, added to 34 *l.* 16 *s.* 9 *d.* 1 *q.* will make 100 *l.*

	l.	*s.*	*d.*	*q.*	
From	100	00	00	00	
Subt.	34	16	09	01	
	65	03	02	03	The answer.

MULTIPLICATION.

BY *Multiplication* we increaſe or multiply one number by another, as oft as there are units in either of the numbers: and it ought to be perfectly underſtood by the learner, who would know any thing of Arithmetic; thouſands of queſtions in a great many parts of the Mathematics being reſolved thereby.

In Multiplication are three numbers or members to be well taken notice of.

Firſt, The multiplicand, or number to be multiplied.

Secondly, The multiplier, or number by which we multiply.

Thirdly, The product or the number proceeding or produced from both.

In Multiplication it holds,
As an unit: To the multiplier : :
So is the multiplicand : To the product.

So if one yard coſt 5 ſhillings, what will 64 yards coſt?

Here one yard bears ſuch proportion to 5 ſhillings, as 64 yards will bear to the product.

To work this queſtion, place your numbers, in order; as in the example following.

Yd.	*s.*	*Yds.*	
If 1 :	5 : :	64	Multiplicand
		5	Multiplier
Product		320	*Facit* 320 ſhillings, or 16 pounds.

Here I multiply 64 by 5, ſaying, 5 times 4 is 20, ſet down a cipher and carry 2; then five times 6 is 30, and 2 I carried is 32; which ſet down to the left hand, the cipher makes the ſum 320 for the product: and ſo many ſhillings will 64 yards coſt, at 5 ſhillings the yard.

But before we proceed any farther, it will be convenient

venient to give you a table of Multiplication, which the learner ought to get perfectly by heart.

A Table of MULTIPLICATION.

1	2	3	4	5	6	7	8	9	12
2	4	6	8	10	12	14	16	18	24
3	6	9	12	15	18	21	24	27	36
4	8	12	16	20	24	28	32	36	48
5	10	15	20	25	30	35	40	45	60
6	12	18	24	30	36	42	48	54	72
7	14	21	28	35	42	49	56	63	84
8	16	24	32	40	48	56	64	72	96
9	18	27	36	45	54	63	72	81	108
12	24	36	48	60	72	84	96	108	144

The foregoing table contains the multiplication of the 9 digits, one by another, or by themselves, to which we have added a column of 12 by the digits, for the usefulness thereof; the reading whereof is easy: for suppose the product of 7 by 9 were required, look for a number at the top, as suppose 9, and the other, to wit 7, in the side, and in the angle or meeting is 63, the product required; so 8 times 6 will be 48, look 8 in the top, and 6 in the side, and in the angle of meeting you will find 48, and so of any other.

In *multiplication* it mattereth not which of the numbers is made the multiplicand, or whether the multiplier; for the product is the same.

Only it is more convenient to make the less the multiplier, and then proceed to the work by the following rule:

First, Set down the greater number, and under it the leſs, units under units, tens under tens: then drawing a line under them, begin with the firſt figure of the multiplier towards the right hand, and by it multiply each figure of the multiplicand, obſerving for every ten to carry one to the next place; then proceed to the ſecond figure of your multiplier, doing as before, only you muſt place your product a figure nearer to the left-hand; and ſo proceed to every figure, doing as before, and removing every product a place nearer to the left-hand; then drawing a line under them, add them as they ſtand, and you will have the true product, which may better be underſtood by obſerving the work of the following examples.

Example the firſt.

By one figure. *Mul.* 1728 *Multiplicand.*
by 7 *Multiplier.*

12096 *Product.*

Having placed your numbers as in the margin, ſay, 7 times 8 is 56, ſet down 6 and carry 5; then 7 times 2 is 14, and 5 is 19, ſet down 9 and carry 1; then 7 times 7 is 49, and 1 I carried is 50, ſet down a cipher, and carry 5; then 7 times 1 is 7, and 5 I carried is 12, which ſet down, and the product is 12096.

This queſtion is the ſame as if one had demanded, In 1728 weeks, how many days?

Or, in 1728 *Lancaſhire* perches, how many yards?

Or, in 7 foot of timber, how many ſolid inches? The anſwer would have been alike in all.

Example the ſecond.

By two figures. *Multiply* 3421 *Multiplicand.*
by 36 *Multiplier.*

20526
10263
123156

Firſt ſay, 6 times 1 is 6, which ſet down; then 6 times 2 is 12, ſet down 2, and carry 1; then 6 times 4 is 24, and one is 25, ſet down 5, carry 2; then 6 times 3

is

is 18 and 2 is 20, which set down; then beginning with the 2d figure of the multiplier, say, 3 times 1 is 3, which set down under the second figure from the right-hand; then 3 times 2 is 6, which set down; then 3 times 4 is 12, set down 2, carry 1; then 3 times 3 is 9 and 1 is 10, which set down, and your multiplication is finished. But now you must add the two products as they stand, as before taught in addition of integers, and the sum is the true product, *viz.* 123156. When you had multiplied by 6, instead of multiplying by 3, you might have taken half the product of 6, setting it one place nearer the left hand, as you may see. This question is the same as if one should ask, in 3421 yards, how many inches?

Example the third.

By 3 figures. *Multiply* 1642 *Multiplicand.*
by 231 *Multiplier.*

```
  1642
 4926
3284
------
379302 Product.
```

First say, once 2 is 2, once 4 is 4, once 6 is 6, once 1 is 1. Secondly, 3 times 2 is 6, 3 times 4 is 12, set down 2, carry 1; and 3 times 6 is 18, and 1 I carried is 19, set down 9, and carry 1; then 3 times 1 is 3, and 1 is 4. Then begin with the last figure, and say, 2 times 2 is 4, and 2 times 4 is 8, and 2 times 6 is 12, go 1. Lastly, 2 times 1 is 2, and 1 is 3. These three products placed and added as in the example, give 379302 for the true product.

This question is the same as if one should ask, In 1642 gallons of wine how many solid inches?

These examples being understood, it will be needless to explain any more; only take two or three for practice.

Other examples for practice.

(I.) *Mult.*	41265	And (II.)	462725
by	1728	*by*	2007
	330120		3239075
	82530		925450
	288855		928689075 *Prod.*
	41265		
	71305920 *Prod.*		

(III.) *Mult.*	46725	And (IV.)	123456
by	2400	*by*	1000
	18690000	*Prod.*	123456000
	93450		
Prod.	112140000		

In the ſecond example I contracted my work by omitting the ciphers, only keeping their places vacant.

In the third example I multiplied by 24, adding two ciphers to the product.

In the fourth example I added three ciphers to the multiplicand, for 1 neither multiplies nor divides; and ſo of any other.

Multiplication may be performed without any charge to the memory, by ſetting down the whole product of the multiplication of every ſingle figure, whereby the carriage of the tens will be ſaved; but the trouble of addition will be the greater, as in the work of the following examples will be manifeſt.

EXAMPLE I.

Let it be required to multiply 7825 *by* 7

First, 7 times 5 is 35, which set down; then 7 times 2 is 14, which set down, 1 before 3 and 4 under it, and 7 times 8 is 56, set 5 before 1, and 6 under it. Lastly, 7 times 7 is 49, set 4 before 5, and 9 under it, as may be seen in the work; which numbers added as they stand, will be the true product; which may be proved as in the common way.

```
        7825
  by       7
  ----------
       45135
        964
  ----------
       54775 Prod.
```

```
Multiply   7825
      by      7
      ---------
Prod.     54775
```

EXAMPLE II.

The work in this is the same as the last, only it is three times repeated; and when the product of any figure will not make 10, place a cipher in the place, where, if it had made 10 or above, the figure above 10 must have stood, which may be seen in the work itself: so we will not trouble ourselves, or the learner, with any more explication.

```
Multiply 4215
      by  879
     --------
       31045
        689
      21035
       847
     31040
      268
     --------
     3704985
```

PROOF.

```
Multiply   4215
      By    879
     ----------
          37935
         29505
        33720
     ----------
        3704985
```

Before

Before we make an end of multiplication, it will be convenient to say something concerning multiplication of numbers of divers denominations And first, when one is of divers denomination, and the other an integer.

EXAMPLE I.

If a pack of yarn cost 13 *l.* 17 *s.* 9 *d.* what will 5 packs cost?

Begin first, with the least denomination, multiplying by the integer, so proceeding from one denomination to another, till you come to the greatest; carrying still from one denomination the parts belonging to the next greater.

See the work.

	l.	*s.*	*d.*
	13	17	9
			5
Ans.	69	08	9

So in the *Example*, I say first, 5 times 9 pence is 45 pence, or 3 shillings and 9 pence, set down 9 pence and carry 3 shillings; then 5 times 7 shillings is 35, and 3 shillings is 38 shillings; set down 8 shillings and carry 3 angels; then 5 times 1 angel is 5 and 3 is 8 angels, set down a cipher, and carry 4 pounds; then going to the pounds, work as in integers, saying, 5 times 3 is 15 and 4 is 19, set down 9 and carry 1; then 5 times 1 is 5 and 1 I carried is 6; which set down as in the work, and the answer will be found to be 69 *l.* 8 *s.* 9 *d.*

EXAMPLE II.

If 1 *C.* of Tobacco cost 3 *l.* 15 *s.* 9 *d.* 1 *q.* what will 35 *C.* cost?

Here, because it will be too tedious to multiply by 35 at once, I multiply by the ratio's of 35; to wit, by 5 and 7, for 5 times 7 is 35.

Answer, 132 *l.* 11 *s.* 11 *d.* 3 *q.*

See

So in the *Example*, I multiply 3 *l*. 15 *s*. 9 *d*. 1 *q*. by 7, the product is 26 *l*. 10 *s*. 4 *d*. 3 *q*. and this product I multiply by 5. The product will be as in the example, 132 *l*. 11 *s*. 11 *d*. 3 *q*. which is the answer to the question.

See the work.

l.	*s.*	*d.*	*q.*
3	15	09	1
			7
26	10	04	3
			5
132	11	11	3

EXAMPLE III.

At 12 *s*. 7 *d*. 1 *q*. the gross, what will 78 gross of incle cost?

Because I cannot find two numbers, which multiplied together make 78: I take two which will make as near 78 as possible, to wit, 9 and 8, which multiplied together make 72, which wants 6 of 78. Then multiplying the first number given by 6, adding the product to the last product before found, gives the answer to the question.

So here I multiply 12 *s*. 7 *d*. 1 *q*. by 9 first; and that product, to wit, 5 *l*. 13 *s*. 5 *d*. 1 *q*. I multiply by 8, which make 45 *l*. 7 *s*. 6 *d*; and to this I add the product of 12 *s*. 7 *d*. 1 *q*. multiplied by 6; to wit, 3 *l*. 15 *s*. 7 *d*. 2 *q*. and the sum is 49 *l*. 3 *s*. 1 *d*. 2 *q*. the answer.

See the work.

l.	*s.*	*d.*	*q.*
	12	07	1
			9
5	13	05	1
			8
45	07	06	0
3	15	07	2
49	03	01	2

EXAMPLE IV.

At 6 *l*. 12 *s*. 5 *d*. the bag, what will 80 bags of cotton cost?

See

First, I multiply by 8, and that product by 9, which makes 72, which wants 8 of 80; and seeing the first product was the number given, multiplied by 8, I add the two products together for the answer to the question, which is 529 *l.* 13 *s.* 4 *d.*

See the work.

6	12	5
		8
52	19	4
		9
476	14	0
529	13	4

This example might have been wrought as under, by multiplying by 9, and that product by 9 again, which makes 81, too much by one; wherefore if from the last product you subtract the first given number, the answer will be found as before.

* Multiply by 8, and that product by 10, gives the answer 529 *l.* 13 *s.* 4 *d.* much shorter and easier; and will for any number from 20 to 120 arising by tens, to multiply by the figures, and then that product by 10.

Take an example, where both numbers are of divers denominations, but of contrary kinds.

EXAMPLE V.

At 3 *l.* 14 *s.* 5 *d.* the *C.* what will 36 *C.* 2 *q.* 14 *lb.* cost?

First, I multiplied by 9, and that product by 4, which makes 36; then for the ½ *C.* I took ½ the first number, which is the price of one hundred; and for the 14 *l.* I took the 4th part of the ½ *C.* which 3 numbers added together, make 136 *l.* 5 *s.* 6 *d.* 0 *q.* ½.

See the work.

l.	*s.*	*d.*	*q.*
3	14	05	
		9	
33	09	09	
		4	
133	19	00	
1	17	02	2
	09	03	2½
136	05	06	0½

When

When both are of unlike denominations, but of the ſame kind, as pounds, ſhillings, and pence, by pounds, ſhillings, and pence, you muſt take good notice of the following directions:

Firſt, Pounds, multiplied by pounds, produce pounds.

Secondly, Pounds multiplied by ſhillings, every 20 is one pound, the reſt ſhillings.

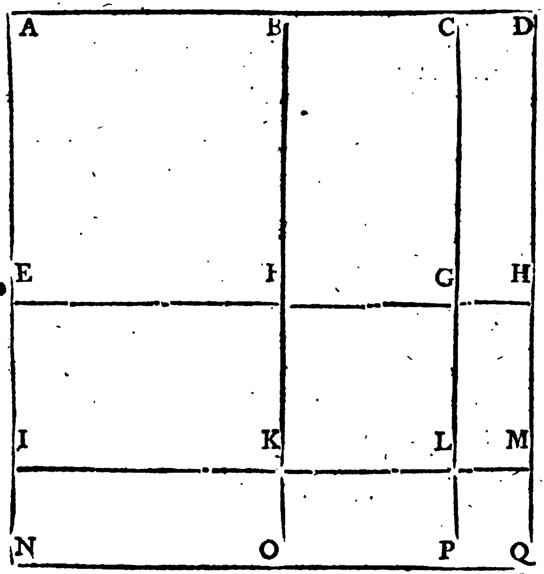
Thirdly, Pounds multiplied by pence, every 12 is one ſhilling, the reſt pence.

Fourthly, Shillings multiplied by ſhillings, every 20 is 1 ſhilling, every 5 is 3 pence, and each one is 2 farthings, and 4 tenth parts of a farthing.

Fifthly, Shillings multiplied by pence, every 5 is a farthing, and each one 2 tenth parts of a farthing.

Sixthly, and *laſtly*, Pence multiplied by pence, every 60 is a farthing, and every 6 one tenth part.

The reaſon whereof is plain in the following *diagram*.



1. Let

1. Let there be two numbers of three denominations given, and let A F be the square or rectangle, made of the greatest denomination in both numbers, E K and B G two rectangles, made by multiplying the first denomination by the 2d; the product divided by an integer of the greatest denomination reduced into the parts of the 2d, the quotient shall be of the same denomination with the greatest, and the remainder of the same denomination with the 2d.

2. F L is the square of the 2d denomination, which being divided by an integer of the greatest, reduced into the parts of the 2d, the quotient shall be of the same denomination with the 2d; and if there be any remainder, it must be multiplied by a number, which, in the 3d denomination, is equal to an integer in the 2d, the quotient shall be of the 3d denomination: and if there be yet a remainder, it must be multiplied by a number which in the 4th denomination is equal to an integer in the 3d, and divided as before, the quotient shall be of the 4th denomination: and so forward till the remainder cannot be reduced into lesser terms. And thus we have done with the square or rectangle A C I L.

3. C H and I O are two rectangles made by the multiplication of the sum of the greatest denomination given, by the sum given, which is of the 3d inferiour denomination: the product shall be of the same denomination with the 3d; and therefore if that product be greater than an integer of the 2d denomination, reduced into the parts of the 3d, it must be divided by a number, which in the 3d denomination is equal to an integer in the 2d; the quotient shall be of the 2d denomination, and the remainder of the 3d.

4. G M and K P are two rectangles made by multiplying the sum of the 2d denomination by the 3d; and the product being divided by one of the integers in the greatest denomination, reduced into the parts of the 2d, the quotient shall be of the same denomination

nation with the 3d, and the remainder must be multiplied by a number which in the 4th denomination is equal to an integer; in the 3d the quotient shall be of that 4th denomination, and the remainder shall be the numerator of a fraction whose denominator is that former divisor.

5. *Lastly*, L Q is the square of the 3d denomination, which must be divided, if it may be by one integer of the greatest denomination, reduced into the parts of the 3d; the quotient shall be of the 4th inferiour denomination, and the remainder shall be the numerator of a fraction, whose denominator is the same divisor.

This *Diagram* being well understood, the multiplication of pounds, shillings and pence, by pounds, shillings and pence, will be easy; as may better be seen in the following examples.

EXAMPLE.

Let it be required to multiply 3 *l.* 5 *s.* 6 *d.* by 2 *l.* 12 *s.* 9 *d.*

See the work.

l.	*s.*	*d.*		*tenth Pts.*
3	05	6		
2	12	9		
6				
2	6			
	3	3		
	3	0	*q.*	
		5	3	4
				9
8	12	9	0	3

First, I say, 2 *l.* by 3 makes 6 *l.* which set down.

Secondly, 2 *l.* by 5 *s.* is 10, and 3 *l.* by 12 *s.* is 36, whose sum is 46; which by direction the 2d will be 2 *l.* 6 *s.* which set down.

Thirdly, 2 *l.* by 6 *d.* is 12, and 3 *l.* by 9 *d.* is 27, whose sum is 39; which by direction the 3d will be 3 *s.* 3 *d.*

Fourthly, 12 *s.* by 5 *d.* is 60, which by direction the 4th will be 3 *s.* which set down.

Fifthly, 12 *s.* by 6 *d.* is 72, and 5 *s.* by 9 *d.* is 45, whose sum is 117, which by direction the 5th will be 5 *d.* 3 *q.* and 4 tenths, which set down.

Sixthly,

Sixthly, and laſtly, 6 *d.* by 9 *d.* is 54, which by direction the 6th will be 9 tenths; and adding all as they ſtand, the ſum will be the true product; *viz.* 8 *l.* 12 *s.* 9 *d.* 0 *q.* 3 tenths, as may be ſeen in the work itſelf.

You may likewiſe obſerve by the way, that when I multiplied by contrary denominations, I multiplied croſs-ways both ways, which in the like caſe the learner is to take notice of.

EXAMPLE VI.

Let it be required to multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* one pound being taken for the integer.

2 Shillings by 2 Shillings makes 2 *d.* 1 *q.* 6 *tenths;* then 2 Shillings by 6 pence makes 12, and 2 ſhillings by 6 makes 12 likewiſe; the ſum is 24, equal to 1 *d.* 0 *q.* 8 *tenths:* laſtly, 6 pence, by 6 pence, makes 36, equal to 6 *tenths;* which 3 numbers added together, produce 3 *d.* 3 *farthings* for the true product, and the anſwer of the queſtion. Thus you ſee fractions multiplied become leſs in the ſame proportion as integers by multiplying become greater.

See the work.

s.	*d.*	*q.*	*tenth Pts.*
2	6		
2	6		
	2	1	6
	1	0	8
			6
	3	3	0

But if it were required to multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* and making a ſhilling the integer, then the former directions will not fit, but the diagram holds for any.

But for this caſe take the directions following.

I. Shillings by ſhillings produce ſhillings.

II. Shillings by pence, every 12 is a ſhilling, the reſt pence.

III. Shillings by farthings, each one is a farthing.

IV. Pence by pence, every 12 is a penny, and each 3 a farthing.

V. Pence by farthings, each 12 is a farthing, and every three is a quarter of a farthing.

VI. Laſtly, farthings by farthings, each 12 is a quarter of a farthing.

See

See the work.

EXAMPLE III.

	s.	d.
	2	6
2 Shillings by 2 s. makes 4 shillings, and 2 s. by 6 d. is 12, and 2 s. by 6 d. is 12, sum is 24, which is 2 shillings; then 6 d. by 6 d. is 36=to 3 d. So the product will be 6 shillings 3 d.	2	6
	4	
	2	0
		3
	6	3

Whereby you may see the value of your product altereth according as you take your integer.

These directions will not only fit for this, but may very well serve for the measuring of board, glass, *&c.* For seeing a foot is divided into 12 inches, and every inch into 4 quarters, the same directions will fit, if instead of shillings, pence and farthings, you account feet, inches, and quarters.

EXAMPLE IV.

A piece of wainscot is 8 foot 6 inches and ½ long, and 2 foot 9 inches ³ broad. The content of this piece of wainscot is required.

See the work.

Ans. 24 feet, and something more, as in the work.

These rules will prove of excellent use for those persons that understand not vulgar nor decimal fractions, in measuring superficial measure.

f.	in.	q.
8	6	2
2	9	3
16		
7		
	7	0
	4	2
		3
		0⅛
24	0	1⅛ *The Ans.*

Contractions in MULTIPLICATION.

The foregoing examples being well conſidered, are ſufficient for the induſtrious learner. We will here annex a contraction or two, and conclude the rule with ſome practical queſtions.

To multiply by an unit with ciphers was ſhewn before, together with another contraction or two; ſo we ſhall forbear thoſe, and name ſome others.

1. And firſt to multiply by 11 : 12 : 13, *&c.* at one operation.

To multiply by 11, is but to ſet down the multiplicand twice, the lower being removed one place either towards the right or left hand.

EXAMPLE.

Multiply 4721 by 11, the product will be 51931.
Place your numbers thus,

4721,	or thus 4721.
4721	4721
Prod. 51931	*Prod.* 51931

To multiply by any of the reſt, is no more but to multiply by 2, 3, 4, 5, *&c.* and as you multiply, to add that figure of the multiplicand which ſtands on the right hand.

EXAMPLE.

Multiply 12345 by 13.

See the work.

12345	*Md.*
13	*Mr.*
160485	*Prod.*

Firſt, I ſay, 3 times 5 is 15, ſet down 5, carry 1, and 3 times 4 is 12, and 1 I carried is 13, and 5 on the right-hand is 18, ſet down 8, and carry 1; then 3 times 3 is 9, and 1 I carried is 10, and 4 on the right-hand is 14, ſet down 4, carry 1; then

3 times

3 times 2 is 6, and 1 I carried is 7, and 3 on the right-hand is 10, set down 0, carry 1; and 3 times 1 is 3, and 1 I carried is 4, and 2 is 6, set down 6; and lastly annex 1, being the first figure in your multiplicand, and your work is finished.

Other examples.

Mul. 6729004 *Md.*	*Mul.* 54321 *Md.*
by 19 *Mr.*	*by* 16 *Mr.*
127851076	869136

2. To multiply by 111, 112, 113, 114, 115, 116, *&c.* at one operation.

To do which you must multiply by 1, 2, 3, 4, 5, *&c.* and as you multiply, add those two figures of your multiplicand which stand to the right-hand.

EXAMPLE.

Multiply 654321 by 115.

See the work.

654321 *Md.*
115 *Mr.*
75246915 *Prod.*

First, I say, 5 times 1 is 5, which set down; and 5 times 2 is 10, and 1 is 11, set down 1, carry 1; then 5 times 3 is 15, and 1 I carried is 16, and 2 on the right-hand is 18, and 1 beyond that is 19; set down 9 and carry 1; then 5 times 4 is 20, and 1 I carried is 21, and 3 on the right-hand is 24, and 2 beyond that is 26, set down 6 and carry 2; then five times 5 is 25, and 2 is 27, and 4 on the right-hand is 31, and 3 is 34, set down 4, carry 3; then 5 times 6 is 30, and 3 is 33, and 5 is 38, and 4 is 42, set down 2, carry 4; which four I add to 6, and that to 5, makes 15, set down 5, carry 1; which I added to the 6, makes 7, which set down, as in the work.

Other examples.

Mult. 4246 by 111; and 642341 by 119.

4246 *Md.*	642341 *Md.*
111 *Mr.*	119 *Mr.*
471306 *Prod.*	76438579 *Prod.*

3. To multiply by 101, 102, 103, 104, 105, 106, &c. is no more than to multiply by 1, 2, 3, 4, 5, &c. and as you multiply, add that figure of your multiplicand that ſtandeth next the right-hand, except one. As you may ſee in the example.

Mul. 4321 *Md.*
by 106 *Mr.*

458026

Say, 6 times 1 is 6, which ſet down; and 6 times 2 is 12, ſet down 2, carry 1; then 6 times 3 is 18, and 1 is 19, and 1, which is the next but one to the right-hand, is 20, ſet down 0, carry 2; then 6 times 4 is 24, and 2 I carried is 26, and 2 which is next but one, is 28, ſet down 8, carry 2; to which add the next but one, which is 3, makes 5, which ſet down; to which add 4, and your work is finiſhed: and the product is 458026.

Other examples.

Mul. 427005 *Md.*	And 604150
by 101 *Mr.*	*by* 109
43127505	65852350

Many more contractions might be added; but theſe being ſufficient, we ſhall deſiſt, and ſpeak ſomething concerning the proof of the rule.

Proof of MULTIPLICATION.

There are ſeveral ways to prove Multiplication; but the only proof is by diviſion; but that being not learned, we ſhall forbear that way for the preſent.

Another

Another way mentioned by several authors is, by casting away the nines, both in the multiplicand, multiplier, and product. But this way being erroneous, we shall mention it no farther.

A third way, and that which we shall use at present, is by making Multiplication to prove itself, thus: Make that which was your multiplicand your multiplier; then multiplying as usual, if your product be the same, your work is right, else not.

EXAMPLE.

Let it be required to multiply 1234 *Md.*
by 123 *Mr.*

```
  3702
 2468
1234
151782 Prod.
```

By the work, I find the product to be 151782

To prove which, multiply 123 *Md.*
by 1234 *Mr.*

```
   492
  369
 246
123
151782 Prod.
```

Here you may see the work is contrary, but the product the same, which is the proof of the work; and thus of any other.

We shall here annex a question or two to exercise Multiplication, and so conclude this rule.

Questions

Questions in MULTIPLICATION.

I. How many feet and tails have 30 thrave of dogs, when 24 dogs make one thrave?

See the work.

Mul.	24
By	30
And	720
By	5
Facit	3600 Feet and tails.

II. How many sparrows at 10 a penny, will buy a yoke of oxen of 10 *l.*

First, bring 10 *l.* into shillings, by 20, and then into pence, by 12; and because 10 sparrows are equal to one penny, multiply that product by 10, and your work is finished; and the answer will be 24000 sparrows.

See the work.

10 *l.*
20

200
12

400
200

2400
10

24000 *Sparrows.*

III. If one yard cost 2 shillings and 3 pence, what will 60 yards cost?

Answ. 6 *l.* 15 *s.*

See

First, Multiply 2 *s.* 3 *d.* by 8, and that product by 7; and because 8 times 7 makes but 56, which is less than 60 by 4, therefore multiply 2 *s.* 3 *d.* by 4; add this to the last product by 7, and it gives the answer.

2 *s.* 3 *d.* multiplied by 4 makes

See the work.

	l.	*s.*	*d.*
Mul.		2	3
By			8
		18	0
			7
	6	6	0 } Add
		9	0 }
Facit	6	15	0

This method of finding the value of any number of yards, ells, pounds, hundreds, *&c.* at any price *per* yard, ell, pound, hundred, *&c.* is of excellent use for all numbers under a hundred, and so will be beneficial for such as buy or sell by retail.

But in great numbers we shall shew you another method in the rule of practice following.

Yet sometimes it may so happen, that your number, though a considerable great number, may be wrought by this method, as may more plainly be seen in the following questions.

IV. If a pack of yarn cost 8 *l.* 16 *s.* 5 *d.* what will 336 packs cost? *Answ.* 2963 *l.* 16 *s.*

First, I multiplied by 8, and that product by 7, for 56, and that product by 6, for 336, for 6 times 56 is 336.

And so of many other.

See the work.

l.	*s.*	*d.*
8	16	5
		8
70	11	4
		7
493	19	4
		6
2963	16	0

V. If

V. If a hogshead of tobacco cost 3 *l.* 7 *s.* 9 *d.* 1 *q.* what will 729 hogsheads cost?

Answ. 2470 *l.* 4 *s.* 11 *d.* 1 *q.* *See the work.*

First, I multiplied by 9, and that product by 9 again, for 81; then because 9 times 81 is equal to 729, the number given, I multiply that product by 9 again, and it gives the answer, as in the work.

l.	*s.*	*d.*	*q.*
3	7	9	1
			9
30	9	11	1
			9
274	9	5	1
			9
2470	4	11	1

VI. How many changes may be rung on 6 bells?

Facit 720 *See the work.*

This question is wrought by that sort of multiplication, which some call *continued;* which is nothing else but what numbers you have given to be multiplied this way, you must multiply the first by the second, and that product by the third, and that product again by the fourth; so continuing till you have multiplied all your given numbers, one into another.

1 . 2 . 3 . 4 . 5 . 6.
2
2 changes on 2 bells.
3
6 changes on 3 bells.
4
24 changes on 4 bells.
5
120 changes on 5 bells.
6
720 changes on 6 bells.

Take another question in continued multiplication.

VII. What number is that, which divided by 1, 2, 3, 4, 5, 6, 7, 8, 9, will leave no remainder?

Answ.

Answ. 362880, found by multiplication of 1, 2, 3, 4, 5, 6, 7, 8, 9, continually one into another, the last product is the answer.

VIII. In 1694 years, how many months, weeks, days, hours, and minutes?

When 13 months of 28 days a-piece make one year, 4 weeks make one month, 7 days one week, 24 hours one natural day, and 60 minutes one hour, *facit* 22022 months, 88088 weeks, 616616 days, 14798784 hours, and 887927040 minutes.

See the work.

The answer is as in the work. If you would find the minutes in so many years more exact, you must note, that in a complete year are 365 days, 5 hours, and 49 minutes, according to the computation of the best astronomers; and that is the reason that every fourth year is called *Leap* year, consisting of 366 days; but the work annexed may serve well enough for the practice of the rule.

```
     1694 Years.
       13
     ----
     5082
    1694
    -----
    22022 Months.
        4
    -----
    88088 Weeks.
        7
   ------
   616616 Days.
       24
  -------
  2466464
 1233232
 --------
 14798784 Hours.
       60
---------
887927040 Min.
```

IX. In 205 miles, the measured distance between *Manchester* and *London*, how many furlongs, perches, yards, feet, inches, and barley-corns?

When

When 8 furlongs make a mile, 40 perches make a furlong, 5 yards and $\frac{1}{2}$ make one perch or rod, 3 feet one yard, 12 inches one foot, and 3 barley-corns one inch. *See the work.*

When I came to multiply by $5\frac{1}{2}$ I multiplied by 5 first, and for the $\frac{1}{2}$ I took the number of perches, placing it orderly under the product I had before; and the sum of those two were the yards sought.

205	*Miles.*
8	
1640	*Furlongs.*
40	
65600	*Perches.*
$5\frac{1}{2}$	
328000	
32800	
360800	*Yards.*
3	
1082400	*Feet.*
12	
2164800	
1082400	
12988800	*Inches.*
3	
38966400	*Barley-corns.*

X. If one yard cost 12 shillings, what will 142 yards cost? *Answ.* 85 *l.* 4 *s.*

Mul. 142
By 12

170|4

l. *s.*
Facit 85 4

Here

Here I multiplied by 12, the contracted way, and from the product cutting off the last figure, half the rest will be pounds, and the remainder shillings.

XI. At 6 *s.* 6 *d.* a yard, what will 142 yards cost?

Ans. 46 *l.* 3 *s.*

$$\begin{array}{r} 142 \\ 6\frac{1}{2} \\ \hline 852 \\ 71 \\ \hline 92|3 \\ \hline l. \quad s. \\ 46 \quad 3 \end{array}$$

Here I multiplied first by 6, for 6 shillings, and for 6 pence took $\frac{1}{2}$ of 142, which I add to the last product; and from the sum cutting off the last figure, half of that sum is pounds, and the figure cut off shillings.

DIVISION.

BY *DIVISION* we discover how often one number is contained in another.

In *Division* are three principal parts to be taken notice of.

First, The *Dividend*, or number to be divided.

Secondly, The *Divisor*, or number by which we divide.

Thirdly, The *Quotient*, or number proceeding from the other two.

Sometimes by accident there is a fourth number, called a *Remainder*.

In *Division* it holds

As the Divisor : To an Unit : :

So the Dividend : To the Quotient.

So if 4 yards coſt 32 ſhillings; what will 1 yard coſt?

Here 4 yards the diviſor bears ſuch proportion to an unit, or 1 yard, as 32 yards the dividend doth bear to the quotient; which will be the anſwer to the queſtion.

To work this queſtion, place your numbers as underneath.

	Yd.		Sh.		Yd.		Sh.
If	4	:	32	: :	1	*Facit*	8
			1				

4) 32 (8 ſhill.
32
——
00

Becauſe 1 doth not multiply, I divide 32 by 4, ſaying, How oft 4 in 32? *Anſ.* 8 times; which I place in my quotient, as you ſee.

Diviſion is either ſingle, or compound.

Single diviſion is when the diviſor is but one figure, and the dividend but two at the moſt, as in the queſtion before-going. And this ſort of diviſion may either be performed by the memory, or at moſt by the table of multiplication, by ſeeking your diviſor on the top of the table, running down the ſame till you find the dividend; and over-againſt it, in the firſt column, is your quotient ſought.

Compound diviſion is when the dividend conſiſteth of many places, and the diviſor of one or more places.

When a queſtion of compound diviſion is propounded to be brought, you muſt proceed according to the work of the following rule.

Firſt write down your dividend, making a crooked line at either end thereof, that on the left-hand to contain the diviſor, and that on the right-hand for the

the quotient; and having placed your divisor in its place, distinguish with a point so many places of your dividend towards your left-hand, as are equal, or next exceeding your divisor; and asking how oft your divisor is contained in the said sum, the answer must be placed in your quotient on the right-hand the dividend: then multiply your divisor by the figure last placed in your quotient, setting your product under your aforesaid distinguished sum; and drawing a line under both, take the lower from the higher, and to the remainder point and bring down your next figure of the dividend, with which proceed as you did with your distinguished number, and so on, till you have pointed and brought down all the figures of your dividend. And, *Note*, That as many points as you have made in your dividend, so many figures will be in the quotient; all which will be made more plain in the work of the following examples.

EXAMPLE I.

By one figure.

Divide 12096
By 7

See the work.

7) 12096 (1728
....
7
—
50
49
—
19
14
—
56
56
—
00

Having placed the numbers as you see in the work, make a point under the 2d figure of your dividend, and ask how oft 7 in 12? *Ans.* once; then placing 1 in your quotient, say once 7 is 7, which placed under 12, and subtracted from it, rest 5, to which point, and bring down the next figure 0: then how oft 7 in 50? *Ans.* 7 times; place 7 in your quotient, saying, 7 times 7 is 49, which subtracted from 50, rest 1, to which point, and bring down the next figure, 9: then how oft 7 in 19? *Ans.* 2 times;

set 2 in your quotient, saying, 2 times 7 is 14, which subtract from 19, rest 5; to which point and bring down the last figure of your dividend 6, asking how oft 7 in 56? *Ans.* 8 times; place 8 in your quotient, saying, 8 times 7 is 56, which subtract from 56, rest 0, and your work is finished, and your quotient is 1728, as in the work.

This question is the same, as if one should ask, in 12096 days, how many weeks?

EXAMPLE II.

By two figures.

Let it be required to divide 123156
by 36

See the following work.

Having placed your numbers as in the work, make a point under 3, the third figure of your dividend, because you cannot have 36 in 12, the two first figures thereof.

Then ask how oft 36 in 123? or which is better, how oft 3 in 12? Suppose 4 times; but I cannot have 4 times 6 in 3, for you must take the first figure of your divisor no oftener in the first figure, or first and 2d figure of your dividend, than you can have the 2d figure of your divisor in the remainder of the first, or first and second joined to the second or third. Suppose therefore 3 times; but can I have 3 times 6 in 23? that I can; wherefore place 3 in your quotient, and multiplying my divisor thereby, the product (*viz.*) 108, I place under 123, and to the remainder 15, point and bring down the next figure 1, of your dividend: then how oft 36 in 151, or how oft 3 in 15? *Ans.* 4 times; place 4 in your quotient, and multiplying the divisor thereby, your product, to wit, 144 subtracted from

from 151, leaves 7, to which point and bring down the next figure 5: then how oft 36 in 75, or 3 in 7? *Ans.* 2 times; place 2 in your quotient, multiplying your divisor thereby, your product (*viz.* 72) brought from 75, to the remainder 3, point and bring down your next and last figure of your dividend, to wit, 6, asking how oft 36 in 36, or 3 in 3? *Ans.* 1 time; place 1 in your quotient, saying, 1 time 36 is 36, which subtracted from 36, rest 0, and your work is finished; and the quotient is found to be 3421.

See the work.

```
36)123156(3421
    ....
   108
   ---
    151
    144
    ---
      75
      72
      --
       36
       36
       --
       00
```

This question is the same, as if one should ask, in 123156 inches, how many yards?

EXAMPLE III.

By three figures.

Let it be required to divide 379302
by 231

See the work.

```
231) 379302 (1642
      ....
     231
     ---
     1483
     1386
     ----
       970
       924
       ---
        462
        462
        ---
         00
```

The manner of working being the same as in the last, we shall forbear the explication thereof; for the operation by two figures being well understood, the work in any other will be easy. In this question, the quotient you see will be 1642; and is the same as if it were asked, in 379302 solid inches, how many wine gallons?

EXAMPLE IV.

By four figures.

Divide 746321
By 6142

See the work.

$$
\begin{array}{r}
6142)\ 746321\ (121\frac{3139}{6142} \\
6142 \\
\hline
13212 \\
12284 \\
\hline
9281 \\
6142 \\
\hline
3139
\end{array}
$$

In this example, after your division is finished, you see there is a remainder of 3139, which is the numerator of a fraction, and the divisor is a denominator thereunto, and the entire quotient is $121\frac{3139}{6142}$, as in the work.

The value of this fraction, or any other in the parts of the integer, may be found as in the work of the following example.

EXAMPLE V.

Let it be required to divide a prize worth 368424 pounds, among 1728 men; and let each man's part in pounds, shillings and pence, be demanded.

After the division is finished, there is a remainder of 360 pounds, which multiplied by 20, brings them into shillings, to wit, 7200; which divided by the former divisor, quotes 4 shillings, and leaves a remainder of 288 shillings; which multiplied by 12, brings them into 3456 pence; and divided by the former divisor, quotes 2 pence, and nought remains; but if any thing had remained, it must have been multiplied by 4 for farthings; and the like of any other. So the quotient, or each man's part will be 213 *l.* 4 *s.* 2 *d.*

```
See the work.               l.  s. d.
            1728) 368424 (213 . 4 . 2
                  3456
                  ----
                   2282
                   1728
                   ----
                    5544
                    5184
                    ----
                     360 remainder.
   Shil. in a pound   20
                    ----
                    7200 (4 shil.
                    6912
                    ----
                     288 rem.
      Pence in shil.  12
                    ----
                     576
                    288
                    ----
                    3456 (2 pence.
                    3456
                    ----
                       0
```

Division may be performed without charge to the memory, by making a tablet of your divisor multiplied into the 9 digits, and may prove of good use to the learner; not only in great numbers, but by practising a few. This way he will attain to a good knowledge of division, and be enabled to work easily without such a table.

EXAMPLE I.

Let the dividend be 67254, and the divisor 19. Make a table by duplication, or addition, or by multiplication.

Over-

Over-against 1 place your divisor; for 2 double your divisor, or first number; for 3, add the 1st and 2d number; the 4th is the 2d doubled; the 5th is the sum of the 2d and 3d; the 6th is the double of the 3d; the 7th is the sum of the 3d and 4th; the 8th is the 4th doubled; the 9th is the sum of the 4th and 5th; otherwise the divisor multiplied by any of the 9 digits gives the number in the table over-against such digit. And after this way may a table be made for any divisor whatever.

1	19
2	38
3	57
4	76
5	95
6	114
7	133
8	152
9	171

To work the former question, place your numbers as usual; then seek how far your divisor will reach into your dividend which will be to two places, which is 67; how oft 19 in 67? and by the table seeking for that number, or next less to it, I find it 3 times, (*viz.*) 57, which I set under 67, and subtract, and it leaves 10, to which point, and bring down the next figure 2; then how oft 19 in 102? and by the table I find 5 times, (*viz.*) 95 which subtracted from 102, leaves 7, to which point, and bring down 5, the next figure of your dividend, asking how oft 19 in 75? and by the table I find 3 times, (*viz.*) 57, which subtract from 75, leaves 18, to which point, and bring down the next and last figure of your dividend 4, asking how oft 19 in 184, and by the table I find 9 times, (*viz.*) 171, which subtracted from 184, leaves 13 for a remainder, and your work is finished, and the quotient will be $3539\frac{13}{19}$.

See the work.

```
19) 67254 (3539 13/19
      ...
    57
    ---
    102
     95
    ---
      75
      57
     ---
     184
     171
     ---
      13 Rem.
```

EXAMPLE

EXAMPLE II.

Let it be required to divide 14672865 by 6425.

See the work.

1	6425
2	12850
3	19275
4	25700
5	32125
6	38550
7	44975
8	51400
9	57825

```
6425) 14672865 (2283
      ....
      12850
      -----
       18228
       12850
       -----
        53786
        51400
        -----
         23865
         19275
         -----
          4590
```

The making of the table and the working of the example is the same as before; so we shall not trouble ourselves to explain it, but leave it to the consideration of the learner.

Divisions of divers Denominations.

As in *Multiplication*, so here likewise in *Division*, we shall say something of division of numbers of divers denominations.

And, first, when the dividend is a number of divers denominations, and the divisor an integer; and for example take the following question.

I. If a man in 12 months spend 64*l.* 18*s.* 9*d.* 3*q.* what will his expenses be for one month?

Place

Place your numbers as in the work you see done, and ask how oft 12 in 64? *Ans.* 5 times, place 5 in the quotient for pounds; then 5 times 12 is 60, from 64, rest 4, to which point, and bring down 18 shillings, and ask how oft 12 in 4 *l.* 18 *s.* or 98 shillings, *facit* 8 times, which place in your quotient for shillings; then 8 times 12 is 96, or 4 *l.* 16 *s.* which subtract from 4 *l.* 18 *s.* rest 2 shillings, to which point, and bring down 9 *d.* then how oft 12 in 2 *s.* 9 *d.* or 33 pence? *facit* 2 times, place two in your quotient for 2 pence, saying, 2 times 12 is 24 pence, or 2 shillings from 2 shillings 9 pence, rest 9 pence; to which point, and bring down 3 farthings; then how oft 12 in 9 *d.* 3 *q.* or 39 *q.*? *facit* 3 times; place 3 *q.* in your quotient, saying, 3 times 12 is 36, or 9 *d.* from 9 *d.* 3 *q.* rest 3 farthings which is $\frac{3}{12}$ or $\frac{1}{4}$ of a farthing.

See the work.

```
     l.  s.  d. q.   l. s. d. q.
12) 64 . 18 . 9 . 3 (5 . 8 . 2 . 3
    60
    ----
     4  18
     4  16
     ------
        2  9
        2  0
        ------
           9  3
           9  0
           ------
```

$\frac{3}{12}$ or $\frac{1}{4}$ of a *q.*

So the true quotient, or the monthly expense, is 5 *l.* 8 *s.* 2 *d.* 3 *q.* $\frac{1}{4}$.

Take another example.

II. Divide 4 *l.* 15 *s.* 6 *d.* among 6 men.

See the work.

```
   l.  s.  d.   l.  s.  d.
6) 4  15   6  (0  15  11
   4  10   0
   ---------
       5   6
       5   6
   ---------
           0
```

And so of any other.

An example or two where both the dividend and divisor are numbers of divers denominations.

I. Divide 31 *l.* 17 *s.* 6 *d.* 3 *q.* by 12 *l.* 12 *s.* 6 *d.*

See the work.

```
  l.   s.  d.    l.   s.  d. q.   l.  s.  d.
                                 .  .
 12 . 12 . 6)  31 . 17 . 6 . 3 (2 . 10 . 6
               25 . 05 . 0 .
               ------------
                6   12   6   3
                6    6   3   0
               ------------
                     6   3   3
                     6   3   3
                    -------
                             0
```

Your numbers being placed as in the work, ask how oft 12 pound 12 *s.* 6 *d.* in 31 *l.* 17 *s.* 6 *d.* or how oft 12 in 31? *Answ.* 2 times; place 2 *l.* in your quotient, and by it multiply your divisor by the rules in multiplication, and the product 25 *l.* 05 *s.* 0 *d.* subtract from 31 *l.* 17 *s.* 6 *d.* rest 6 *l.* 12 *s.* 6 *d.* to which point and bring down 3 *q.* saying, how oft 12 in 6 *l.* 12 *s.* or 132 shillings? *Ans.* 10 times; place 10 shillings in your quotient, and by it multiply your divisor, by the rules laid down in multiplication of divers denominations, and the product will be 6 *l.* 6 *s.* 3 *d.* which subtract from 6 *l.* 12 *s.* 6 *d.* 3 *q.* leaves 6 *s.* 3 *d.* 3 *q.* Lastly, how oft 12 in 6 *s.* 3 *d.* or 75 pence? *Answ.* 6 times; place 6 *d.* in your quotient, and by it multiply your divisor, which by the rules before mentioned will be 6 *s.* 3 *d.* 3 *q.* which subtract from 6 *s.* 3 *d.* 3 *q.* resteth 0, and the quotient I find to be 2 *l.* 10 *s.* 6 *d.*

Take another example.

II. Divide 11 *l.* 6 *s.* 8 *d.* by 3 *l.* 5 *s.*

See the work.

```
 l.  s.   l.  s.  d.   l.  s.  d.  q.
3 . 5)  11 . 6 . 8  (3 . 9 . 8 . 3.
         9  15
        ---------
         1  11  8
         1   9  3
        ---------
             2  5
             2  2
           ------
                3
```

Contractions in DIVISION.

For the learner's advantage, we will here annex two or three contractions in division.

I. And, first, to divide by any of the 9 digits, without setting down any figures but the quotient itself.

Example.

Divide 7642, by 2

$\frac{1}{2}$ 3821 Quot.

To divide by 2, is but to halve the number, setting down the figures of the quotient orderly under the dividend; so in the example, $\frac{1}{2}$ of 7 is 3, set down 3, carry the 1 that remains; then $\frac{1}{2}$ of 16 is 8, set down 8; and $\frac{1}{2}$ of 4 is 2, and $\frac{1}{2}$ of 2 is 1; so the quotient is 3821.

Divide 7426, by 3

$\frac{1}{3}$ is 2475$\frac{1}{3}$

To divide by 3 is to take $\frac{1}{3}$ of the number given; so $\frac{1}{3}$ of 7 is 2, of 14 is 4, of 22 is 7, of 16 is 5, and 1 remains, which is $\frac{1}{3}$, and the quotient is 2475$\frac{1}{3}$.

So

So to divide by 5, is to take $\frac{1}{5}$ of the number given; so here $\frac{1}{5}$ of 6 is 1, of 17 is 3, of 24 is 4, of 45 is 9, and the quotient is 1349, and so of any other.

Divide 6745, by 5

$\frac{1}{5}$ is 1349 Quot.

After this method may any number be divided, if the divisor be contained in your multiplication-table: so $\frac{1}{12}$ of 67246 is 5603 $\frac{10}{12}$ for $\frac{1}{12}$ of 67 is 5, of 72 is 6, of 4 is 0, of 46 is 3, rest $\frac{10}{12}$.

II. When your divisor is an unit, with any number of ciphers annexed to the right-hand, cut from your dividend the same number of places, the remainder is the quotient, and those out of a decimal fraction: so if 46575 pounds Sterling were to be divided amongst 100 men, every man's part would be 465 *l.* 15 *s.*; if amongst 1000 men, every man's part would be 46 *l.* 11 *s.* 6 *d.* and so of any other.

III. When your dividend and divisor consist of ciphers to the right-hand, cut off an equal number of ciphers in both, with the rest proceed as by the rules before given. So if 7962000 must be divided 6000, cut off three ciphers on both, $\frac{1}{6}$ part of the remainder (*viz.*) 1327 is the quotient sought.

IV. If your divisor have ciphers annexed, and your dividend none, cut off as many figures in your dividend as there are ciphers in your divisor, with the remainder proceed as before; so if 498565 must be divided by 12000, the quotient would $39\frac{565}{12000}$.

498|565

$\frac{1}{12}$ 39 $\frac{565}{12000}$

V. If your dividend and divisor may be equally divided by any number, without leaving any remainder, your dividend and divisor may be brought into less numbers. And if your divisor dwindle to an unit, your last dividend will be the quotient sought. So if 672 were to be divided by 48, your quotient will be 14; for seeing they are even numbers, I halve them both as long as I can. So $\frac{1}{2}$ of 48 is 24, of 672 is 336: again, $\frac{1}{2}$ of 24 is 12, of 336 is 168: again, $\frac{1}{2}$ of 12 is

is 6, of 168 is 84; again, $\frac{1}{2}$ of 6 is 3, of 84 is 42. Lastly, $\frac{1}{3}$ of 3 is 1, of 42 is 14, the quotient sought, because my divisor is become an unit.

See the work.

The higher num. are all divis.	48	24	12	6	3	1
The lower num. are dividends	672	336	168	84	42	14

Proof of DIVISION.

Multiplication and Division mutually prove each other: for in Multiplication, if you divide your product by your multiplier, the quotient will be your multiplicand: likewise in Division, if you multiply your quotient by your divisor, the product will be your dividend.

Examples in both.

```
Multiply 41725
By         632
       -------
         83450
        125175
       250350
       -------
632) 26370200 (41725 Proof of the multiplication.
     ....       632
     2528     ------
     ----      83450
      1090    125175
       632   250350
      ----   --------
      4582   26370200 Proof of the division.
      4424
      ----
      1580       And so of any other.
      1264
      ----
       3160
       3160
       ----
          0
```

QUEST.

QUEST. I.

I. Divide 46462 yards among 246 men.

See the work.

```
246) 46462 (188 yards.
     246
     ----
     2186
     1968
     -----
      2182
      1968
      ----
       214
         4
       ---
       856 (3 quarters.
       738
       ---
       118
         4
       ---
       472 (1 nail.
       246
       ---
       226 remain.
```

Every man must have 188 yards, 3 quarters, and a nail, together with $\frac{226}{246}$ parts of a nail.

QUEST. II.

II. If a *C.* of tobacco, or 112 *lb.* cost 2 *l.* 11 *s.* 4 *d.* what will one pound weight cost?
Answ. 5 *d.* $\frac{1}{2}$.

See the following work.

First, I brought 2 *l.* 11 *s.* 4 *d.* into pence, and divided by 112, gives 5 *d.* in the quotient; the remainder multiplied by 4, for farthings, and divided by 112, gives 2 farthings.

```
         l.  s.  d.
         2   11  4
         20
         ---
         51
         12
       -----
  112)  616 (5 pence
        560
        ---
         56
          4
        ---
        224 (farth.
        224
        ---
          0
```

QUEST. III.

III. A square acre contains 160 perches: now the side of a field is 12 perches, how much on the other side will go to make an acre?

Answ. 13 perches and $\frac{1}{3}$.

See the work. $\frac{160}{12} \; 13\frac{1}{3}$

QUEST. IV

IV. A captain and 160 soldiers gain a prize worth 362 pounds, of which the captain had $\frac{1}{5}$ for his share, the rest was divided equally among the soldiers; what was each man's part?

Answ. $\left\{\begin{array}{l}\text{The captain's share}\\ \text{Each soldier's share}\end{array}\right\}$ $\begin{array}{l}72\ l.\ 8\ s.\\ 1\ l.\ 16\ s.\ \frac{1}{5}\end{array}$

See

See the following work.

$$
\begin{array}{rl}
 & l. \\
\text{The prize } & \underline{362} \\
 & \tfrac{1}{5}\ 72 \quad \overset{s.}{8} \text{ The captain's share.} \\
 & 289\ \ 12 \\
 & \underline{\quad 20} \\
160) & 5792\ (36 \text{ shillings and } \tfrac{32}{160} \text{ or } \tfrac{1}{5} \text{ each soldier's share.} \\
 & \underline{480} \\
 & 992 \\
 & \underline{960} \\
 & 32
\end{array}
$$

And thus much shall suffice for Division.

REDUCTION.

BY *REDUCTION* we change money, weight, measure, &c. out of one denomination into another, whereby we know how many of one denomination are equal to so many of the other.

We shall (though contrary to other authors) divide Reduction into three parts.

First, *Reduction* by *Multiplication*.
Secondly, *Reduction* by *Division*.
Thirdly, *Reduction* by *Multiplication* and *Division*.

Of these in their order.

Reduction by *Multiplication*, is when we bring a greater denomination into a less, as pounds into shillings, pence into farthings, yards into quarters or nails, *&c*.

See the work of the following questions.

Reduction by MULTIPLICATION.

QUEST. I.

Reduce 36 *l.* 7 *s.* 9 *d.* 1 *q.* into farthings.

First, I multiplied by 20, and as I multiplied took in the 7 shillings, and brought the pounds into shillings; those shillings, to wit, 727, I multiplied by 12, bringing in the 9 *d. facit*, 8733 pence; which multiplied by 4, and adding 1 to the product, brings them all into farthings, and your work is finished.

l.	*s.*	*d.*	*q.*
36.	7.	9.	1.
20			
727 shillings.			
12			
1463			
727			
8733 pence.			
4			
34933 farthings.			

QUEST. II.

In 672 yards, how many quarters and nails?

See the work.

672 yards.
4
2688 quarters.
4
10752 nails.

QUEST.

QUEST. III.

Reduce 21 *C.* 2 *q.* 11 *lb.* into ounces.
Answ. 38704 ounces.

See the work.

```
C.  q.  lb.
21  2   11
    4
    ---
    86
    28
    ---
    689
    173
    ---
    2419 pounds.
      16
    -----
    14514
    2419
    -----
    38704 ounces.
```

QUEST. IV.

Reduce 72467 nobles into groats.
Answ. 1449340 groats.

See the work.

```
72467 nobles.
   20
-------
1449340 groats.
```

QUEST. V.

In 6 bags of pepper, weighing each 3 *C.* 2 *q.* 11 *lb.* how many pounds?

Answ. 2418 *lb.*

See the following work.

C.	q.	lb.		C.	q.	lb.
				Or thus.		
3	2	11		3	2	11
		6		4		
21	2	10		14		
4				28		
86 quarters.				113		
28				29		
688				403		
173				6		
2418 pounds.				2418 pounds.		

QUEST. VI.

In 142 hogsheads, how many gallons and pints?

See the work.

	142 hogsheads.
	63
[63 gallons make a hogshead,	426
and 8 pints a gallon.]	852
	8946 gallons.
	8
	71568 pints.

REDUC-

REDUCTION by Division.

Reduction by Division, is when we bring a less denomination into a greater, as farthings into pence, shillings into pounds, and nails into quarters or yards, *&c.*

In this part of the rule we will use the converse of those questions we had in the former part of the rule, which will be a proof of the work.

QUEST. I.

In 34933 farthings, how many pounds?

Instead of dividing the farthings by 4, I took $\frac{1}{4}$ part, and so $\frac{1}{12}$ of that, to bring them into shillings; and from the shillings I cut off the last figure, and took half the rest, instead of dividing by 20; and the *answer* is 36 *l.* 7 *s.* 9 *d.* 1 *f.*

34933 farthings.
$\frac{1}{4}$ 8733 . 1 farthing.
$\frac{1}{12}$ 627 . 9 pence.
$\frac{1}{20}$ 36 . 7 shillings.

QUEST. II.

In 10752 nails, how many yards and quarters?

See the work.

$\frac{1}{4}$ 10752 nails.

Ans. 672 yards $\frac{1}{4}$ 2688 quarters.

672 yards.

QUEST. III.

In 38704 ounces, how many hundreds, quarters, and pounds?

See

See the work.

16) 38704 (2419 pounds.
32
——
67
64
——
30
16
——
144
144
——
0

28) 2419 (86
224 ¼ 86
——
179 21 *C.* 2 *q.*
168
——
11 *lb.*

Anſ. 21 *C.* 2 *q.* 11 *lb.*

QUEST. IV.

Reduce 1449340 groats into nobles.
Facit 72467 nobles.

$\frac{1}{20}$ 1449340 groats.
——
72467 nobles

QUEST. V.

Six equal bags of pepper weigh 2418 pounds, the weight of one bag is demanded.

$\frac{1}{6}$ 2418
——
403
Anſ. 3 *C.* 2 *q.* 11 *lb.*

28) 403 (14 quarters.
28
——
123
112
——
11 pounds.

¼ 14
——
3 *C.* 2 *q.* 11 *lb.*

QUEST.

QUEST. VI.

In 71568 pints, how many hogsheads?
Anſ. 142.

$\frac{1}{8}$ 71568 pints.

8946 gallons.

```
63) 8946 (142
    63
    ---
    264
    252
    ---
     126
     126
     ---
       0
```

Reduction by MULTIPLICATION *and* DIVISION.

Under this head may be compriſed the method of exchanging coins, weights and meaſures that have not that immediate reference to one another, as thoſe ſpoken of before; and is of excellent uſe to moſt perſons, as well merchants as meaner chapmen.

We ſhall make all plain in the work of the following examples.

QUEST. I.

In 672 nine-pences, how many thirteen pence halfpennies?

In this or the like queſtion, I conſider how many of one coin are equal to any number of the other; and I find that 3 nine-pences are equal to two thirteen-pence half-pennies. And which, to make my multiplicator,

plicator, I consider whether the answer, or the number sought, will be greater or less than the number given; and accordingly I make the greater, or less, my multiplicator.

So here I consider, that in 672 nine-pences there are less thirteen pence half-pennies; so the less number, to wit, 2, is my multiplicator, and 3 my divisor.

```
   672                    Answer. 448 thirteen pence
     2                                  [half-pennies.
  ----
3) 1344 (448
   ..
   12      Or thus,        672
  ----     Its double     1344
   14                     ----
   12      1/3 is - - -    448 the answer.
  ----
   24
   24
  ----
    0
```

The method how these divisors and multiplicators are found in this and the following questions, is thus; bring both your coins, weights, measures, &c. into their least equal known parts; which may be brought into lesser, by dividing them both by some known divisor; or by taking $\frac{1}{2}$ or $\frac{1}{3}$, &c. as oft as you can.

So in 9 *d.* are - - - - - 36 farthings.
And in 13 $\frac{1}{2}$ *d.* are - - - 54 farthings.

Say $\frac{1}{2}$ of 36 is 18, of 54 is 27; $\frac{1}{3}$ of 18 is 6, of 27 is 9; $\frac{1}{3}$ of 6 is 2, of 9 is 3; so are 2 and 3 the numbers sought.

$$\text{Then}\ \frac{36}{54}\Big|\frac{18}{27}\Big|\frac{6}{9}\Big|\frac{2}{3}$$

The

The other numbers are multiplicators and divisors, as well as these two; but these being the least, are most fit for use.

QUEST. II.

In a 100 thirteen-pence half-pennies, how many nine-pences?

This is but the converse of the last, and by the work you may see the answer is 150.

$$\begin{array}{r} 100 \\ 3 \\ \hline \frac{1}{2}\ 300 \\ \hline \end{array}$$

$\frac{1}{2}$ 150 nine-pences.

QUEST. III.

In 672 *English* ells, how many yards *English*? *Ans.* 840.

$$\begin{array}{r} 672 \\ 5 \\ \hline \frac{1}{4}\ 3360 \\ \hline 840 \end{array}$$

840 the answer.

QUEST. IV.

In 672 ells *English*, how many ells *Flemish*, when a yard $\frac{1}{4}$ is an ell *English*, and $\frac{3}{4}$ of a yard an ell *Flemish*?

Ans. 1120, or as 3 to 5.

QUEST. V.

In 642 nobles, how many crowns? *Ans.* 856, or as 4 to 3.

```
        642
          4
      -----
  3) 2568 (856
     24
     ---
      16
      15
      ---
       18
       18
       ---
        0
```

QUEST. VI.

In 856 crowns, how many nobles? *Ans.* 642, the converse of the last.

QUEST. VII.

In 672 little hundred, how many great hundred? *Ans.* 600.

A little hundred is 100; a great hundred is 112.

```
112) 67200 (600
     672
     ---
       0
```

QUEST. VIII.

In 600 great hundred, how many little hundred? *Ans.* 672, the converse of the last.

QUEST.

QUEST. IX.

In 672 guineas, at 1 *l.* 1 *s.* 8 *d.* *per* piece, how many pounds *Sterling?* *Ans.* 728.

See the work.

```
      672            Or thus, 672
       13            Add 1/12  56
      ----                     ---
     2016            The sum  728
     672
     ----
  12) 8736 (728
     84
     --
      33  Pence in a pound     | 24 | 0 | 12
      24                       | -- | - | --
      --  Pence in a guinea    | 26 | 0 | 13
       96
       96
       --
        0
```

In this second operation, because 1 *s.* 8 *d.* is the $\frac{1}{12}$ of a pound, I take $\frac{1}{12}$ of 672, which added to 672, gives the *answer* 728, as before.

QUEST. X.

In 728 pounds, how many guineas, at 1 *l.* 1 *s.* 8 *d.* each? *Ans.* 672.

The converse of the last.

QUEST. XI.

A merchant at *Amsterdam* is indebted to a merchant at *London* in 642 pounds, and would pay it in *Spanish guilders* at 2 shillings *per* piece: How many must the *English* merchant receive? *Ans.* 6420.

 Two

Two ſhillings being the tenth part of a pound, I only add a cipher to the pounds, and it gives the *anſwer*, to wit, 6420.

QUEST. XII.

In 672 *Spaniſh guilders*, how many *French piſtoles*, at 17 ſhillings 6 pence *per* piece?

Anſ. $76\frac{28}{35}$, or 76 *piſtoles* and 14 ſhillings *Sterling*.

See the work.

```
          672     24 | 8 | 4
            4     -- | - | --
         ----    210 | 0 | 35
     35) 2688 (76
         245
         ---
          238
          210
          ---
           28
```

The Golden Rule; or, Rule of THREE.

IT is called *The Golden Rule*, for the excellency thereof: ſometimes it is called *The Rule of Three*, or *Rule of Proportion*, becauſe there are always three numbers given to find a fourth, which muſt bear ſuch proportion to the third, as the ſecond bears to the firſt.

The chiefeſt difficulty lies in ſtating your queſtion; which that you may do, obſerve the following rule.

The Queſtion Firſt and Third are of the ſame,
The middle number has another name,
And which to make the Fourth you cannot miſs,
The unknown quantity it always is.

EXAMPLE.

EXAMPLE.

If 16 yards cost 5 pounds, what will 144 yards cost?

Yd. *l.* *Yd.*

Thus stated, If 16 : 5 : : 144 :

Here you see the first and the third number is yards, the middle number is pounds; and because I wanted the price of 144 yards, I put it in the last or third place.

This being understood, we will give you such another rule, for the working of any such question, which is this that follows:

If the Fourth the Second must exceed, then see
By the Great Extream it multiplied be;
But if it must be less than Second, aim
To multiply it by the less extream.

EXAMPLE.

If 13 packs cost 326 pounds, what will 39 packs cost? *Ans.* 978 *l.*

```
     P.    l.     P.
  If 13 : 326 : : 39 :
              39
            ----
            2934
            978
            ----
   13) 12714 (978 l. the answer.
       117
       ---
        101
         91
        ---
         104
         104
         ---
           0
```

 Having

Having stated your question, as before, it may be easily seen, that the 4th number will exceed the 2d; for 39 packs must needs cost more than 13 packs; wherefore I multiply the 2d, or middle number, by the greater of the two extreams, *viz.* 39; then must the less extream, to wit, 13, be my divisor.

So multiplying 326 by 39, the product 12714, I divide by 13, and the quotient is 978 pounds, the *Ans.*

QUEST. II.

If 64 yards of broad cloth cost 38 pounds 8 shillings, what will 5 yards of the same cloth cost? *Ans.* 3 *l.*

Because your numbers ought to be of one denomination, before any work can be done, you must reduce 38 pounds 8 shillings into shillings; then state and work your question as underneath.

Yd.	*s.*	*Yd.*	*l.*	*s.*
If 64 :	768 : :	5	38	8
	5		20	
64)	3840 (60 shill.		768 shill.	
	[or 3 *l.* the answer.			
	384			
	00			

Here I multiplied the middle number by the less extream, because the fourth must be less than the second; the reason is evident.

Note likewise, That your fourth number must be of the same denomination with your second; so here your second number being shillings, your fourth number, or the answer to the question, is shillings likewise, to wit, 60 shillings, which divided by 20, gives 3 pounds, the answer to the question.

QUEST. III.

If 6 yards of holland cost 3 pounds 12 shillings and 6 pence, what will $64\frac{1}{4}$ cost?

Before

Before you state your question, bring the first and third number into quarters, and your second into pence, as underneath.

```
Yd.        l.   s.  d.        64¼
 6         3    12  6          4
 4         20                 ---
---        ------             257 quart.
24 quart.  72
           12
          ----
          150
          72
          ----
          870 pence.
```

Then state and work your question as in the following work.

```
   Qr.  d.     Qr.                 l.   s.  d. q.
If 24 : 870 :: 257 :         Ans. 38 . 16 . 4 . 1
            257
          ------
           6090
          4350
         1740
         -------
24) 223590 (9316 pence.
        1/12  776|6=4 pence.
    216         l.   s.  d. q.
    ----   1/20 38 . 16 . 4 . 1 the answer.
     75
     72
    ----
      39
      24
     ----
      150
      144
     -----
      6/24 = 1/4 or 1 farthing.
```

The

The fourth number being pence, I reduce them into ſhillings, by taking $\frac{1}{12}$ part, which makes 776 ſhillings and 4 pence, and 776 ſhillings divided by 20, which is done by cutting off the laſt figure, and taking half of the reſt.

QUEST. IV.

If for 3 weeks diet, I pay 11 ſhillings 3 pence, what is that a-year? Or, which is the ſame, if 21 days require 11 *s.* 3 *d.* what will 365 days require?

Anſ. 9 *l.* 15 *s.* 6 *d.* $\frac{3}{7}$

See the work.

```
  D.   d.    D.                       s.  d.
If 21 : 135 :: 365 :                  11  3
            135                       12
         -------                     ----
            1825                      25
           1095                       11
           365                       ----
         -------                      135
21) 49275 (2346 d.
    ...
    42    1/12 195=6 d.
   ----        l.  s.  d.
    72         9 . 15 . 6 3/7 the anſwer.
    63
   ----
     97
     84
   ----
     135
     126
   ----
   9/21 or 3/7
```

QUEST. V.

How many yards of velvet at 13 *s.* 4 *d.* the yard, will 136 *l.* 12 *s.* buy? *Anſ.* 204 yards, and $\frac{9}{10}$ parts.

See

See the following work.

s. d.	l. s.
13 . 4	136 . 12
12	20
30	2732
13	12
160	5464
	2732
	32784

Then say,

pen. *yd.* *pence.*
If 160 : 1 : : 32784 :

160) 32784 (240 $\frac{9}{10}$
320
784
640

$$\frac{144}{160}\Big|\frac{36}{40}\Big|\frac{18}{20}\Big|\frac{9}{10}$$

QUEST. VI.

A soldier's weekly pay, which is 3 shillings and 5 pence, is forborne for 3 years, 9 months, and 10 days; what is then due? *Ans.* 33 *l.* 2 *s.* 4 *d.* $\frac{1}{7}$.

See the work.

s. *d.*
3 5
12
41 pence

3 *Y.* : 9 *M.* : : 10 *Days.*
365 28
1095 252
252
10
1357 days.

Then say,
D. *d.* *Days.*
If 7 : 41 : : 1357
41
1357
5428
55637
$\frac{1}{7}$=7948 $\frac{1}{7}$

Pence
7948 $\frac{1}{7}$
$\frac{1}{12}$ 66|2 4 *d.*
Ans. 33 *l.* 2 *s.* 4 *d.* $\frac{1}{7}$

QUEST.

QUEST. VII.

If 20 men do a piece of work in 60 days, in how many days will 30 men do the same work? *Ans.* 40 days. *See the work.*

M.	*D.*	*M.*
20 :	60 : :	30 :

$$
\begin{array}{r}
60 \\
20 \\
\hline
30)\ 1200\ (40 \\
120 \\
\hline
00
\end{array}
$$

This question, and some that follow, are by most authors esteemed as questions of *the Rule of Three Inverse*: but we will not confound the learner with such needless differences, for we shall make no distinction between *the Rule of Three Direct*, and *Inverse*. The rule you have for working of your question being sufficient in all cases: For here I consider that the fourth number sought will be less than the second, because 30 men will needs require less time than 20 men; wherefore I multiply the middle number by the less extream, and divide by the greater, and the answer is as in the work.

QUEST. VIII.

How many yards of stuff $\frac{3}{4}$ of a yard broad, will line a cloak containing 5 yards $\frac{1}{2}$ in length, and is 1 yard $\frac{1}{4}$ broad? *See the work.*

	Qr.	*Qr.*	*Qr.*
If	5 :	22 : :	3 :

$$
\begin{array}{r}
22 \\
5 \\
\hline
3)\ 110\ (36\tfrac{2}{3},\ \text{or 9 yards and } \tfrac{1}{6}. \\
9 \\
\hline
20 \\
18 \\
\hline
2 \\
\hline
3
\end{array}
$$

QUEST.

QUEST. IX.

If 360 men be in garrison, and have provision for 6 months; but expecting no relief till the end of 9 months, how many men must be turned out, that the provision may last so much longer? *Ans.* 120 men.

Mo. *Men* *Mo.*
Say, If 6 : 360 : : 9 :
6

9) 2160 (240 Men to be retained, and the remainder to 360, *viz.* 120 must be turned out.
18

36
36

00

QUEST. X.

If a traveller go 160 miles in 7 days, when the day is 16 hours long; how many days will he go the same journey when the day is 12 hours long? *Ans.* In 9 days and 4 hours.

See the work.
H. *D.* *H.*
Say, If 16 : 7 : : 12 :
16

12) 112 ($9\frac{1}{3}$
108

$\frac{4}{12}$ or $\frac{1}{3}$

But many times you may have a question of the *Rule of Three* proposed, that may require some preparation before you can state your question, either by *Addition,* *Subtraction,*

Subtraction, *Multiplication*, or *Division*, &c. as may be seen in the examples following.

QUEST. XI.

A merchant at *London* buys 64 tun of *French* wine, for 460 *l.* the freight thereof from *France* to *London* cost 220 *l.* for loading and unloading 10 *l.* for custom 15 *l.* the charge of the cellar 8 *l.* and would gain 250 *l.* by the bargain.

A gentleman comes and demands the price of 24 tun of the said wine.

The question is, what he must give?

Answer 361 *l.* 2 *s.* 6 *d.*

By *Addition* find the total sum of the freight, with all the expenses and gain; which is 963 pounds.

```
                       Tun.  l.   Tun.
          Then say, If 64 : 963 :: 24:
                             24
460                         ----
220                         3852
 10                        1926
 15                        ----
  8                    64) 23112 (361 l. 2 s. 6 d.
250                         ..
---                        192
963                        ---
                            391
                            384
                            ---
                             72
                             64
                             --
                              8/64 or 1/8
```

QUEST. XII.

If 60 gallons of water, in 1 hour's time, fall into a cistern,

ciſtern, containing 200 gallons; and by a pipe in the ſame ciſtern there runs out 45 gallons in an hour; in how many hours will it be filled?

Anſ. In 13 hours and 20 minutes.

Find how much it fills more than it empties, by Subtraction, which is 15 gallons.

Gall. H. Gall.

Then ſay, If 15 : 1 : : 200 :

60 Filling cock.
45 Emptying cock.
——
15 Difference.

15) 200 ($13\frac{1}{3}$
15
——
50
45
——
$\frac{5}{15}$ or $\frac{1}{3}$

QUEST. XIII.

A butcher ſends his man with 216 pounds to a fair, to buy cattle; Oxen at 11 *l.* Cows at 40 *s. per* piece; Colts at 1 *l.* 5 *s. per* piece; Hogs at 1 *l.* 15 *s. per* piece; and of each a like number: How many of each muſt he buy?

Anſ. 13 of each ſort, and he would have 8 *l.* remaining.

Bring the price of each ſort of cattle into ſhillings, and by *Addition* find the ſum of thoſe ſhillings, which will be 320 *s.* and 216 *l.* is 4320 *s.* as you may ſee.

	11 *l.*	the price of an Ox, equal to	220 ſhil.
216	2	the price of a Cow, equal to	40
20		the price of a Colt, equal to	25
——		the price of a Hog, equal to	35
ſhil. 4320			——
			320

Then

	Sh.	*Cat.*	*Sh.*
Then say, If	320 :	1 : :	4320 :

320) 4320 (13 *Facit.*
320

1120
960

160

QUEST. XIV.

Two persons, *A* and *B*, depart from one place, and both go one road; but *A* goes 3 days before *B*, and travels 30 miles a-day; *B* follows after, and travels 50 miles a-day: How many miles, and in how many days travel will *B* overtake *A*?

First, find by *Subtraction* how much *B* exceeds *A* daily; then by *Multiplication* find how many miles *A* hath travelled at *B*'s setting out; as under.

30 *A*'s daily travel.
3

90 Miles which *A* hath travelled at *B*'s setting out.

B 50
A 30

20 Excess

	M.	*D.*	*M.*
Then say, If	20 :	1 : :	90 :

He will overtake him at the end of 4 days and a half; and will have travelled 225 miles.

9.0
4 $\frac{1}{2}$ the answer.

QUEST. XV.

In 460 pounds, how many shillings, six-pences, four-pences, two-pences, and pence of each a like number, may there be found therein? *Ans.* 4416.

Then

Then say, If 25 : 1 :: 110400 :

	l.	25) 110400 (4416
	460	...
d.	20	100
12	——	——
6	9200	104
4	12	100
2	——	——
1	18400	40
—	9200	25
25	——	——
	110400	150
		150
		——
		0

The proof is easy.

The Golden Rule; or, Rule of THREE *repeated.*

The aforegoing questions being well understood and considered, are sufficient for the understanding both how to state and work any question in the *Rule of Three:* In this place therefore we will treat of the *Golden Rule Compound;* or, *Rule of Three repeated.*

We shall in the practice thereof work the examples given, by so many single *Rules of Three* as the question admits of; it being most consentaneous to reason, most intelligible to the young learner, and many times as quick; besides a great many questions not admitting of any other way of work, whereby the rules before given, without troubling the learner with any more, will be sufficient for the purpose: yet here and there, for the reader's satisfaction, we shall name the common way also.

QUEST. I.

If the carriage of 20 packs from *Manchester* to *London*, which is 136 miles, cost 16 pounds; what will the carriage of 12 packs from *Manchester* to *Chester* cost, being 28 miles?

See the following work.

First, I say, if 20 packs cost 16 pounds, what will 12 packs cost? *Ans.* 9 pounds, 12 shillings?

```
 P.   l.    P.
20 : 16 : : 12 :
      12
     ---
      32
     16
     ---
20) 192 (9l. 12s.
    180
    ---
     12
```

Then say again, if 136 miles cost 9 *l.* 12 *s.* or 192 shillings, what will 28 miles cost? *Ans* 1 *l.* 19 *s.* 6 *d.* $\frac{6}{17}$.

```
  M.    s.    M.
136 : 192 : : 28 :
         28
       ----
       1536
       384
       ----
136) 5376 (3|9 s.
             1 l. 19 s. 6 d. 6/17
     408
     ----
     1296
     1224
     ----
       72
       12
     ----
      144
      72
     ----
      864 (6 d.
      816
 ---  -----
  48     6
 --- or ---
 136    17
```

If

If you would work this queſtion at one operation by the rules commonly given, you muſt take notice there are five numbers given to find a ſixth in proportion thereto.

Which numbers muſt be ſo placed as the three firſt may contain a ſuppoſition, and the two laſt a demand; which that you may place right, let the firſt term be of the ſame denomination with the fourth, the ſecond of the ſame denomination with the fifth, and the third with the term required.

So in the foregoing queſtion the numbers will be diſpoſed in the following order.

	P.	*M.*	*l.*	*P.*	*M.*			
If	20 :	136 :	16 : :	12 :	28.			

Or thus,

	P.	*l.*	*P.*
If	20 :	16 : :	12
	M.		*M.*
	136		28.

Then will your two firſt numbers multiplied together be the firſt number in the ſingle *Rule of Three*, the third will be the ſecond, and the two laſt multiplied together will be the third; then will your numbers ſtand thus, as you ſee.

If 2720 : 16 :: 336

336	136	28
16	16	12
2016	2720	56
336		28
		336

2720) 5376 (1 *l.*
2720

2656
20

53120 (19 *ſh.*
2720

25920
24480 The *Anſwer* is 1 *l.* 19 *s.* 6 *d.* $\frac{6}{17}$ as before.

1440
12

2880
1440

17280 (6 *d.*
16320

$\frac{96}{272}$	$\frac{0}{0}$	$\frac{48}{136}$	$\frac{24}{68}$	$\frac{12}{34}$	$\frac{6}{17}$

QUEST. II.

If 20 pounds gain 16 pounds in 15 months, what ſum of money will gain 24 pounds in 3 months?

Firſt ſay, If 16 pounds come from 20 pounds, what will 24 pounds come from? *Anſw.* 30 pounds.

If

$$\begin{array}{l} \quad l. \quad l. \quad l. \\ \text{If } 16 : 20 :: 24 : \\ \qquad\qquad 20 \\ \hline 16)\ 480\ (30\ l. \\ \quad 48 \\ \hline \quad 00 \end{array}$$

Then I ſay again, If 15 months require 30 *l.* what will 3 months require? *Anſ.* 150 *l.* the number ſought.

$$\begin{array}{l} M. \quad l. \quad M. \\ \text{If } 15 : 30 :: 3 \\ \qquad 15 \\ \hline 3)\ 450\ (150\ l. \\ \quad 3 \\ \hline \quad 15 \\ \quad 15 \\ \hline \quad 00 \end{array}$$

If 15 months require 30 pounds, 3 months will require more money, becauſe it is leſs time; therefore I multiply by the greater extream, and divide by the leſs; and the anſwer will be found to be 150 pounds, as in the work.

This queſtion may be otherwiſe ſtated, yet wrought by two operations, as before; for you may ſay, If 15 months come from 20 *l.* what will 3 months? From more money, becauſe leſs time. *Anſw.* 100 *l.*

$$\begin{array}{l} 15 : 20 :: 3 \\ \qquad 15 \\ \hline 3)\ 300\ (100 \\ \quad 3 \\ \hline \quad 00 \end{array}$$

Say again, If 16 pounds come from 100 pounds, what will 24 come from? *Anſw.* 150. the anſwer as before.

l.

l.		*l.*		*l.*
16	:	100	::	24

16) 2400 (150.
16
80
80
00

But if this question were to be wrought at one operation, you must place your numbers as before directed; and seeing the operation before was after such manner, as the one operation was direct, and the other inverse, the fashion of your work will be altered, for you must multiply them cross-wise, *viz.* the first number of the first rank by the second number of the third rank; and the latter term of the first rank by the first term of the last rank, setting the products under their own multiplicators; so will the product, standing under the first rank be your first number in a *Single Rule of Three Direct*; and the product under the last rank will be your third number, and your middle number the second.

See the work.

	l.		*M.*		*l.*		*l.*		*M.*
If	16	:	15	:	20	::	24	:	3 :
	3						15		
	48						120		
							24		
							360		

Then

Then, If 48 : 20 : : 360
20

48) 7200 (150 *l.* The answer as before.
48

240
240

00

The Golden Rule COMPOUND.

I have been the more large in the foregoing questions, not only to shew the variety of the work, but that I might be the shorter in the following examples: in most of which I shall only state the question, and give the answer; leaving the rest to the exercise of the learner.

QUEST. III.

What is the interest of 672 *l.* for 7 years, at 6 *per cent.* simple interest? *Ans.* 282 *l.* 4 *s.* 9 *d.* 2 *q.*

l. *l.* *l.*
First, I say, If 100 : 6 : : 672
6

40.32 Interest for one year.

Secondly, If 1 : 40.32 : : 7
7

l. 282.24
20

s. 4.80
12

160
80

d. 9.60
4

q. 2.40

QUEST.

QUEST. IV.

What is the interest of 21 *l.* for 5 months and 11 days, or 151 days, at 6 *per cent.* simple interest? *Ans.* 10 *s.* 5 *d.*

	l.	*l.*	*l.*			*D.*	*l.*	*D.*
First, if	100 :	6 : :	21	Secondly,	if	365 :	1.26 : :	151

First:

6

Facit 1.26

Secondly:

151

126
630
126

365) 190.260 (521
1825
776
730
460

.521
20
10.420
12
5.040

QUEST. V.

If 12 rod of ditching be wrought by 2 men in 6 days, how many rod shall be wrought by 8 men in 24 days? *Answ.* 192 rod.

	M.	*R.*	*M.*		*R.*
First, If	2 :	12 : :	8 :	*Facit*	48
	D.	*R.*	*D.*		*R.*
Second, If	6 :	48 : :	24 :	*Facit*	192

QUEST. VI.

If 20 dogs, for 30 groats,
Go 40 weeks to grass;
How many hounds for 60 crowns,
May winter in that place?
Ans. 2000 hounds.

The

The operation.

C.	*D.*	*C.*		*W.*	*D.*	*W.*
If 2 :	20 : :	60	Then if	40 :	600 : :	12
	60				40	

2) 1200 (600
12

00

12) 24000 (2000

This question is the same as if one should say, If 20 dogs for 2 crowns be kept 40 weeks, how many dogs will 60 crowns keep the remainder of the year, or winter-quarter, which is 12 weeks?

QUEST. VII.

If 12 men build a wall 30 foot long, and 6 foot high, and 3 foot thick, in 15 days; in how many days will 60 men make a wall 300 foot long, 8 foot high, and 6 foot thick? *Answ.* In 80 days.

	F. l.	*D.*	*F. l.*	*D.*
First say, If	30 :	15 : :	300 :	150
	F. h.	*D.*	*F. h.*	*D.*
Then, If	6 :	150 : :	8 :	200
	F. t.	*D.*	*F.*	*D.*
Again, If	3 :	200 : :	6 :	400
	M.	*D.*	*M.*	*D.*
Lastly, If	12 :	400 : :	60 :	80

QUEST. VIII.

If 35 ells at *Vienna* make 24 at *Lyons*, and 3 ells at *Lyons* make 5 ells at *Antwerp*, and 100 ells at *Antwerp* 125 at *Frankfort*; how many ells at *Frankfort* make 42 at *Vienna*? *Answ.* 60 ells at *Frankfort*.

Ant.

Ant. Frank. Ant. Frank.

1. If 100 : 125 : : 5 : $6\frac{1}{4}$

Ly. Frank. Ly. Frank.

2. If 3 : $6\frac{1}{4}$: : 24 : 50

Vien. Frank. Vien. Frank.

3. If 35 : 50 : : 42 : 60

(Or,)

Vien. Ly. Ly. Ant. Ant. Frank. Vien.

35 . 24 . 3 . 5 . 100 . 125 . 42

Then you may see the 1st and 7th term are of one denomination, the 2d and 3d of another, the 4th and 5th of another, and the 6th and 8th of another; wherefore multiply the 2d, 4th, 6th and 7th continually for your dividend, and the 1st, 3d, and 5th multiplied together, will be your divisor, and the quotient will be the answer to the question, (*viz.*) 60 at *Frankfort*.

```
   24          35
    5           3
  ---      ------
  120      10500 Divisor.
  125
  ---
  600
 240
120
 -----              105|00) 6300|00(60
 15000                      630
    42                      ---
 -----                       00
 30000
60000
------
630000 Dividend.
```

By the way we may observe, that in the working the questions in the *Rules of Three* aforegoing, we have continually used Multiplication before Division, though it be not out of any necessity: but seeing fractions

fractions are not yet learned, and in dividing there is often a remainder, we do it to avoid the trouble of having a fraction to multiply, or else we may as well divide the middle number by that number which is the dividend, and multiply that quotient by the other extreme; the product is the answer to the question, as in the other way.

EXAPMLE.

If 13 packs of wool cost 65 pounds, what will 142 packs of the same wool cost? *Answ.* 710 *l.*

P. l. P.	142
If 13:65::142:	5
13) 65 (5	
65	710
0	

Here I divided 65 by 13, the quotient 5 I multiplied 142 by; and the product, *viz.* 710 pounds, is the *Answer*.

Contractions in the Rule of THREE.

This being considered, you may oftentimes contract your work in *Questions of the Rule of Three*.

And first, if at any time you have a question given in *the Rule of Three repeated*, and the first and second number in every operation be the same, you may contract your work by dividing the second number by the first, and the quotient will be a common multiplicator, by which you may multiply all your third numbers, and those products will be the numbers sought. And thus will all your divisions be saved, except one, and that commonly a small one.

EXAMPLE.

A, B, C, and *D,* have 100 pounds Sterling to be divided among them, in such sort, that as oft as *A* hath

3 pounds, *B* muſt have 5 pounds; and as oft as *B* hath 5 pounds, *C* muſt have 7 pounds; and as oft as *C* hath 7 pounds, *D* muſt have 10 pounds: what muſt each have?

Add the proportions into one ſum, and ſay,

3		*l.* *l.* *l.*	
5		25 : 100 :: 3	
7	If	25 : 100 :: 5	
10		25 : 100 :: 7	
—		25 : 100 :: 10	
25			

Or ſhorter, thus, if $\overset{l.}{25} : \overset{l.}{100} :: \left\{ \begin{array}{l} l. \\ 3 \\ 5 \\ 7 \\ 10 \end{array} \right.$

$$\begin{array}{l} 25)\,100\,(4 \\ \underline{100 \quad 3} \\ \;\;0 \quad 12 \text{ for } A. \end{array} \qquad \begin{array}{r} 4 \\ \underline{5} \\ 20 \text{ for } B. \end{array}$$

$$\begin{array}{l} A\ 12 \\ B\ 20 \\ C\ 28 \\ \underline{D\ 40} \\ 100 \text{ for proof.} \end{array} \qquad \begin{array}{r} 4 \\ \underline{7} \\ 28 \text{ for } C. \end{array} \qquad \begin{array}{r} 4 \\ \underline{10} \\ 40 \text{ for } D. \end{array}$$

In this example you may ſee there are 3 diviſions ſaved, and that diviſion you have, but a ſmall one; and therefore will prove of excellent uſe in the rule of *Fellowſhip* and *Alligation*, where the firſt and ſecond number in every operation is commonly the ſame; ſo hereafter, we will call this method the contracted way in *Fellowſhip*.

Sometimes a queſtion of the *Rule of Three* may be performed by *Multiplication* only.

Some-

Sometimes by *Division* only.

Sometimes by *two Multiplications*.

Sometimes by *two Divisions*.

And, ſometimes by a *ſmaller Multiplication* and *Diviſion* than your given numbers allow of.

If at any time your diviſor, with either of the other numbers, may be ſeverally divided by ſome common meaſure, without leaving any remainder, and your diviſor come to be an unit, the anſwer will be given by Multiplication only; as in the following example.

If 6 groſs of incle coſt 15 *l.* what will 48 coſt?
Anſw. 120 *l.*

$$\begin{array}{ccccc} Gr. & & l. & & Gr. \\ 6 & : & 15 & :: & 48 \\ \frac{1}{6}\ 1 & : & 15 & :: & 8 \\ & & 8 & & \\ \hline \end{array}$$

Anſw. 120

Here $\frac{1}{6}$ of 6 is 1; of 48 is 8, and 8 times 15 is 120 pounds, the anſwer.

But if either of the other numbers comes to be an unit, the work may be performed by diviſion only; as in the following example.

If 18 groſs coſt 12 *l.* what will 6 groſs coſt?
Anſw. 4 *l.*

$$\begin{array}{ccccc} Gr. & & l. & & Gr. \\ 18 & : & 12 & :: & 6 \\ \frac{1}{6}3 & : & 12 & :: & 1 \\ 3) & & 12 & & (4 \\ \hline & & 0 & & \end{array}$$

Here I take $\frac{1}{6}$ of 18, which is 3, and a 6th part of 6 is 1; then divide 12 by 3, gives 4 pounds, the anſwer to the queſtion.

If your divisor be exactly contained in both your other numbers, the question may be answered by two multiplications; as in the example following.

If 3 gross cost 9 *l.* what will 12 gross cost?
Answ. 36 *l.*

Gr.	*l.*	*Gr.*
3 :	9 : :	12
3 :	3 : :	4
	4	
	12	
	3	
	36	

Here the 3d part of 9 is 3, of 12 is 4; multiply 3 by 4 gives 12, and that by 3 gives 36, the answer.

If you would use two divisions, divide your divisor by your second number, and the 3d by that quotient, which gives the answer.

EXAMPLE.

If 9 gives 3, what will 45 give?
Answ. 36 *l.*

```
3) 9 (3) 45 (15
   9     3
   --    --
   0     15
         15
         --
          0
```

So dividing 9 by 3, quotes 3, and by that dividing 45, quotes 15, the answer.

Sometimes you must use both Multiplication and Division, yet but small ones.

EXAM-

EXAMPLE.

If 48 yards of linen-cloth cost 3 *l.* 12 *s.* what will 112 yards cost? *Answ.* 8 *l.* 8 *s.*

Yd.	s.	Yd.	l.	s.
48 :	72 ::	112	3	12
24 :	72 ::	56	20	
12 :	72 ::	28	—	
6 :	72 ::	14	60	
3 :	72 ::	7	12	
	7		—	
	—		72	

3) 504 (16|8
3   8 *l.* 8 *s.*
—
20
18
—
24
24
—
0

You might have contracted your work more still, by taking $\frac{1}{8}$ of the first and last, at first, and then $\frac{1}{2}$ of that; which would have been the same, as you may see.

The *Double* or *Compound Rule of Three*, may also be greatly contracted by a method of [illegible]aging vulgar fractions, which the reader will find explained in *page* 121 following.——Let us, for example, take the first question in the *Double Rule of Three*, which is resolved at one operation, *page* 89 foregoing. The two first numbers are there multiplied together for a first number or divisor, and the two last for a third number, which, by the *Golden Rule*, is to be multiplied by the second; and consequently, the three last numbers multiplied together form the dividend, and the quo-

tient arising from thence answers the question. This being premised, we shall proceed, according to this short method, to solve the question, which in *page* 89 stands thus.

P.	*l.*	*P.*
20 :	16 :	: 12
M.		*M.*
136		28

The two first numbers, *viz.* 20 and 136, are to be multiplied for a divisor, and the three last, *viz.* 16, 12, and 28, for a dividend, and the quotient arising from thence is the answer. That is $\frac{16\times12\times28}{20\times136}$ will answer the question. This fraction reduced, according to the rule given in *page* 121, will be $\frac{6\times28}{5\times17}$ or $\frac{168}{85}$; the value of which is easily found as follows.

85) 168 (1 [That is 1 *l.* 19 *s.* 6 *d.* $\frac{30}{85}$ or $\frac{6}{17}$ the same as in *page* 90.
85
———
83
20
———
1660 (19 *s.*
85
———
810
765
———
45
12
———
90
45
———
540 (6 *d.*
510
———
30

Proof

Proof of the Golden RULE.

We will ſhew you how to prove the *Rule of Three*, and ſo conclude this rule.

If 4 numbers be proportional, the product of the two means is equal to the product of the two extremes.

Hence, to prove your work, multiply the 4th number found by the firſt number; and if that product be equal to the product of the 2d by the 3d, the work is right, elſe not.

So if 8 *yds.* coſt 16 *l.* what will 45 *yds.* coſt? *Anſ.* 90 *l.*

```
Y.  l.   Y.
8 : 16 : : 45
        45
       ---
        80
       64
       ---
  8) 720 (90
     72
     ---
      00
```

Then the 4 proportional numbers will be,

```
8 : 16 : : 45 : 90
        45       8
       ---      ---
        80      720 the prod. of the 1ſt and 4th.
       64
       ---
       720 the product of the 2d and 3d.
```

You ſee the product of the 1ſt and 4th is equal to the product of the 2d and 3d, which ſhews your work, to be right.

Hence, if of 4 numbers, the 1ſt be to the 2d as the 3d is to the 4th, ſo thoſe 4 numbers ſhall be proportional.

But if your third number be lefs than the firft, and require more; or more, and require lefs; then the product of your firft and fecond will be equal to the product of your third and fourth.

EXAMPLE.

If 12 men do a piece of work in 16 days, in how many days will 24 men do the fame piece of work?

```
   M.   D.   M.
If 12 : 16 : : 24
        12
      ----
        32
       16
     -----
24) 192 (8 days the anfwer.
     ---
      0
```

```
                            M.   D.    M.   D.
Then the 4 numbers will be  12 : 16 : : 24 : 8
                                16          8
                               ---        ---
                                72        192
                               12
                               ---
                               192
```

Here you may fee the product of the 1ft by the 2d is equal to the product of the 3d and 4th; which fhews the work to be right.

The Rule of PRACTICE.

THis rule is only a contraction of the *Golden Rule* aforegoing: for when the price or value of one yard, ell, hundred, *&c.* is given, and the price or value of any other quantity of yards, ells, hundreds, *&c.* were required; the first number being always an unit, the question may more quickly be wrought by the aliquot parts of a shilling or pound, as the nature of the question requires. It will be convenient to treat first of such questions as may be wrought by the aliquot parts of a shilling; wherefore it will be necessary to annex a table for that purpose.

A table of the aliquot parts of a shilling.

	d.			*th.*	
For	1	Take	one	12	Part.
	$1\frac{1}{2}$		one	8	
	2		one	6	
	3		one	4	
	4		one	3	
	6		one	$\frac{1}{2}$	

The use of this table is easy; for you may see that for 1 penny you must take one 12th part; for 1 penny, halfpenny, one 8th part; for 2 pence, one 6th part; and so of any other.

Example I.

At 1 penny the yard, what will 672 yards cost?

$\frac{1}{12}$ 5|6 shillings.

An. 2 *l.* 16 *s.*

Here I said, $\frac{1}{12}$ of 67 is 5, of 72 is 6; then cutting off the last figure, $\frac{1}{2}$ the rest is pounds, and the answer is 2 *l.* 16 *s.*

In

IV.

At 11 *d.* 1 *q.* the yard, what will 752 yards cost?

½ 376
½ 188
½ 94
½ 47

70|5

Ans. 35 *l.* 5 *s.*

Here first I took one half for 6 *d.* and one half of that for 3 *d.* and ½ of 3 *d.* for 1 *d.* ½, and ½ of 1 *d.* ½ for 3 *q.* which is just 11 *d.* 1 *q.* which sums added together, and divided by 20 as before, give 35 *l.* 5 *s.* the answer.

We will now proceed to the questions that consist of a shilling, and some number of pence and farthings beside, that the learner may understand all varieties.

I.

At 13 *d.* ½ the yard, what will 144 yards cost?

⅛ 18

16|2

Ans. 8 *l.* 2 *s.*

In this question I let the number given stand for a shilling, and only take an ⅛ part for 1 *d.* ½, and adding them together gives 162 shillings, equal to 8 *l.* 2 *s.* the answer.

II.

At 16 *d.* ½ the yard, what will 144 yards cost?

35 9
17 10 2

19|6 7 2

Ans. 9 *l.* 16 *s.* 7 *d.* 2 *q.*

It is evident, it will cost 143 *s.* and 143 4 *d.* ½: wherefore I let it stand for a shilling, and taking ¼ part for 3 *d.* and ½ of 3 *d.* for 3 half-pence, adding all together makes 196 *s.* and 7 *d.* ½, equal to 9 *l.* 16 *s.* 7 *d.* ½ the answer.

Here

Here I let it stand for 1 *s.* then taking $\frac{1}{2}$ for 6 *d.* and $\frac{1}{2}$ of 6 *d.* for 3 *d.* and $\frac{1}{2}$ of 3 *d.* for 1 *d.* $\frac{1}{2}$, the sum of all, which is 270 shillings, equal to 13 *l.* 10 *s.* the answer.

III.

At 22 *d.* $\frac{1}{2}$ the yard, what will 144 yards cost?

72
36
18

27|0

Answ. 13 *l.* 10 *s.*

Here follow some more questions of divers natures, for the further exercise of the learner.

For 3 *d.* I took $\frac{1}{4}$ part, and for a farthing $\frac{1}{12}$ of that, and for $\frac{1}{2}$ a farthing, I took $\frac{1}{2}$ of the farthing, and for the quarter of the farthing, $\frac{1}{2}$ of the $\frac{1}{2}$ farthing, which added together, makes 41 *s.* and 3 *d.* equal to 2 *l.* 1 *s.* 3 *d.*

I.

At 3 *d.* farthing, $\frac{1}{2}$ farthing, $\frac{1}{4}$ farth. the yard, what will 144 yards cost?

$\frac{1}{4}$	36	
$\frac{1}{12}$	3	
$\frac{1}{2}$	1	6
$\frac{1}{2}$	0	9
	4|1	3

Answ. 2 *l.* 1 *s.* 3 *d.*

Here I took for 3 *d.* twice, and for 1 farthing $\frac{1}{12}$ part of 3 *d.* which added, makes 21 *s.* 4 *d.* 1 *q.* or 1 *l.* 1 *s.* 4 *d.* 1 *q.*

II.

At 6 *d.* 1 *q.* a yard, what will 41 yards cost?

10	3	
10	3	
00	10	1
2|1	4	1

Answ. 1 *l.* 1 *s.* 4 *d.* 1 *q.*

III.

At 3 farthings the ell, what will 6725 ells cost?

		d.
$\frac{1}{8}$ 840	3	3
$\frac{1}{2}$ 42|0	3	3

Answ. 21 *l.* 0 *s.* 3 *d.* 3 *q.*

Here I took $\frac{1}{8}$ part for 1 *d.* $\frac{1}{2}$, and $\frac{1}{2}$ of that is equal to 420 *s.* and the answer will be 21 *l.* 0 *s.* 3 *d.* 3 *q.*

IV.

At 3 *d.* $\frac{1}{2}$ the ell, what will 728 yards cost?

4

2912

$\frac{1}{5}$ 582 $\frac{2}{5}$=ells.

	d.	q.
197		
72	9	2$\frac{4}{5}$
		2$\frac{4}{5}$

Ans. 169 *s.* 10 *d.* 1$\frac{3}{5}$ or 8 *l.* 9 *s.* 10 *d.* 1 *q.* $\frac{3}{5}$

First, I reduced the yards into ells, *Facit* 582 $\frac{2}{5}$ ells; then for 2 *d.* I took a 6th part, and for the 1 *d.* $\frac{1}{2}$, $\frac{1}{8}$ part: lastly, for the $\frac{2}{5}$ I took $\frac{1}{5}$ of the price of the ell twice; and the answer will be, as in the work, *viz.* 8 *l.* 9 *s.* 10 *d.* 1 *q.* $\frac{3}{5}$

We shall now proceed to questions relating to *Aliquot parts of a pound*: to which purpose take the following table.

Aliquot parts of a pound.

	Shil.	*Pence.*			*th.*
For	1	0	Take	one	20
	1	8		one	12
	2	0		one	10
	2	6		one	8
	3	4		one	6
	4	0		one	5
	5	0		one	4
	6	8		one	3
	10	0		one	$\frac{1}{2}$

Examples

Examples follow.

(1.)
At 1 s. yard, what
$\frac{1}{20}$ 14|4 yards?
Answ. 7 *l.* 4 *s.*

(2.)
At 1 s. 8 *d.* the yard, what
$\frac{1}{12}$ 144 yards?

Facit 12 *l.* the answer.

(3.)
At 2 *s.* yard, what 67|2
yards?
$\frac{1}{10}$ part is 67 $\frac{2}{10}$
67 *l.* 4 *s.*

(4.)
At 2 *s.* 6 *d.* what 172 yards?

$\frac{1}{8}$ is 21 $\frac{4}{8}$
Answ. 21 *l.* 10 *s.*

(5.)
At 3 *s.* 4 *d.* the yd, what 751
yards?

$\frac{1}{6}$ part is 125 $\frac{1}{6}$
Answ. 125 *l.* 3 *s.* 4 *d.*

(6.)
At 4 *s. per* yard, what 176
yards?

$\frac{1}{5}$ part is 35 $\frac{1}{5}$
Answ. 35 *l.* 4 *s.*

(7.)
At 5 *s.* yd, what 735 yards?

$\frac{1}{4}$ part is 183 $\frac{3}{4}$
Answ. 183 *l.* 15 *s.*

(8.)
At 6 *s.* 8 *d.* the yd, what 176,
yards?

$\frac{1}{3}$ part is 58 $\frac{2}{3}$

(9.)
At 10 *s.* yd, what 144 yds?

$\frac{1}{2}$ part is 72
Answ. 72 *l.*

If your question consist not of aliquot parts, divide it into such, the sum of which will be the answer to the question; as in the following examples may more fully appear.

(1.)

At 3 *s.* *yd.* what will 144 *yds.* cost?

$\frac{1}{10}$ for 2 shil. is 14*l.* 8*s.*
$\frac{1}{20}$ for 1 shil. or $\frac{1}{2}$ of 2 *s.* for 1 *s.* } is 7*l.* 4*s.*

The sum is the *Ans.* 21*l.* 12*s.*

(2.)

At 7 *s.* yd, what will 144 yards cost?

$\frac{1}{4}$ part for 5 shillings is 36
$\frac{1}{10}$ for 2 shillings 14 8

The sum is the *Ans.* 50.8

(3.)

At 15 *s.* 6 *d.* *C.* what 721 *C.* cost?

$\frac{1}{2}$ for 10 shil. is 360 10
$\frac{1}{2}$ of 10*s.* for 5*s.* is 180 5
$\frac{1}{10}$ of 5*s.* for 6*d.* is 18 0 6

The sum is the *Ans.* 558 15 6

(4.)

At 11*s.* 4*d.* the *gr.* what 150 *gross* cost?

$\frac{1}{2}$ for 10 shillings is 75
$\frac{1}{10}$ of 10*s.* for 1 *s.* is 7 10
$\frac{1}{3}$ of 1 *s.* for 4 *d.* is 2 10

The sum is the *Ans.* 85 *l.*

(5.)

At 2 *s.* 4 *d.* $\frac{1}{2}$ *p.* what 141 *pounds* cost?

$\frac{1}{10}$ for 2 *s.* is 14 2 0
$\frac{1}{6}$ of 2 *s.* for 4*d.* is 2 7 0
$\frac{1}{8}$ of 4 *d.* for 2*q.* is 0 5 10$\frac{1}{2}$

The sum is the *Ans.* 16 14 10$\frac{1}{2}$

(6.)

At 17 *s.* 6*d.* the *bund.* what 375 *bundles* cost?

$\frac{1}{2}$ for 10 *s.* is 187 10 0
$\frac{1}{2}$ of 10*s.* for 5*s.* is 93 15 0
$\frac{1}{2}$ of 5*s.* for 2*s.* 6*d.* is 46 17 6

The sum is the *Ans.* 328 02 6

If your question consist of shillings and pence, as in the last, you may multiply by the number of shillings, and take the correspondent aliquot parts for the pence, according to the first table; and from the sum cutting off the last figure, and taking half the rest, the answer will be the same as in the foregoing method; and in some particular cases may be more convenient, and oftentimes more easy.

E X-

EXAMPLES.

(1.)
At 7 s. 1 d. the *ell*, what will 144 *ells* cost?

Multiply by	7
Product	1008
$\frac{1}{12}$ of 144 is 12 for 1 penny,	
Sum is	102|0
$\frac{1}{20}$.	51 *l.* the answer.

(2.)
At 17*s.* 4*d.* *yd.* what 172 *yds?*

Multiply by	17
Product	2924
$\frac{1}{3}$ of 172 for 4 *d.* is	57 4
	298|1 4
Answer.	149 *l.* 1 *s.* 4 *d.*

(3.)
At 9*s.* 9*d.* what 141 *yards?*

Multiply by	9
Product is	1269
$\frac{1}{2}$ of 141 for 6*d.* is	70 6
$\frac{1}{2}$ of that for 3*d.* is	35 3
Sum is	137|4 9
Answer.	68 *l.* 14 *s.* 9 *d.*

(4.)
At 1 *l.* 14 *s.* 9 *d.* $\frac{1}{2}$ the *yard*, what will 144 *yds.* cost?

	144
Multiply by	34
	576
	432
Product is	4896 *s.*
$\frac{1}{2}$ of 144 for 6 *d.* is	72
$\frac{1}{2}$ of last for 3 *d.* is	36
$\frac{1}{2}$ of that for 2 *q.* is	6
Sum is	501|0
Answer.	250 *l.* 10 *s.*

If your question consist of shillings only, you may contract your work thus. If your shillings be even, multiply your number given by half the number of shillings; the first figure to the right hand in your product, is a double number of shillings; and in your operation ought to be set apart in the place of shillings, the rest is pounds.

 (1.) At

(1.)

At 16*s.* the yard, what will 672 yards coſt?

Multiply	672
By ½ of 16 (*viz*)	8
Pr. with Sh. apart is	537.12
Anſwer.	537 *l.* 12 *s.*

(2.)

At 6*s.* the yard, what will 172 yards coſt?

	172
	3
Pr. with Sh. apart is	51.12
Anſwer.	51 *l.* 12 *s.*

(3.)

At 12*s.* the yard, what will 172 yards coſt?

	172
Multiply by	6
Pr. with Sh. apart is	103.4
Anſwer.	103 *l.* 4 *s.*

(4.)

At 14*s.* the yd, what will 125 yards coſt?

	125
Multiply by	7
Pr. with Sh. apart is	87.10
Anſwer.	87 *l.* 10 *s.*

Or you may multiply as uſual; and when you have finiſhed your operation, cut off your laſt figure, doubling it for ſhillings, and making the reſt pounds.

Examples follow.

(1.)

At 12*s.* the yd, what will 144 yards coſt?

	144
Multiply by	6
	86|4
Anſwer.	86 *l.* 8 *s.*

(2.)

At 8*s.* the bundle, what will 172 bundles coſt?

	172
Multiply by	4
	68|8
Anſwer.	68 *l.* 16 *s.*

But if your number of ſhillings be odd, work for the greateſt even number of ſhillings therein, and for the odd ſhilling take the 20th part of the given number; thoſe reſults added together, give the anſwer.

EXAM.

EXAMPLE.

(1.)

At 17 *s.* the yard, what will 172 yards coſt?

	172	
Mult. for 16 *s.* by	8	
Product	137	12
$\frac{1}{10}$ for 1 *s.* is	8	12
Sum and anſwer	146 *l.*	4 *s.*

(2.)

At 19 *s.* the yard, what will 144 yards coſt?

	144
Mult. for 18 *s.* by	9
Product	129.6
$\frac{1}{10}$ for 1 *s.* is	7.2
Sum	136.8
Anſwer	136 *l.* 16 *s.*

If your queſtion conſiſt of pounds, ſhillings and pence; for the pounds multiply, and for the ſhillings and pence work by the former rules.

EXAMPLE.

At 2 *l.* 17 *s.* 5 *d.* the hundred, what will 144 hundred coſt?

	144	
Multiply by	2	
Product	288	
$\frac{1}{2}$ of 144 for 10 *s.* is	72	
$\frac{1}{2}$ of that for 5 *s.* is	36	
$\frac{1}{10}$ of 144 for 2 *s.* is	14	8
$\frac{1}{12}$ of 5 *s.* for 5 *d.* is	3	0
The ſum	413	8
Anſwer	413 *l.*	8 *s.*

If a pack of cotton coſt 11 *l.* 11 *s.* 11 *d.* what will 111 packs coſt?

	111
Same one fig. nearer	111
Product for 11 *l.* is	1221
$\frac{1}{10}$ for 11 *s.* is	61.1
$\frac{1}{12}$ of 11 *s.* for 11 *d.* is	5.1.9
	1287.2.9
Anſwer	1287 *l.* 2 *s.* 9 *d.*

If the price of one be given, and the price of any other number be required, together with $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, or any other part, you muſt work as before for the integral part, and for the fractional part take ſuch a part of the given price; the total ſum of which parts will be the anſwer to the queſtion, as in the following examples may appear.

EXAM-

EXAMPLE.

(1.)

At 4 *l.* 16 *s.* 3 *d.* the hundred, what will 34 *C.* ¾ cost?

	34¾		
For 4 *l.* multiply by	4		
Product	136		
$\frac{1}{2}$ of 34 for 10 *s.* is	17		
$\frac{1}{2}$ of 10 *s.* for 5 *s.* is	8	10 *s.*	
$\frac{1}{4}$ of 5 *s.* for 1 *s.* 3 *d.*	2	2	6
$\frac{1}{2}$ of the given price	2	8	$1\frac{1}{2}$
$\frac{1}{2}$ of that	1	4	$0\frac{3}{4}$
The sum	167	4	$8\frac{1}{4}$

Answer 167 *l.* 4 *s.* 8 *d.* $\frac{1}{4}$

(2.)

At 3 *l.* 17 *s.* 6 *d.* the hundred, what will 144 *C.* 2 *q.* 31 *lb.* cost?

	144	02	12
For 3 *l.* mult. by	3		
Product	432		
$\frac{1}{4}$ of 3 *l.* for 15 *s.*	108		
$\frac{1}{6}$ of that for 2 *s.* 6 *d.*	18		
$\frac{1}{2}$ of the given price	1	18	9
$\frac{1}{4}$ of that for 14 *lb.*		9	$8\frac{1}{4}$
$\frac{1}{2}$ of that for 7 *lb.*		4	10
Sum	560	13	$03\frac{1}{4}$

Answer 560 *l.* 13 *s.* 3 *d.* 1 *q.*

But if the fractional parts cannot conveniently be taken, the quickest, easiest, and best way is performed by the *Decimal Rule of Practice* following.

The Doctrine of VULGAR FRACTIONS.

Notation of VULGAR FRACTIONS.

WHat a *Vulgar Fraction* is, was shewed in the *Introduction*, and so needs no repetition.

A vulgar fraction is either single or compound.

A single vulgar fraction hath only one numerator, and one denominator, and is either *Proper* or *Improper.*

A proper single fraction hath its numerator always less than its denominator; as $\frac{3}{4}$ $\frac{11}{18}$ $\frac{25}{79}$, *&c.*

An improper single fraction is when the numerator is greater than its denominator, as $\frac{4}{3}$, $\frac{15}{11}$, $\frac{79}{25}$, *&c.*

A

A compound vulgar fraction is ſuch as hath more numerators and denominators than one, as $\frac{3}{4}$ of $\frac{4}{5}$, and is eaſily known by having this word *of*, placed betwixt them; ſo $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$, is a compound fraction. The formation is eaſy: for 7 pence will be $\frac{7}{12}$ of $\frac{1}{20}$ of a pound, and three farthings is $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound.

From hence proceeds another number, called a mixed number, and conſiſteth of two parts, the one whole, the other broken; ſo 3 yards and 3 quarters is expreſſed in a mixed number thus, $3\frac{3}{4}$; others are $11\frac{7}{19}$, $144\frac{71}{176}$, &c.

Things commonly expreſſed by fractions or broken numbers, are the parts of coins, weight, meaſure, time, &c. As ſhillings, pence, or farthings are fractions in reſpect of a pound; ſo quarters, pounds and ounces are fractions in reſpect of a hundred.

Reduction of Vulgar FRACTIONS.

Becauſe *Addition* and *Subtraction* of *Vulgar Fractions* cannot well be performed without the knowledge of *Reduction*, we will firſt treat of it, and then of the reſt in order.

By *Reduction* we bring fractions into their leaſt equivalent parts.

And into common denominators,

Or into one denomination.

By *Reduction* we find the value of any fraction in the known parts of the integer.

Reduce whole or mixed numbers into improper fractions, *et contra*.

As likewiſe compound fractions into ſingle.

Of theſe in their order.

I. To bring fractions into their leaſt equivalent parts, may be performed ſeveral ways; a general rule for which is either of theſe that follow.

Firſt, Divide the denominator by the numerator, and the diviſor by the remainder, if any be; thus doing till you find nothing remain; your laſt diviſor is the greateſt common meaſure ſought: or divide the denominator by the numerator, and likewiſe by the remainder as long as there is any; the laſt diviſor is your greateſt common meaſure ſought as before.

By which dividing your numerator and denominator, reduceth your given fraction into its leaſt parts.

Note,

Note, If your last divisor be an unit, the fraction is in its least terms already.

EXAMPLE.

Let us find the greatest common measure of $\frac{555}{629}$. Here I divide 629 by 555, remains 74; by which dividing 555, rest 37; by which dividing 74, nothing remains: so is 37 my last divisor, the common measure sought.

See the work.

```
                 555) 629 (1
                      555
                      ---
                  74) 555 (7
                      518
The first way         ---
                  37)  74 (2
                       74
                       --
                        0
```

Facit 37 for the greatest common measure.

```
                 555) 629 (1
                      555
                      ---
                  74) 629 (8
                      592
The second way        ---
                  37) 629 (17
                      37
                      ---
                      259
                      259
                      ---
                        0
```

Facit 37, as before.

Then if you divide 555 and 629 severally by 37, the two quotients will be 15 and 17, which placed fractionally, thus, $\frac{15}{17}$, will be equal in value to the former fraction, but in its least terms.

So the greatest common measure of $\frac{637}{819}$ will be found to be 91; by which dividing both the numerator and denominator, reduceth the fraction into its least parts, *viz.* $\frac{7}{9}$, and so of any other.

But

But fractions may more quickly be abbreviated, if you can descry any number that will evenly divide both your numbers, without leaving any remainder; which in all even numbers may be done, by halving both as often as you can: if your numbers end with 5, or a cipher, it may be done by taking $\frac{1}{5}$ part, or $\frac{1}{10}$ part, and so in many other.

So $\frac{144}{240}$ by halving, will be abbreviated into $\frac{9}{15}$, and $\frac{9}{15}$ by taking $\frac{1}{3}$ will become $\frac{3}{5}$ the least parts required, as you may see in the work.

$$\frac{144}{240} \Big| \frac{72}{120} \Big| \frac{36}{60} \Big| \frac{18}{30} \Big| \frac{9}{15} \Big| \frac{3}{5}$$

II. When several fractions are given to be reduced into other equivalent fractions, having a common denominator, use this rule:

Multiply every numerator into each denominator continually, except its own, which shall be new numerators; then multiply all the denominators into one another, for a common denominator, and your work is finished.

EXAMPLE.

Let $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$ be reduced into other fractions, which shall have one common denominator.

Multiply 1, 3, and 4 together, *Facit* 12; and 2, 2 and 4, *Facit* 16; and 3, 3 and 2, *Facit* 18; so have you three new numerators. Next multiply 2, 3 and 4 into one another. *Facit* 24 for a common denominator to the former numerators.

So $\frac{12}{24}$, $\frac{16}{24}$, $\frac{18}{24}$ will be equal to $\frac{1}{2}$, $\frac{2}{3}$ $\frac{1}{4}$.

Reduce $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{11}$, and $\frac{24}{75}$, into a common denominator, and you will find $\frac{22275}{29700}$ equal to $\frac{3}{4}$, $\frac{16500}{29700}$ equal $\frac{5}{9}$, $\frac{18900}{29700}$ equal to $\frac{7}{11}$ and $\frac{9504}{29700}$ equal to $\frac{24}{75}$, and thus of any other.

III. Fractions of divers denominations may be brought into one denomination, by involving the less into the parts of the greater, whereby it will become a compound fraction.

EXAMPLE.

So if $\frac{1}{4}$ of a shilling, and $\frac{4}{7}$ of a pound must be brought into the fraction of a pound, you may observe that $\frac{1}{4}$ of a shilling is $\frac{1}{4}$ of one $\frac{1}{20}$ of a pound, because one shilling is $\frac{1}{20}$ of a pound; which compound fraction, when reduced by one of the following rules, will be $\frac{1}{80}$ of a pound; so have you both in one denomination, as was required.

So

So $\frac{3}{7}$ of an ounce reduced into the fraction of a *C.* weight, will be $\frac{3}{7}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of a *C.* weight, equal to $\frac{3}{784}$ *C.* and so of any other.

An expression of the following form, *viz.* $\frac{6\times20\times5}{40\times15\times9}$, which implies the continued multiplication of 6, 20, and 5, divided by the continual product of 40, 15, and 9, may be abbreviated by dividing any of the factors, or parts of the numerator and denominator by some number that is a common measure of those parts. Thus, because 20 will measure 20 (a part of the numerator) and 40 (a part of the denominator) the fraction will be reduced to $\frac{6\times1\times5}{2\times15\times9}$; likewise because three will measure 6, and 9, it will be $\frac{2\times1\times5}{2\times15\times3}$; and since 2 is a factor both in the numerator and denominator, it will be farther reduced to $\frac{1\times5}{15\times3}$; also because 5 will measure both 5 and 15, it will become $\frac{1\times1}{3\times3}$, or $\frac{1}{9}$, the fraction in its lowest terms. And after the same manner may any other fraction of the same kind be abbreviated.

IV. To find the value of any vulgar fraction in the known parts of the integer, do thus;

Multiply the numerator of the fraction given, by the known parts of the next inferiour denomination; which product divided by the denominator, quotes the parts of that denomination sought; the remainder, if any, multiplied by the parts of the next inferiour denomination, and divided as before, gives the parts of the next denomination; and thus you must do, till you have it brought into the least known parts, or till nothing remains.

EXAM-

EXAMPLE.

What is $\frac{133}{480}$ of a pound sterling? *Ans.* 5 *s.* 6 *d.* 2 *q.*

$$
\begin{array}{rrl}
 & 133 & \text{numerator} \\
 & 20 & \text{shillings in a pound} \\
\hline
\text{Denominator}=480) & 2660 & (5 \text{ shillings} \\
 & 2400 & \\
\hline
 & 260 & \text{Remains} \\
 & 12 & \text{Pence in a shilling} \\
\hline
 & 520 & \\
 & 260 & \\
\hline
\text{Denominator}=480) & 3120 & (6 \text{ pence} \\
 & 2880 & \\
\hline
 & 240 & \\
 & 4 & \text{Farthings in a penny} \\
\hline
\text{Denominator}=480) & 960 & (2 \text{ farthings} \\
 & 960 & \\
\hline
 & 0 &
\end{array}
$$

After the same manner the value of $\frac{74}{195}$ of an hundred will be one quarter, 14 pounds, and 8 $\frac{8}{195}$ ounces.

V. To reduce whole or mixed numbers into improper fractions, do thus:

If your number given be an integer, it is but making an unit denominator thereunto; so 7 reduced into an improper fraction, will be $\frac{7}{1}$.

If your denominator be fixed, the product of it, and your integer given, will be the numerator.

So if 7 were to be reduced into an improper fraction, whose denominator should be 11, the improper fraction answering, would be $\frac{77}{11}$; and so in any other.

But if it be a mixed number, then multiply the integral part of your mixed number by the denominator of your fractional part, and to the product add the numerator of the said fractional part, the sum will be the numerator to the former denominator.

So $2\frac{3}{5}$ will be $\frac{13}{5}$, and $7\frac{11}{17}$ will be $\frac{130}{17}$, *&c.*

On the contrary, if you would reduce any improper fraction into its equivalent whole or mixed number, do thus: Divide your numerator by the denominator, the quotient is the whole or integral part; and the remainder, if any, is numerator to the former denominator.

So if $\frac{25}{5}$ were reduced, it would be a whole number, (*viz.*) 5: and if $\frac{45}{13}$ were reduced, it would be a mixed number, to wit, $3\frac{6}{13}$; and so of any other.

VI. To reduce a compound fraction into a single fraction.

Multiply all the numerators one into another for a new numerator, and the denominators one into another for a new denominator, so have you the single fraction sought.

So if $\frac{3}{4}$ of $\frac{5}{9}$ were reduced into a single fraction, it would be $\frac{15}{36}$; and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ would be $\frac{6}{24}$ or $\frac{1}{4}$.

And thus much shall suffice for Reduction.

Addition in Vulgar FRACTIONS.

Addition of Fractions, (after they are reduced or abbreviated, if occasion be), is very easy, and consisteth only in adding together their numerators, the total of which is the numerator to the given denominator, and is the sum of the fractions sought. And this happens either in fractions with fractions, whole numbers with fractions, mixed with fractions, mixed numbers with mixed, or mixed with integers.

I. *First*, Fractions with fractions.

EXAMPLE.

Add $\frac{2}{7}$ to $\frac{3}{7}$ the sum is $\frac{5}{7}$, and the sum of $\frac{5}{11}$, $\frac{2}{11}$, and $\frac{4}{11}$, is $\frac{11}{11}$, or 1.

So if $\frac{2}{13}$ and $\frac{5}{9}$ were to be added, their sum would be found to be $\frac{83}{117}$, for $\frac{2}{13}$ and $\frac{5}{9}$ will be reduced into $\frac{18}{117}$, and $\frac{65}{117}$, and then by addition will be $\frac{83}{117}$.

And if $\frac{3}{5}$ of a pound were to be added to $\frac{3}{7}$ of a shilling, the sum will be found to be $\frac{87}{140}$.

First, $\frac{3}{7}$ of a shilling, reduced into the fraction of a pound, will be $\frac{3}{140}$. Secondly, $\frac{3}{5}$ and $\frac{3}{140}$ will be reduced into $\frac{420}{700}$ and $\frac{15}{700}$; and by addition, Thirdly, the sum is $\frac{435}{700}$; and, Fourthly, by abbreviation, into $\frac{87}{140}$.

II. In whole numbers with fractions.

EX-

EXAMPLES.

Add 7 and $\frac{3}{5}$ together, the sum will be $\frac{38}{5}$.

So if 5, 17, $\frac{5}{12}$ and $\frac{3}{7}$ of $\frac{9}{13}$ were to be added, the sum would be $\frac{24803}{1092}$.

First, $\frac{3}{7}$ of $\frac{9}{13}$ will be reduced into this single fraction $\frac{27}{91}$.

Secondly, By reduction $\frac{5}{12}$ and $\frac{27}{91}$ will become $\frac{455}{1092}$ and $\frac{324}{1092}$.

Thirdly, By addition the sum of those two is $\frac{779}{1092}$.

Fourthly, By addition, 5 and 17 makes 22.

Lastly, 22 added to $\frac{779}{1092}$, is $\frac{24803}{1092}$.

And 11 *l.* and $\frac{5}{8}$ of a shilling added together, is $\frac{1325}{120}$.

III. In mixed numbers with integers.

EXAMPLES.

Add 7 and $5\frac{3}{5}$ together, the sum is $12\frac{3}{5}$ or $\frac{63}{5}$.

So if 3, 9, $2\frac{11}{15}$, and $5\frac{2}{7}$ were added, the sum will be $\frac{2102}{105}$.

For $2\frac{11}{15}$ and $5\frac{2}{7}$ will be $\frac{41}{15}$ and $\frac{37}{7}$, and these two again will become $\frac{287}{105}$ and $\frac{555}{105}$; and these added to the sum of 3 and 9, (*viz.*) 12, become $12\frac{842}{105}$, or $\frac{2102}{105}$.

IV. In mixed numbers with mixed.

EXAMPLES.

Add $2\frac{3}{5}$ to $7\frac{5}{9}$, the sum will be $4\frac{57}{45}$; for $2\frac{3}{5}$ and $7\frac{5}{9}$ will be reduced to $\frac{117}{45}$ and $\frac{340}{45}$, and by addition into $\frac{457}{45}$.

And if $5\frac{4}{9}$ were to be added $4\frac{3}{9}$ the sum will be $9\frac{7}{9}$; these being of like bases, are very easy, being performed without any reduction, by addition only.

V. Fifthly, and lastly, in mixed numbers and fractions.

EXAMPLES.

Add $\frac{3}{5}$ and $7\frac{2}{5}$ into one sum, *Facit* 8; for $\frac{3}{5}$ and $\frac{2}{5}$ is $\frac{5}{5}$, or 1, and 7 and 1 is 8.

So the sum of $2\frac{5}{7}$, $13\frac{2}{9}$, $\frac{3}{11}$, and $\frac{27}{25}$, will be $\frac{295378}{17325}$ which will be equal to $17\frac{853}{17325}$.

And thus of any other.

Subtraction in Vulgar FRACTIONS.

As in *Addition* we took the sum of numerators, after the work of *Reduction* (if any) was performed; so in *Subtraction* (after such work, if need be) we must take the

difference of the numerators, observing all the cases in *Addition*. Of which in their order.

I. *First*, Where both are fractions.

EXAMPLES.

So if the difference of $\frac{4}{7}$ and $\frac{3}{7}$ were required, it would, by subtracting the less numerator from the greater, be found to be $\frac{1}{7}$.

Again, if the difference betwixt $\frac{2}{13}$ and $\frac{5}{9}$ was sought, these two fractions, because of unequal bases, by *Reduction*, would become $\frac{18}{117}$, and $\frac{65}{117}$; and then, by subtracting the less numerator from the greater, the difference sought will be $\frac{47}{117}$.

II. *Secondly*, Where one is an integer, and the other a fraction.

EXAMPLE.

If the difference betwixt 7 and $\frac{3}{5}$ were sought, it would be $6\frac{2}{5}$, for 1 from 7, rest 6; which 1 reduced into a fraction whose denominator is 5, is $\frac{5}{5}$; then $\frac{3}{5}$ from $\frac{5}{5}$ rest $\frac{2}{5}$, in all $6\frac{2}{5}$; and the difference of 11 pounds and $\frac{5}{6}$ of a shilling, will be $10\,l.\ \frac{23}{24}$.

III. *Thirdly*, Where one is an integer, and the other a mixed number.

EXAMPLE.

From 7 let us subtract 2, and $\frac{3}{5}$, the remainder will be $4\frac{2}{5}$. So if from 13 we subtract $11\frac{3}{7}$, the remainder is $1\frac{4}{7}$.

IV. *Fourthly*, Where both are mixed numbers.

EXAMPLE.

From $16\frac{2}{3}$ subtract $11\frac{9}{13}$, the remainder is $\frac{194}{39}$, or $4\frac{38}{39}$. And from 13 pounds and $\frac{5}{18}$ subtract $8\frac{7}{11}$ shillings, rest $12\frac{5}{8}\frac{42}{72}$ of a pound.

V. *Fifthly* and *lastly*, Where one is a mixed number, and the other a fraction.

EXAMPLE.

From $7\frac{2}{5}$ subtract $\frac{3}{5}$, the remainder will be $6\frac{4}{5}$.

From $16\frac{9}{11}$ subtract $\frac{21}{95}$, the remainder is $\frac{17344}{1045} = \frac{624}{1045}$; by taking the $\frac{21}{95}$ from $\frac{9}{11}$, and the remainder is $16\frac{624}{1045}$.

Here it may be observed, That if one cannot distinguish the greater of two fractions, by reducing them both into equal bases the greater or lesser is easily known.

In-

Multiplication in Vulgar FRACTIONS.

In multiplication of Vulgar Fractions, reduce mixed numbers into improper fractions; whole numbers, like fractions, and compound fractions into single, abbreviating where occasion is; then the rule is, Multiply the numerators together for a new numerator, and the denominators together for a new denominator, which numerator and denominator is the product sought.

EXAMPLES.

I. Let us multiply $\frac{4}{13}$ by $\frac{2}{9}$, the product will be $\frac{8}{117}$, for 2 times 4 is 8, and 9 times 13 is 117, which placed fractional-wise, is the product sought.

As if it were required to multiply 2 *s.* 6 *d.* by 2 *s.* 6 *d.* as the fraction of a pound, 2 *s.* 6 *d.* being $\frac{1}{8}$ of a pound: multiply $\frac{1}{8}$ by $\frac{1}{8}$, *Facit* $\frac{1}{64}$ of a pound, equal to 0 *l.* 0 *s.* 3 *d.* 3 *q.* By which it is evident, that the multiplication of fractions decreases the value in the same proportion as whole numbers increase it, as is intimated further in multiplication of decimals.

So $\frac{1}{2}$ multiplied by $\frac{1}{2}$ becomes $\frac{1}{4}$. See this demonstrated in Mr *Leybourn*'s *Cursus Mathematicus*, pag. 38.

II. If one be an integer and the other a fraction, as if we would multiply $\frac{2}{5}$ by 7, the product will be $\frac{14}{5}$, $2\frac{4}{5}$, for 7 made like a fraction is $\frac{7}{1}$; then as before.

So if $\frac{5}{18}$ were to be multiplied by 12, the product would be $\frac{10}{3}$, or $3\frac{1}{3}$; for $\frac{5}{18}$ and $\frac{12}{1}$, may, by abbreviating the cross terms 12 and 18, be brought into $\frac{2}{3}$ and $\frac{5}{1}$; and by multiplication into $\frac{10}{3}$; or $3\frac{1}{3}$.

III. If both be mixed numbers, as if $2\frac{3}{5}$ must be multiplied by $5\frac{3}{7}$, the product would be $\frac{494}{35}$, or $14\frac{4}{35}$.

So if 21 *l.* 16 *s.* and 9 *d.* were to be multiplied by 3 *l.* 12 *s.* 6 *d.* the product would be $\frac{50663}{640}$ equal to 79 *l.* $\frac{103}{640}$; for first 21 *l.* 16 *s.* 9 *d.* would be made $21\frac{67}{80}$, and 3 *l.* 12 *s.* 6 *d.* would be $3\frac{5}{8}$, and those two again would become $\frac{1747}{80}$ and $\frac{29}{8}$, and then by multiplication would be $\frac{50663}{640}$, or 79 *l.* $\frac{103}{640}$.

IV. If you would take the parts of any fraction, or mixed number, it is easily done by multiplication: Thus if you would take $\frac{1}{3}$ of $\frac{3}{4}$, the same would be $\frac{1}{4}$; for $\frac{1}{3}$ multiplied by $\frac{3}{4}$ produceth $\frac{3}{12}$, or $\frac{1}{4}$, the part sought: So $\frac{11}{13}$ of $15\frac{3}{7}$ will be $13\frac{5}{91}$, which is nothing but the product of one by the other.

Division in Vulgar FRACTIONS.

In *Division of Vulgar Fractions*, as in *Multiplication*, we must reduce mixed numbers into improper fractions; whole numbers like fractions, and compound fractions into single, abbreviating where may be needful; and then the rule will be, To multiply the denominator of the divisor by the numerator of the dividend, for the numerator of the quotient; and the numerator of the divisor by the denominator of the dividend, for the denominator of the quotient, and your work is finished; or invert the divisor, then work as in multiplication.

EXAMPLES.

I. Let it be required to divide $\frac{8}{117}$ by $\frac{4}{13}$, the quotient will be found to be $\frac{2}{9}$; for 13 times 8 is 104, for a numerator, and 4 times 117 is 468, for a denominator; which fraction abbreviated by 4, becomes $\frac{26}{117}$, and that again by 13, becomes $\frac{2}{9}$, as in the work.

The work.

$$\frac{4}{13})\frac{8}{117}(\frac{104}{468}|\frac{26}{117}|\frac{2}{9}$$

Divide $\frac{36}{114}$ by $\frac{6}{19}$, the quotient will be $\frac{684}{684}$ equal to 1 integer; by which it doth appear the fractions were equal one to the other, and had been the same as if I had divided $\frac{6}{19}$ by $\frac{6}{19}$; for any fraction divided by itself quotes unity.

$$\frac{6}{19})\frac{36}{114}(\frac{684}{684}=1$$

II. If one be an integer and the other a fraction, as if we would divide $\frac{2}{5}$ by 7, the quotient would be $\frac{2}{35}$.

$$\frac{7}{1})\frac{2}{5}(\frac{2}{35}$$

But if you must divide 7 by $\frac{2}{5}$, the quotient will be $\frac{35}{2}$ or $17\frac{1}{2}$.

$$\frac{2}{5})\frac{7}{1}(\frac{35}{2}=17\frac{1}{2}$$

III. If both be mixed numbers, or one a fraction and the other a mixed number, as if $5\frac{3}{7}$ must be divided by $2\frac{3}{5}$, the quotient would be $\frac{190}{91}$, or $2\frac{8}{91}$; for $2\frac{3}{5}$ would by reduction become $\frac{13}{5}$, and $5\frac{3}{7}$, would be $\frac{38}{7}$, which would quote $\frac{190}{91}$, or $2\frac{8}{91}$.

See the work.

$$\frac{13}{5})\frac{38}{7}(\frac{190}{91}=2\frac{8}{91}$$

Divide $\frac{21}{37}$ by $5\frac{7}{11}$, the quotient would be $\frac{231}{2292}$.

You may note, If a fraction be divided by a whole number, the denominator multiplied by that number, the product is the new denominator, and the numerator the same as before.

The Rule of Three in Vulgar FRACTIONS.

In the *Rule of Three*, or *Golden Rule in Vulgar Fractions*, if any of your terms be integers, mixed or compound fractions, they must be reduced, as hath been before shewn; then stating your question, as shewn in the *Golden Rule* aforegoing, and multiplying and dividing, as in *Multiplication* and *Division of Vulgar Fractions*, your work is finished, and the quotient gives your answer.

E X A M P L E S.

I. If $\frac{2}{3}$ of a *yard* cost $\frac{5}{8}$ of a pound, what will $\frac{11}{17}$ of a *yard* cost?

Thus stated

$$\begin{array}{llllll} Yd. & l. & Yd. & \frac{11}{8} & \frac{17}{8} \\ \text{If } \frac{2}{3} & : \frac{5}{8} & :: \frac{11}{17} & \frac{55}{136} & 136 \end{array}$$

$\frac{2}{3}$) $\frac{55}{136}$ ($\frac{165}{272}$ equal to 12 *s.* 1 *d.* 2 *q.* $\frac{6}{17}$.

II. If $\frac{3}{4}$ of a yard of velvet cost 7 shillings and 3 pence, what will 9 yards and $\frac{1}{3}$ cost?

Your numbers reduced and stated as afore-taught, appear as in the work.

$$\begin{array}{lll} Yd. & l. & Yd. \\ \text{If } \frac{3}{4} & : \frac{87}{240} & :: \frac{28}{3} \end{array}$$

Contracted thus; If $\frac{3}{1} : \frac{29}{15} :: \frac{7}{1}$. *Facit* $\frac{203}{45}$ *l.* = 4 *l.* 10 *s.* 2 *d.* 2 *q.* $\frac{2}{3}$.

In this question, seeing the numerators of the two last terms, and their altern denominators, may be severally abbreviated; one, (*viz.*) the number of the last term, and the denominator of the 2d by 4, and the number of the 2d term and denominator of the last by 3; the contracted terms of which are $\frac{29}{60}$ for the second term, and $\frac{7}{1}$ for the third or last term, then the work will stand thus; If $\frac{3}{4} : \frac{29}{60} :: \frac{7}{1}$. And seeing again the denominator of the last term is an unit, and the denominator of the two first terms may be abbreviated by 4, after which the 3 terms offer themselves thus; If $\frac{3}{1} : \frac{29}{15} :: \frac{7}{1}$. And the 4th term is easily found by multiplying the numerator of the quotient and the numerator of the 1st term by the denominator of the 2d, for the denominator of the said quotient; so the 4th term sought will be $\frac{203}{45}$, equal to 4 *l.* $\frac{23}{45}$, or 4 *l.* 10 *s.* 2 *d.* 2 *q.* $\frac{2}{3}$, as in the work.

III. If $\frac{3}{4}$ of a pound of flax cost 8 pence, what will 1 pound cost? *Facit* 10 *d.* $\frac{2}{3}$.

The work.

lb. d.

If $\frac{3}{4}$: $\frac{8}{1}$: : $\frac{1}{1}$

$\frac{3}{4}$) $\frac{8}{1}$ ($\frac{32}{3}$ = 10 *d.* $\frac{2}{3}$.

If either of the extremes be a fraction and the other not, as here, reduce it to a like denomination, cancel the denominators, and work as in integers. So if 3 *q.* : 8 *d.* : : 4 *q.* *Facit* 10 *d.* $\frac{2}{3}$ as before.

IV. If 3 men do a piece of work in $4\frac{1}{2}$ hours, in how many hours shall 10 men do the same work? *Facit* 1 hour 21 min.

The work.

M. H. M.

If $\frac{3}{1}$: $\frac{9}{2}$: : $\frac{10}{1}$

$\frac{10}{1}$) $\frac{27}{2}$ ($\frac{27}{20}$ equal to one hour and $\frac{7}{20}$.

In this question the last term was my divisor, because more men require less time.

V. If the penny white loaf weigh 7 ounces, when a bushel of wheat costs 5 *s.* 6 *d.* what is the bushel worth, when the penny white loaf weighs but 2 ounces and $\frac{1}{2}$? *Ans.* 15 *s.* $\frac{2}{5}$.

l. s. l.

Say, If $\frac{7}{12}$: $\frac{11}{2}$: : $\frac{5}{24}$

$\frac{5}{24}$) $\frac{77}{24}$ ($\frac{77}{5}$ = 15 *s.* $\frac{2}{5}$ the answer.

Seeing the denominators of the dividend and divisor are both the same, throw them away, the numerator of the dividend is the numerator of the quotient, and the numerator of the divisor, denominator thereto.

Double Rule of Three in Vulgar FRACTIONS.

Take a question or two in the *Double Golden Rule in Vulgar Fractions*, and so finish *Vulgar Fractions*.

Question I. If 13 *l.* 6 *s.* 8 *d.* in $\frac{3}{4}$ of a year gain 1 *l.* $\frac{1}{12}$ what will 50 *l.* gain in 5 months?

l. l. l.

First I say, If $\frac{40}{3}$: $\frac{13}{12}$: : $\frac{50}{1}$. *Facit* 4 *l.* 1 *s.* 3 *d.*

$\frac{40}{3}$) $\frac{650}{2}$ ($\frac{65}{16}$ = $4\frac{1}{16}$.

Yd. l. Yd.

Say again, If $\frac{3}{4}$: $\frac{65}{16}$: : $\frac{5}{12}$. *Facit* 2 *l.* 5 *s.* 1 *d.* 2 *q* $\frac{2}{3}$

$\frac{3}{4}$) $\frac{325}{192}$ ($\frac{325}{144}$ = 2 *l.* $\frac{37}{144}$.

Question II. If 50 pounds in five months gain 2 *l.* 5 *s.* 1 *d.*

1 *d.* 2 *q.* $\frac{2}{3}$, or 2 *l.* $\frac{37}{144}$, what time will 13 *l.* 6 *s.* 8 *d.* or 13 *l.* $\frac{1}{3}$ require to gain 1 *l.* 1 *s.* 3 *d.* or 1 *l.* $\frac{1}{12}$?

l. *yr.* *l.* *yr.*

First say, If $\frac{50}{1} : \frac{5}{12} :: \frac{40}{3}$, *Facit* $\frac{25}{16}$, or 1 year and $\frac{9}{16}$. Say again, If $\frac{325}{144} : \frac{25}{16} :: \frac{13}{12}$, *Facit* $\frac{144}{192}$, or $\frac{3}{4}$ of a year, or 9 months.

Note, The former proportion was inverse, and the 2d was direct. This shall suffice for the *Golden Rule in Fractions*.

Questions to exercise Vulgar FRACTIONS.

Question I. The difference of two numbers is 21$\frac{75}{97}$, the lesser 17$\frac{3}{7}$, what is the greater? *Answer* 39$\frac{137}{679}$, found by *Addition*.

Question II. There is in 3 bags 56 *lb.* $\frac{3}{8}$; in the first bag 12 pounds and $\frac{5}{8}$; in the 2d, 21$\frac{7}{12}$, what is in the third bag? *Answer* 22$\frac{47}{120}$, found by *Addition* and *Subtraction*.

Question III. What number added to 11$\frac{5}{7}$ will produce 36$\frac{337}{616}$? *Answer* 24$\frac{71}{77}$, found by *Subtraction*.

Question IV. What is $\frac{7}{11}$ of $\frac{19}{37}$? *Answer* $\frac{133}{407}$, found by *Multiplication*.

Question V. What number multiplied by $\frac{3}{7}$ produceth 11$\frac{9}{17}$? *Answer* 26$\frac{46}{51}$, found by *Division*.

Arith-

Arithmetical Progression.

PROGRESSION consisteth of two parts, *Arithmetical* and *Geometrical*.

Arithmetical Progression is, when a rank of numbers above two, increase or decrease equally, by the continual addition or subtraction of some equal number.

So 1, 3, 5, 7, 9, 11, and 42, 35, 28, 21, 14, 7, are two ranks of numbers in *Arithmetical Progression*; the first increasing by the continual addition of two, and the second decreasing by the continual subtraction of seven; and so of any other.

In *Arithmetical Progression* these five things are to be considered:

(1.) The first term commonly the least term.
(2.) The last term commonly the greatest.
(3.) The number of terms.
(4.) The equal difference or common excess.
(5.) The sum of all the terms, or total aggregate.

Any three of these five being given, the other two may be found; which will admit of 20 propositions, as may be seen in Mr *Oughtred's Clavis Mathematica*, Chap. 29. Prob. 4. either in the *Latin*, or late *English* translation. But we shall not concern ourselves with them all, but only such as may be of common use.

But in the first place we will lay down some *Theorems*, for the better understanding of what follows after.

THEOREM I.

Any term of an *Arithmetical Progression* contains the first (that is, the least) term, together with the product of the common excess and number of terms before it.

So

So in this *Arithmetical Progression*, 2, 5, 8, 11, 14, 17, the term 17 is equal to the first term 2, added to the product of 5, the preceding number of terms by 3, the common excess.

Hence may arise this *Corollary*;

That if the common excess be multiplied by the number of terms *minus unity*, and to the product the least term be added, the sum is equal to the greatest.

THEOREM II.

If three numbers be in *Arithmetical Progression*, the double of the mean is equal to the sum of the extremes.

So 2, 4, 6, are three numbers in *Arithmetical Progression*, and the double of the mean 4, is equal to the sum of the two extremes 2 and 6.

THEOREM III.

If four numbers are in *Arithmetical Progression*, the sums of the two means is equal to the sum of the two extremes.

So 7, 11, 15, 19, are four numbers in *Arithmetical Progression*, and the sum of the two means 11 and 15, is equal to the sum of the two extremes 7 and 19.

THEOREM IV.

In an *Arithmetical Progression*, any term doubled is equal to the sum of any other two terms equally distant.

EXAMPLE.

3, 8, 13, 18, (23), 28, 33, 38, 43.

In the annexed *Arithmetical Progression*, the double of 23 is equal to the sum of 3 and 43, or of 8 and 38, or of 13 and 33, or of 18 and 28, all numbers which are equally distant.

THEOREM V.

In any *Arithmetical Progression*, the sum of any two terms is equal to the sum of any other two terms of like distance from them.

EXAM-

EXAMPLE.

2, 5, 8, 11, 14, 17, 20, 23, 26, 29.

In the annexed *progression*, the sum of 14 and 23, is equal to the sum of 8 and 29, or of 11 and 26, or of 17 and 20; all being alike distant.

THEOREM VI.

1. In any *Arithmetical Progression* whatsoever, if the sum of the greatest and least terms be multiplied by the numbers of terms, and the product divided by 2, the quotient is equal to the sum of all the terms.

2. Or if the sum of the greatest and least be multiplied by $\frac{1}{2}$ the number of terms, the product is equal to the sum of all the terms.

3. Or if the half sum of the greatest and least terms be multiplied by the number of terms, the product is equal to the sum of all the terms.

4. Or the middle number (when the progression is odd) multiplied by the number of terms, gives the sum of all the terms.

EXAMPLE.

3, 6, 9, 12, 15, 18, 21.

(1.)	(2.)	(3.)	(4.)
21	21	$12\frac{1}{2}$ Sum	12
3	3	7	7
24	24	84 Sum.	84 Sum.
7	3.5		
168	120		
$\frac{1}{2}$ 84 sum	72		
	84|0 Sum		

Every way the same.

THEO

THEOREM VII.

In a progression of natural numbers, as 1, 2, 3, 4, *&c.* if the last term be multiplied by the next greater, one half of the product is equal to the sum of the whole progression.

1, 2, 3, 4, 5, 6, 7.

So the product of 7 by the next greater 8, gives 56; one half of which is 28, which is the sum of the whole progression.

THEOREM VIII.

In a *natural progression* of odd numbers, as 1, 3, 5, 7, *&c.* the sum of the whole is equal to the square of the number of terms.

1, 3, 5, 7, 9, 11, 13.

The number of terms 7 squared is 49, the sum of the whole.

THEOREM IX.

In a *natural progression* of even numbers, the sum of the whole is equal to the product of the number of terms by the number of terms *plus unity*.

2, 4, 6, 8, 10, 12.

Here the number of terms is 6, which multiplied by 7, gives 42, equal the sum of the whole.

THEOREM X.

In any *arithmetical progression* whatsoever, if from the greatest term the least be taken, the remainder divided by the common excess, and to the quotient adding unity, you have the number of terms.

2, 4, 6, 8, 10, 12, 14.

From 14 ſubtracting 2, reſt 12, divided by the common exceſs 2, gives 6, to which add unity, makes 7, equal to the number of terms.

THEOREM XI.

In any *arithmetical progreſſion* whatſoever, if from the laſt term the firſt term be ſubtracted, and the remainder divided by the number of terms *minus unity*, the quotient is the common exceſs.

3, 5, 7, 9, 11, 13.

From 13 ſubtracting 3, reſt 10, which divided by 5, one leſs than the number of terms, quotes 2, the common exceſs.

Let theſe *theorems* ſuffice; we will now return to where we left, in having any three of the five given, to find the other two.

PROP. I.

The firſt or leaſt term, the laſt or greateſt term, and the number of terms being given, to find the common exceſs.

Or, the firſt, ſecond and third given, to find the fourth.

RULE.

From the ſecond ſubtract the firſt, the remainder, divided by the third *minus unity*, quotes the fourth.

By *theorem* the firſt, and the *corollary*.

EXAMPLE.

A man had 12 ſons; the youngeſt was 3 years old, and the elder was 58; they increaſed in *arithmetical progreſſion*; what was the common difference of their ages?

The

The 2d 58
The 1st 3

The 3d—1=11) 55 (5
55

0

Answer, They increase by five years.

PROP. II.

The first, second and third given, to find the fifth.

RULE.

Multiply the half sum of the first and second by the third, the product is the fifth.

By *theorem* the 6th, and third way.

EXAMPLE.

A man buys 17 yards of kersey in *arithmetical progression*; for the first yard he paid two shillings, or 24 pence, and for the last yard ten shillings, or 120 pence; what did the whole amount to?

The first term	24
The last term	120
Sum=	144
½ Sum	72
Multiply by the num. of terms	17
	504
	72
	1224 the *Answ.* in pence.
$\frac{1}{12}$	10|2

Answ. 5 *l.* 2 *s.*

PROP. III.

The firſt, ſecond and fourth given, to find out the third.

RULE.

From the ſecond ſubtract the firſt, the remainder divided by the fourth, the quotient *plus unity*, is equal to the third. By *theorem* the firſt and the *corollary*.

EXAMPLE.

A man going a journey, his firſt day's travel was five miles, his laſt day's travel was 35 miles, he increaſed his journey every day three miles; how many days did he travel? *Anſw.* He travelled 11 days.

The laſt	35
The firſt	5

3) 30 (10+1=11
   30
   ——
   00
   00
   ——
   00

PROP. IV.

The ſecond, third and fourth given, to find the firſt.

RULE.

Multiply the fourth by the third *minus unity*, the product ſubtracted from the ſecond leaves the firſt.

EXAMPLE.

A man in 6 days went to *London* from *Mancheſter*, every day's journey was greater than the day before by

by four miles, his last day's journey was 40 miles; what was the first? *Ans.* 20 miles.

The 4th is	4
The 3d—1 is	5

The product 20, which subtract from the second 40, leaves 20, the first day's journey.

40
20

20 the *Answer.*

PROP. V.

The first, third and fourth given, to find out the fifth.

RULE.

From the product of the third into the fourth, subtract the fourth, and to the remainder add the double of the first, ½ the product of that sum multiplied by the third, gives the fifth.

EXAMPLE.

An hundred eggs are placed in a right line, a yard distant one from another, and the first a yard distant from a basket: it is required to know how far one must go before he brings the eggs one by one into the basket without breaking any?

The third	100
The fourth	2
	200
The fourth	2
Rest	198
The doub. of the 1st	4
Multip. by the 3d=	20200
½	10100

Sir Jonas Moore *makes the distance run but* 10000 *yards, which is too little by* 100 *yards.* Moore's Arith. p. 324. Last edition.

Ans. 10100 yards, or 5 miles and ¾ wanting 20 yards.

PROP. VI.

The second, third and fifth given, to find the first.

RULE.

Divide the fifth by the third, and from the quotient subtract $\frac{1}{2}$ the product of the fourth into the third *minus unity:* the remainder is the first.

EXAMPLE.

A man is to receive 300 pounds at 12 several payments, each payment to exceed the former by four pounds; he is willing to bestow the first payment on any one that can tell him what it is.

What must the arithmetician have for his pains?

12)	300	(25	11
	24	22	4
	60	3	44
	60		$\frac{1}{2}$ 22
	00		

Ans. Three pounds are the workman's wages.

Many more propositions might have been added, but the foregoing are sufficient in most cases; wherefore we will begin with *geometrical progression.*

Geometrical PROGRESSION.

GEometrical *Progression* is, when a rank of numbers above two increase or decrease by an equal *ratio*; that is, by the continual multiplication or division of some equal number.

So 2, 4, 8, 16, 32, 64, and 1215, 405, 135, 45, 15, 5, are two ranks of numbers in geometrical progression, the first ascending and increasing, by continual multiplying the foregoing term or number by 2, or by a double *ratio*.

And the second descending or decreasing, by continually dividing the preceding term by 3, or in a triple *ratio*.

In any geometrical progression, the same things are to be considered as in arithmetical progression: as first, the first term commonly the least. Secondly, the last term commonly the greatest. Thirdly, the number of terms. Fourthly, the *ratio* or common excess. Fifthly, the total sum of all the terms.

But before we mention any propositions, we will annex some theorems, as preparatory thereunto.

THEOREM I.

If three numbers be in geometrical progression, the square of the mean or middle number is equal to the product of the two extremes.

EXAMPLE.

3, 9, 27, are three numbers in geometrical progression; and the square of 9, the mean is equal to the product of 27 by 3, the two extremes; and so in others.

THEOREM II.

If four numbers be in geometrical progression, the product or *rectangle* of the two means is equal to the product of the two extremes.

EXAM-

EXAMPLE.

3, 15, 75, 375, are four numbers in geometrical progreſſion, and the product of the two means (*viz.*) of 75 by 15, is equal to the product of 375 by 3.

This will likewiſe hold, if the four numbers be diſcontinued, as in theſe four numbers following, 6, 12 : : 18, 36; for the product of 36 by 6, is equal to the product of 18 by 12. And hence proceeds that excellent rule in arithmetic, called, *The Rule of Proportion*; *Rule of Three*; or, *Golden Rule.*

THEOREM III.

If any term of geometrical progreſſion be ſquared; it will be equal to the product of any other two terms of like diſtance from that term either way.

EXAMPLE.

3, 6, 12, 24, (48,) 96, 192, 384, 768,

In the annexed geometrical progreſſion, the ſquare of 48 is equal to the product of 768 by 3, or of 384 by 6, or of 192 by 12, or of 96 by 24; all being terms equally diſtant.

THEOREM IV.

In any geometrical progreſſion whatſoever, the product of the two extremes is equal to the product of any other two immediate terms of like diſtance from both.

EXAMPLE.

5, 20, 80, 320, 1280, 5120.

So in this geometrical progreſſion, the product of the two extremes 5120 by 5, is equal to the product of 1280 by 20, or of 320 by 80, all being alike diſtance from both.

THEOREM V.

Any geometrical progreſſion may be continued *ad infinitum* upwards, and aſcending by Multiplication, and downward, or deſcending by Diviſion, the *ratio* or common exceſs being given, that being your multiplicator

tiplicator upwards, and your divisor downwards; notwithstanding oftentimes the terms will not continue integral numbers, neither in the ascending or descending part thereof, as hereafter declared.

EXAMPLE.

&c. $\frac{32}{9}$, $\frac{16}{3}$, 8, 12, 18, 27, $\frac{81}{2}$, $\frac{243}{4}$, *&c.*

So in the annexed progression, if 8, 12, 18, were to be continued infinitely forward and backward, the common excess being $1\frac{1}{2}$ or $\frac{3}{2}$, suppose forwards; first, I multiply 18 by $\frac{3}{2}$, gives 27 for the next term, and 27 multiplied by $\frac{3}{2}$ gives $\frac{81}{2}$ for the next term; and here the integral parts or terms cease, and multiplying $\frac{81}{2}$ by $\frac{3}{2}$ gives $\frac{243}{4}$, and so as far as you please: then in the descending part, if I divide 8 by $\frac{3}{2}$, the quotient is $\frac{16}{3}$ for the next descending term, and that by $\frac{3}{2}$, gives $\frac{32}{9}$, and so on as far as you please.

THEOREM VI.

Any geometrical progression, where the *ratio* is *multiple*, (that is, where the greater term is exactly measured by the less), may be continued upwards *ad infinitum* in integral numbers, but downwards sometimes not so far as unity.

EXAMPLE I.

1, 2, 4, 8, 16, 32, 64, *&c.*

In the annexed progression, the *ratio* or common excess being two, by which multiplying any term, as 8 gives 16, and that by 2 produceth 32, and that by 2 gives 64, and so *ad infinitum* in integral numbers; and in descending, it will come down as far as unity; for 8 divided by 2 quotes 4, and that by 2 gives 2, and 2 by 2 gives 1, then integers cease.

EXAMPLE II.

$\frac{3}{2}$, 3, 6, 12, 24, 48, 96, *&c.*

But in this progression, though the terms may be continued upwards *ad infinitum*, as in the last, yet it will not descend so far as unity without a fraction, because

cause 3 cannot be divided by the *ratio*, which is 2, without a remainder.

THEOREM VII.

In any geometrical progression; if the *ratio* be not *multiple*, the same can neither be continued upward *ad infinitum* in integral numbers, nor downwards so far as unity.

$\frac{3}{4}$, 27, 36, 48, 64, $\frac{256}{81}$.

In the annexed progression, where the *ratio* is $\frac{3}{4}$, the terms quickly become mixed numbers, both in the ascending and descending part thereof; for seeing 64 and 27, cannot be multiplied or divided evenly by $\frac{3}{4}$, the integral terms cease.

THEOREM VIII.

In any geometrical progression, if the extremes be prime numbers one to another, the same *progression* can be continued no farther, either upwards or downwards in integral numbers; so in the last example, supposing 27 and 64 to be the extreme terms, and they being prime one to another, therefore they can be continued no farther either way in integral numbers.

Numbers are said to be prime one to another, when only unity is their common measure.

THEOREM IX.

In any geometrical progression proceeding from unity, the second term (the first term not being computed) the 4th, 6th, and 8th term, and all the following terms, whose exponents may be divided by 2, are square numbers: the 3d, 6th, 9th, and all the following terms, whose exponents may be divided by 3, are cube numbers. The 6th, 12th, 18th, and the following terms, whose

Exponents are a series of natural numbers proceeding from unity, shewing the places of the terms of the progression.

whose exponents may be divided by 6, are both square and cube numbers. The 5th, 7th, 11th, 13th, and all the following terms, whose exponents are prime numbers, are neither square nor cube numbers.

EXAMPLE.

Numbers are said to be prime, which unity only measureth.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
1 . 2 . 4 . 8 . 16 . 32 . 64 . 128 . 256

This example needs no explication.

Note, That in this and some following theorems, whether proceeding from unity or not, it being commodious, we have annexed their indices or exponents, placing a cipher over the first term of the progression, whereby it may be seen how far any term is distant from unity, or from the first term, if not unity.

THEOREM X.

In any geometrical progression proceeding from unity, if any term be squared or multiplied by itself, it will produce any term of the same progression doubly distant from unity.

EXAMPLE.

0 . 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8
1 . 2 . 4 . 8 . 16 . 32 . 64 . 128 . 256

So in this progression, the square of 8, the 3d term, is equal to 64, which is the 6th term, or doubly distant from the 1st, or unity.

THEOREM XI.

In any geometrical progression proceeding from unity, the rectangle of any two terms is equal to that term of the same progression, signified by the sum of the others exponents.

EX-

EXAMPLE.

0 .	1 .	2 .	3 .	4 .	5 .	6 .	7
1 .	3 .	9 .	27 .	81 .	243 .	729 .	2187

In this progression the product of the 3d and 4th terms (*viz.*) of 81 by 27, or of the 5th and 2d terms (*viz.*) or of 243 by 9, is equal to the 7th term of the same progression, which is 2187, because the sum of either of their exponents make 7.

THEOREM XII.

In any geometrical progression, not proceeding from unity, if any term be squared or multiplied by itself, and the product divided by the first or least term, the quotient gives a term doubly distant from the first.

EXAMPLE.

0 .	1 .	2 .	3 .	4 .	5 .	6 .	7 .	8
3 .	6 .	12 .	24 .	48 .	96 .	192 .	384 .	768

In the annexed progression, if the 4th term (*viz.* 48) be squared and divided by the first term 3, the quote is 768, which is the 8th term and doubly distant from the first.

THEOREM XIII.

In any geometrical progression, not proceeding from unity, if any two terms be multiplied together, and the product divided by the least or first term, the quotient will be equal to that term signified by the sum of the others exponents.

EXAMPLE.

0 .	1 .	2 .	3 .	4 .	5 .	6 .	7
2 .	6 .	18 .	54 .	162 .	486 .	1458 .	4374

In this progression, if the 2d and the 5th be multiplied together, and the product divided by the least term, the quotient will be equal to the 7th term, because the sum of their exponents make 7. *Note,*

These

These four last *theorems* are useful in finding any following term of a geometrical progression, without producing all immediate terms.

THEOREM XIV.

In any finite geometrical progression, where the *ratio* is double, the difference of the greatest and least term is equal to the sum of all the terms, except the greatest.

EXAMPLE.

3 . 6 . 12 . 24 . 48 . 96 . 192

In this progression, if from the greatest term 192, we take the least term 3, the remainder 189 is the sum of all, except the greatest.

THEOREM XV.

In any finite geometrical progression it holds,
As the ratio, or common excess, minus unity:
Is to unity : :
So is the difference of the greatest and least term:
To the sum of all, except the greatest.

EXAMPLE.

3 . 9 . 27 . 81 . 243 . 729 . 2187

So in the annexed progression, I say,
As the ratio, minus unity (*viz.*) 2:
Is to unity (*viz.*) 1 : :
So is 2184, the difference of the greatest and least:
To 1092, the sum of all the rest.

COROLLARY.

Hence it follows, that if the ratio of any geometrical progression be double, the difference of the greatest and least term is equal to all the rest: if the ratio be triple, the excess or difference is double the sum of all the rest: if quadruple, triple: if quintuple, quadruple; and so on.

THEOREM XVI.

In any finite geometrical progression, it holds,
As the difference of the two greatest terms:
Is to the greatest ::
So is the greatest minus the least:
To the total sum of all, excepting the least.

EXAMPLE.

5 . 10 . 20 . 40 . 80 . 160

In this progression, As 80 : to 160 :: So 155 : to 310, is the sum of all but the least.

THEOREM XVII.

In any geometrical progression whatsoever, decreasing and continued *ad infinitum*, it holds,
As the common difference minus unity :
Is to unity ::
So is the first or greatest term :
To the sum of all the following terms, *in infinitum.*

EXAMPLE.

162 . 54 . 18 . 6 . 2 . $\frac{2}{3}$. $\frac{2}{9}$. $\frac{2}{27}$. $\frac{2}{81}$. $\frac{2}{243}$, *&c.*

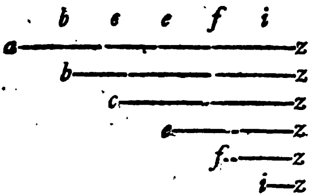
Let the first or greatest term of an infinite decreasing progression be 162, and let the ratio be triple; then will the terms descend as in the *example*: For 162 divided by 3, gives 54; and 54 by 3, quotes 18, and so on, as in the table; and still further, *ad infinitum.* And it will follow,

That as 2: (*viz.* the ratio minus unity) is to unity, or 1 :: So is 162: the greatest or first term to 81, which is the sum of all the remaining terms, *ad infinitum.* This appears plain by the 15th *theorem.* Wherefore in any geometrical progression descending in any given proportion *ad infinitum,* the least term vanisheth; and therefore it holds as in this *theorem.*

This may appear strange to many, how it should be

be possible to give the sum of an infinite progression in numbers; whereas, if the work were actually begun, and the terms continued, it would after a thousand years labour, and after thousands of millions of terms, be never nearer finishing. And yet that the sum of this infinite progression should so easily be found, it appeared to me at first as a notion (if I may so speak) almost divine; but that it may be performed, take the following demonstration.

Let there be several continual proportionals, as *az*, *bz*, *cz*, &c. all which transfer into the first *az*; then will *ab*, *bc*, *ce*, *ef*, &c. be proportional differences, which together with the last quantity *iz*, are equal to the first *az*: because if that proportional number be continued downward *ad infinitum*, the last quantity, as said before, vanisheth; therefore the infinite proportional differences are equal to the whole line *az*. Further,



because it holds, that as *az* to *bz*, so *bz* to *cz*, and so on; and by division, as *ab* to *bz*, so *bc* to *cz*; and by conversion, as *ab* the first difference, to *bz* the first quantity, so *bc* the second difference, to *bz* the second quantity, and so on; therefore as *ab* the first difference to *az* the first quantity, so all the differences to all the quantities, that is, to the whole sum of all the infinite quantities; which was to be demonstrated.

Hence may arise this *corollary*:

That the first term of an infinite descending geometrical progression, where the ratio is double, is equal to the sum of all the rest *ad infinitum*.

But if the ratio be triple, the first term is double the sum of all the rest; in a quadruple progression, triple; in a quintuple, one quadruple; and so on. Hence we

may demonstrate unity not to be the beginning of numbers.

THEOREM XVIII.

In any geometrical progression continued downward *ad infinitum*, it will be, As the difference of the two first, or the greatest terms : Is to the second term : : So is the first or greatest term : To the sum of all the rest *ad infinitum*. So in the last example the difference of the two first terms is 108, the second term is 54, the first 162.

Wherefore, as 108 : to 54 : : so is 162 : to 81, the sum of all the rest *ad infinitum*.

Wherefore the difference of the two first terms, the first term, and the sum of the infinite terms, are continual proportionals, as was demonstrated in the last. Hence may arise this *corollary* :

That when the two first, or greatest terms, differ only by unity, the square of the first term is equal to the sum of all the rest, *ad infinitum*.

Many more *theorems* might be laid down, but these are sufficient; we will only annex a *proposition* or two, and so conclude both *arithmetical* and *geometrical progression*.

PROP. I.

In any geometrical progression proceeding from unity, the ratio being known, how to find any remote term without producing all the intermediate terms.

RULE.

Find a few of the leading terms, over which place their exponents; then, by *theorem* 10, multiply the last found term by itself, which will produce a term double thereto. And this last multiplied by itself, produceth another term doubly distant again: thus do till either you have the term sought, or one that falls a little short; if so, multiply the term last found by that term answering the difference of the exponent of

of the last found term, and that sought, this last product is the term required by *theorem* XI.

EXAMPLE.

A country gentleman going to a fair to buy oxen, meets with a crafty youth, who had a company of very good oxen, in number 23. The gentleman demanding the price, was answered, He should have them for sixteen pounds the piece, one with another. The gentleman bids him fifteen pounds *per* piece, and take all. The young spark tells him, it would not be taken: but, says he, if you will give me what the last ox will come to, by doubling the whole number by a farthing, you shall have all; to which the gentleman assents. The question is, What the gentleman paid for the oxen?

Four or five of the first terms are easily got, as thus,

0 . 1 . 2 . 3 . 4 . 5 . *Exponents.*
1 . 2 . 4 . 8 . 16 . 32 . *Terms.*

Note, You need only to find that term which will answer the exponent 22, which will be the 23d term; because the exponents are less by one than the terms; for in this method we account not the first term, which the learner is desired carefully to observe.

So if I multiply the 5th term 32, by itself, it gives the 10th term 1024, by *theorem* 10; which multiplied again by itself, gives 1048576, which is the 20th term from the first; but taking the first into the number, is the 21st term; and seeing I want two terms more, I multiply this last product by the term under the exponent 2, which is 4, which gives 4194304, the last term, and the price of the oxen in farthings, which makes 4369*l.* 1*s.* 4*d.* a great rate to pay for so many oxen.

PROP. II.

In any geometrical progression not proceeding from

 unity,

unity, the *ratio* being known, and the first term, to find any remote term without producing all the intermediate terms.

RULE.

Find a few of the leading terms, as in the last, and multiply the last by itself, and divide the product by the first or leading term, the quote gives a term doubly distant from the first, by *theorem* 12; and this again multiplied by itself, and divided by the first term, gives a term doubly distant from the last term. Thus do until either you have the term sought, or one that falls a little short; if so, multiply the last term found by that term answering to the difference of their exponents; and this product divided by the first or leading term, quotes the term required by *theorem* 13.

EXAMPLE.

A nobleman dying, left ten sons; to whom, and to his executor, he bequeathed his estate, in manner and form following: (*viz*) *Imprimis*, to his executor, in seeing his will performed, he left 1024 crowns; the youngest son was to have as many, and half as many as the executor; and so every son to exceed the next younger by the equal ratio of $1\frac{1}{2}$. The question is, what the eldest son's portion is?

Calculate five or six of the first terms, as here we have found five.

Executor	Youngest	Next	Next	Next	Next
1024 .	1536 .	2304 .	3456 .	5184 .	7776
0 .	1 .	2 .	3 .	4 .	5

Then multiplying the 5th, 7776 by itself, it will produce 60466176; and this divided by 1024, the first

first term, quotes the 10th term, or what the eldest son must have.

Here the *ratio* being half triple, the difference of the greatest and least is half double the sum of all the rest, excepting the greatest, by *theorem* 15.

If the whole estate had been demanded, it may be found, by *theorem* 15, to be 175099 crowns.

PROP. III.

First number, common excess, and number of places given, to find the total sum of all the places.

RULE.

Find the last term, as in the last proposition; then from the greatest term subtract the least, the remainder divided by the common excess minus unity, quotes the sum of all, excepting the greatest, by *theorem* 15; to which adding the greatest, gives the sum of the whole.

Otherwise, or in other words thus; the difference of the greatest and least terms divided by the excess minus unity, the quotient multiplied by the excess, and to the product adding the first number, the sums are equal the total.

Or, according to *corollary* in *theorem* 15, it holds, That if the *ratio* of your progression be double, the difference of the greatest and least added to the greatest, gives the total sum.

If the *ratio* be triple, $\frac{1}{2}$ the difference added to the greatest is the total. If the *ratio* be quadruple, $\frac{1}{3}$ of the difference added to the greatest is equal to the total sum of the rest. And so on.

EXAMPLE.

A merchant having a soft young man to his son, covetous enough, but scarce able to keep a shop-book, was mind to purchase for him some considerable lands

lands in the country; and bid him enquire out some handsome estate that would be sold, and he would buy it for him. The young man overjoyed at the news, runs to an inn, where he heard divers country-gentlemen lodged; and in all haste, asked them if any of them would sell their estate? Most of them were very angry, and near beating of him; but one of them being a facetious gentleman, resolved to put a trick upon him; and told him, that he had a neat hall, with a goodly park and manor, on the bank of a pleasant river, and a great number of sufficient tenants; all which, with the royalty of a fair, market-town, and the patronage of a parish-church, belonging thereto, should be his, upon condition he would lay him down one penny on the threshold of the porch-door belonging to the hall, twopence at the next door, fourpence at the 3d door, and so on, doubling till he had gone through all the doors, which were 64 in all. I will have it, saith the young man, and here is a piece in earnest; and in all haste tells his father what a purchase he had made, wishing him to give him an hundred pounds, for that he thought could not but abundantly satisfy. Thou calf, quoth his father, the King of *Spain*'s revenues would not pay what thou hast promised, if they were sold at twenty years value; much less can my estate pay for thy purchase, for it will not bring thee past the 24th threshold. The best is, the gentleman knows thee not; and if he did, he could get no advantage of one that has nought; but I will warrant thee, he is making merry with a fool's earnest. Now I desire to know what the sum to be laid down on the 24th threshold was, and what the whole, which he promised, would have come to?

First, The sum to be laid down on the 24th threshold, by *Prop.* 1. will be found to be 8388608 pence. And by this *Proposition* the sum of the whole unto the 24th threshold will be found to be 16777215 pence, equal to 69905 *l.* 1 *s.* 3 *d.* which the father must be worth,

worth, else he could not bring him over the 24th threshold.

2*dly*, The number to be laid down on the 64th threshold, by the said 1st *proposition*, 9223372036854775808 pence; and by this *proposition*, the sum of the whole, which the young man should have given to the purchase, will be 18446744073709551615 pence, equal to 76861433649456465 pounds, one shilling and three-pence: by which it may appear the gentleman spoke within compass; for this sum would purchase the yearly rent of 3843071682022823 *l.* 5 *s.* 0 *d.* $\frac{3}{4}$, which is a great deal more than the King of *Spain*'s revenues are worth: for supposing his revenues were worth one hundred millions *per ann.* (which I think no potentate of the earth is worth), it would be no more considerable to the sum last mentioned, than a red-herring of an ounce weight would be to the loading of 20 ships of 50 ton burden a-piece; which may be thus demonstrated; for allowing 20 hundred to the ton, the whole number of ounces equal to the number of so many ships of such capacity, will be 35840000; and this number of ounces multiplied by one hundred million, is only 3584000000000000, which is less than the aforegoing number by 259071682022823, which is a number large enough to load a great many more ships.

EXAMPLE II.

What will a horse cost by trebling the nails in his shoes (which are 32) with a farthing?

Answer, 96511468169 3 *l.* 13 *s.* 4 *d.*

See

See the work.

Nails 1=	1	The 8th nail=2187	
2=	3	Multiply by	2187
3=	9		
4=	27		15309
5=	81		17496
6=	243		2187
7=	729		4374
8=	2187		

4782969
Trebled is 14348907=the 16th nail.
Multiply by the same 14348907

100442349
129140163
114791256
57395628
43046721
57395628
14348907

205891132094649
Trebled 617673396283947=the 32 nail.

And the whole sum will be 926510094425920 farthings.

EXAMPLE III.

A gentleman having a coat and waste-coat with 12 dozen of silver plate buttons; a baker seeing it, and fancying it, demands of the gentleman the price thereof; who answered, If he would double every button with a barley-corn, proceeding from the first gradually to the last, it should be his. To which the baker assents.

I.

I demand the number of barley-corns, together with the worth and weight of the same?

Observe the following work.

Buttons 1=	1	the 9th but.=256	18th but.=131072
2=	2	256	131072
3=	4	———	———
4=	8	1536	262144
5=	16	1280	917504
6=	32	512	131072
7=	64	———	393216
8=	128	65536	131072
9=	256	18th but. 131072	———

17179869184
The 36 button=34359738368
34359738368
———

274877906944
206158430208
103079215104
274877906944
103079215104
240518168576
309237645312
171798691840
103079215104
137438953472
103079215105
———
1180591620717411303424
The 72d button=2361183241434822606848

Which last number must be multiplied by itself, and then by the common excess, and so you will have what the last button will amount to.

See the rest of the work.

The 72d button=2361183241434822606848
2361183241434822606848

18889465931478580854784
9444732965739290427392
18889465931478580854784
14167099448608935641088
141670994486089356410880
4722366482869645213696
4722366482869645213696
18889465931478580854784
9444732965739290427392
7083549724304467820544
9444732965739290427392
2361183241434822606848
9444732965739290427392
4722366482869645213696
7083549724304467820544
18889465931478580854784
2361183241434822606848
2361183241434822606848
14167099448608935641088
7083549724304467820544
4722366482869645213696

5575186299632655785383929568162090376495104 Button.
11150372599265311570767859136324180752990208=144

22300745198530623141535718272648361505980415 Total Sum.

Which last number is the exact quantity of barley, which the whole 12 dozen of buttons will amount to. Now for the worth.

An ounce *Averdupoise* had been exactly weighed, and found to contain 681 grains of barley; therefore a pound *Averdupoise* would contain 10896 grains: and seeing a bushel of the same barley weighed 50 pounds, the grains in a bushel will be 544800. Wherefore dividing the whole number of barley-corns by 544800, the number of bushels will be as here 40933830114777208409573638532761309665, and above $\frac{3}{4}$; and esteeming barley at 2 shillings the bushel, the value of

of the whole quantity of barley will be 40933830114-7772084095736385327613096 6 *l.* 11 *s.* and 11 *d.* $\frac{1}{2}$, which in words at length is, four millions of millions of millions of millions of millions of millions, ninety three thousand three hundred eighty three millions of millions of millions of millions of millions, eleven thousand four hundred seventy-seven millions of millions of millions of millions, seven hundred twenty thousand eight hundred forty millions of millions of millions, nine hundred fifty seven thousand three hundred sixty three millions of millions, eight hundred fifty-three thousand two hundred seventy-six millions, one hundred and thirty thousand nine hundred sixty-six pounds, eleven shillings and eleven pence half-penny. Which sum is so vastly great, that if the whole globe of the earth and sea, with whatsoever is contained on or therein, were converted into solid gold, and coined into guineas of equal quantity with those we now have, and to be valued at 30 *s. per* piece; a hundred of such guineas would come as near purchasing all the land on the face of the whole earth, as the said quantity of guineas would purchase all that barley; which may seem as a paradox, yet may easily be demonstrated to be true.

For suppose every degree of the meridian circle answer to 80 *English* miles upon the earth, which supposition is too much, none having yet accounted above 73; and Mr *Norwood* by experiment found only 69, and something above $\frac{1}{2}$ to answer to a degree on the earth; but supposing 80 miles, that we may not take too little, the circumference of the earth in miles is 28800, and is in inches 1824768000, and the solidity is 1065658515630637583853 15840 inches; and computing guineas at one pound ten shillings *per* piece, and to weigh five penny-weight nine grains, as they ought to do, a solid inch of gold would be worth 55 *l.* 7 *s.* but according to the account concerning the value of gold given by Sir *Jonas Moore* in his *Mathematical Compendium*, p. 16. a solid inch of angel gold (which

is the best) will be worth 33 *l.* 16 *s.* 4 *d.* by which we may see how guineas are advanced above the worth; but, taking them according to the greater rate, the worth of the whole globe of the earth, converted into such gold, will be 5898702283522221145586952830

: : : .

: .

pounds, 19 shillings and 6 pence.

And, according to the former computation, the square miles on the face of the whole earth will be 2640190464, one third of which being allowed for seas, the remainder will be 1760126976 square miles. And, seeing a square mile contains 640 square acres, the number of square acres on the face of the earth will be 1126481264464; and valuing an acre at 20 *s.* which is too much, accounting one with another, the worth will be 22529625329280 pounds, which may near as soon be purchased with a 100 guineas, as the barley before-named with the whole quantity of the said gold.

Nay, if we suppose the earth and seas, and all contained therein, were converted into fine sand, the number of grains of sand would far come short of the aforesaid number of barley-corns; so that the bulk of barley exceeds some millions of our earth we live upon, if it were possible to be brought into one place.

And, lastly, if the weight be considered, seeing a bushel weighs 50 pounds, the weight of the whole will be 2046691505738860420478681926638065483259 4 pounds, 13 ounces, 3 drams ½. All this may seem impossible to any but an accomptant, who is the best judge of the great and almost incredible power of numbers.

In the last place, we will annex a table of *geometrical progression* fitted to the last question, whereby any question of *geometrical progression* proceeding from unity, and of a *duple ratio*, may be resolved by inspection, if the number of terms exceed not 144.

A TABLE follows.

A TABLE of GEOMETRICAL PROGRESSION, proceeding from Unity, and continued to 144 places, the *ratio*, or common Excess being 2.

1	1	36	34359738368
2	2	37	68719476736
3	4	38	137438953472
4	8	39	274877906944
5	16	40	549755813888
6	32	41	1099511627776
7	64	42	2199023255552
8	128	43	4398046511104
9	256	44	8796093022208
10	512	45	17592186044416
11	1024	46	35184372088832
12	2048	47	70368744177664
13	4096	48	140737488355328
14	8192	49	281474976710656
15	16384	50	562949953421312
16	32768	51	1125899906842624
17	65536	52	2251799813685248
18	131072	53	4503599627370496
19	262144	54	9007199254740992
20	524288	55	18014398509481984
21	1048576	56	36028797018963968
22	2097152	57	72057594037927936
23	4194304	58	144115188075855872
24	8388608	59	288230376151711744
25	16777216	60	576460752303423488
26	33554432	61	1152921504606846976
27	67108864	62	2305843009213693952
28	134217728	63	4611686018427387904
29	268435456	64	9223372036854775808
30	536870912	65	18446744073709551616
31	1073741824	66	36893488147419103232
32	2147483648	67	73786976294838206464
33	4294967296	68	147573952589676412928
34	8589934592	69	295147905179352825856
35	17179869184	70	590295810358705651712

71	1180591620717411303424
72	2361183241434822606848
73	4722366482869645213696
74	9444732965739290427392
75	18889465931478580854784
76	37778931862957161709568
77	75557863725914323419136
78	151115727451828646838272
79	302231454903657293676544
80	604462909807314587353088
81	1208925819614629174706176
82	2417851639229258349412352
83	4835703278458516698824704
84	9671406556917033397649408
85	19342813113834066795298816
86	38685626227668133590597632
87	77371252455336267181195264
88	154742504910672534362390528
89	309485009821345068724781056
90	618970019642690137449562112
91	1237940039285380274899124224
92	2475880078570760549798248448
93	4951760157141521099596496896
94	9903520314283042199192993792
95	19807040628566084398385987584
96	39614081257132168796771975168
97	79228162514264337593543950336
98	158456325028528675187087900672
99	316912650057057350374175801344
100	633825300114114700748351602688
101	1267650600228229401496703205376
102	2535301200456458802993406410752
103	5070602400912917605986812821504
104	10141204801825835211973625643008
105	20282409603651670423947251286016

106	40564819207303340847894502572032
107	81129638414606681695789005144064
108	162259276829213363391578010288128
109	324518553658426726783156020576256
110	649037107316853453566312041152512
111	1298074214633706907132624082305024
112	2596148429267413814265248164610048
113	5192296858534827628530496329220096
114	10384593717069655257060992658440192
115	20769187434139310514121985316880384
116	41538374868278621028243970633760768
117	83076749736557242056487941267521536
118	166153499473114484112975882535043072
119	332306998946228968225951765070086144
120	664613997892457936451903530140172288
121	1329227995784915872903807060280344576
122	2658455991569831745807614120560689152
123	5316911983139663491615228241121378304
124	10633823966279326983230456482242756608
125	21267647932558653966460912964485513216
126	42535295865117307932921825928971026432
127	85070591730234615865843651857942052864
128	170141183460469231731687303715884105728
129	340282366920938463463374607431768211456
130	680564733841876926926749214863536422912
131	1361129467683753853853498429727072845824
132	2722258935367507707706996859454145691648
133	5444517870735015415413993718908291383296
134	10889035741470030830827987437816582766592
135	21778071482940061661655974875633165533184
136	43556142965880123323311949751266331066368
137	87112285931760246646623899502532662132736
138	174224571863520493293247799005065324265472
139	348449143727040986586495598010130648530944
140	696898287454081973172991196020261297061888

141	1393796574908163946345982392040522594123776
142	2787593149816327892691964784081045188247552
143	5575186299632655785383929568162090376495104
144	11150372599265311570767859136324180752990208

Take an example or two in the ufe of the aforegoing table.

QUEST. I.

What will 20 pieces of cloth coft by doubling every piece by a farthing? And fuppofing every piece to contain 50 yards and each yard worth 1 *l.* 1 *s.* 8 *d.* what will the difference be in the price?

RULE.

Subtract unity from the 21ft number, gives 1048575 farthings, which reduced, is 1092 *l.* 5 *s.* 3 *d.* 3 *q.*

And multiplying 20 by 50 gives 1000, the number of yards, which at 1 *l.* 1 *s.* 8 *d.* the yard, will amount to 1083 *l.* 6 *s.* 8 *d.* which fubtracted from the former number, gives 8 *l.* 18 *s.* 7 *d.* 3 *q.* the difference fought.

QUEST. II.

A certain man whofe daughter was married on *New-year's-day*, gave her hufband towards her portion one fhilling, promifing to double it on the firft day of every month for one whole year; I demand what was her portion?

Subtract an unit from the 13th number, being one number more than the number of months, and the remainder is 4095 fhillings, or 204 *l.* 15 *s.* the *Anfwer.* And thus of any other.

Or, fuppofe the progreffion to be *quadruple*, (fuppofing the progreffion proceeds from unity), the laft number may be found by the table, by fubtracting an unit from the double number of terms given.

EXAMPLE.

What will 9 packs of broad cloth coft, by quadrupling every pack by one fhilling?

From the double number of terms fubtracting an unit, leaves 17, and the 17th number in the table is 65536 the number of fhillings the laft pack will coft, and ⅓ of the

the difference of the last and first term added to the last, gives 87381 shillings, or 4369 pounds, one shilling, for the price of the whole, by *theorem* 15th.

More examples and more uses might be shewn; but let these suffice.

The *Combination, Election, Permutation,* and *Composition of Numbers or Quantities.*

Combination of Numbers, is how oft a less number of quantities may be taken out of a great number of quantities, without considering their places.

As if 10 letters of the alphabet, *a . b . c . d . e . f . g . h . i . k* were given, and it were required to know how many combinations of 2 letters; as *ab. ac. ad, &c.* may be taken in the said 10 letters; or how many combinations of 3 letters, as *abc. abd. abe, &c.* may be found in the same letters.

EXAMPLE.

How many combinations of 2 letters in 8, (*viz.*) *a . b . c . d . e . f . g . h?*

First, It is easily seen *a* will combine with all the following letters, *b . c . d . e . f . g . h,* from whence will arise 7 combinations, (*viz.*) *ab. ac. ad. ae. af. ag. ah.*

Secondly, *b* will be combined with all following itself, (but not with *a,* for *ba* is all one as *ab*), as *c . d . e . f . g . h*; whence arise 6 combinations, *viz. bc. bd. be. bf. bg. bh*; and so every letter will combine with those following itself, as may be seen at large in the following *table.*

In

In all 28 combinations, which is the answer.

And if to every binary already found, be added its following letters, it will produce the ternaries, or all the combinations of 3 letters, as in the following synopsis is evident.

ab. ac. ad. ae. af. ag ah.
bc. bd. be. bf. bg. bh.
cd. ce. cf. cg. ch.
de. df. dg. dh.
ef. eg. eh.
fg. fh.
gh.

In all 56 combinations.

Again, If to every ternary already found, be added its following letters, it will produce all the combinations on 4 letters in 8. And so of any other.

abc. abd. abe. abf. abg. abh.
acd. ace. acf. acg. ach.
ade. adf. adg. adh.
aef. aeg. aeh.
afg. afh.
agh.
bcd. bce. bcf. bcg. bch.
bde. bdf. bdg. bdh.
bef. beg. beh
bfg. bfh.
bgh.
cde. cdf. cdg. cdh.
cef. ceg. ceh.
cfg. cfh.
cgh.
def. deg. deh.
dfg. dfh.
dgh
efg. efh.
egh.
fgh.

56

But because this may seem tedious in large numbers, we have here exhibited another method, whereby to find the combinations in any given numbers or quantities with much ease,

THUS:

Having placed the given number of quantities, by itself,

itself, decrease it gradually by an unit so often as there are quantities in the combination; placing them one after another, with a sign of multiplication betwixt them; which numbers must be multiplied into one another for a dividend: then placing an unit with the like number of places, decreasing by unity with the sign of multiplication betwixt each, multiply them continually for a divisor, and the quotient will be the number of combinations sought.

EXAMPLE.

How many combinations of 5 letters in 10? *Facit* 252.

$5\times4\times3\times2\times1)\ 10\times9\times8\times7\times6\ (252$

The product of the divisor is 120.

The product of the dividend is 30240.

And the quotient will be 252.

Which will be the number of combinations of 5 letters in 10.

EXAMPLE II.

A country farmer going to a fair, makes a bargain with a moorlander for 50 sheep, which were to be chosen out of 100; but he thinking him long in chusing them, tells the farmer, that if he would give him a farthing for every parcel of 50 sheep, which may be taken out of the said 100, he should have the whole hundred; to which the farmer assents. The question is, what they will cost?

If you work according to the rule last laid down, the number of combinations of 50 in 100 will be 100891306544874079257172497256; which number of farthings reduced into pounds, shillings, and pence, will be 63428444317577165892888017 pounds, 19 shillings, 6 pence; which sum is so great, that if any man were able to pay a hundred thousand pounds a-day, he would be above sixty thousand millions of millions of years in paying it. Such a vanity may be concluded for want of judgment.

Note, In any given number of quantities, the number of combinations increase gradually, till you come about

about the mean numbers, and so decrease gradually again. So in 8 quantities, there are more combinations of 3 and 5, than of 2 and 6; and more of 2 and 6, than of 1 and 7; as may be seen in the following table.

Note, farther, That if the number of quantities be even, $\frac{1}{2}$ the number of places shews the greatest number of combinations that can be made in those quantities.

So if the number of quantities be 8, the $\frac{1}{2}$ of which is 4, shews the greatest number of combinations in these quantities will be of 4 in 8, as in the table.

1	8
2	28
3	56
4 In 8 =	70
5	56
6	28
7	8
8	1

But if the number of quantities be odd, then those two numbers which are next together, and whose sum is equal to the given number of quantities, shew the greatest number of combinations; so of 7 quantities the greatest number of combinations will be of 3 or 4 quantities in 7, and are equal as in the table.

1	7
2	21
3 In 7 =	35
4	35
5	21
6	7

A question to the former or last note.

EXAMPLE.

How many locks whose wards differ, may be unlocked with a key of 8 several wards?

Answer, 255 locks, 8 whereof may have one single ward, 28 double wards, 56 treble wards, 70 four wards, 56 five wards, 28 six wards, 8 seven wards, and 1 lock eight wards.

Of Election of QUANTITIES.

By *election of quantities* is meant, any number of quantities given, how many several ways I may take them without respect had to their places, as *A. B. C.* may be taken 7 ways, *viz. a. b. c. ab. ac. bc.* and *abc.*

The election of quantities may easily be found out by the geometrical table of progression aforegoing; thus,

to

to the given terms add the ſum; ſeek in the firſt column of the table, and from the number over-againſt it ſubtract an unit, the remainder is the number of elections ſought.

EXAMPLE.

How many elections are there of the letters of the alphabet?

Look in the firſt column of the table for 25, and over againſt it is 16777216=the 24th power of 2.

Subtract 1

Reſt 16777215, the number of elections of 24 letters.

Of Variations of QUANTITIES.

By *variation of quantities* is meant how many ſeveral ways any given number of quantities may be changed, as in reſpect to their places.

As *a b* may be changed into *b a*, and *a*, *b*, *c*, may be changed 6 ways, (*viz.*) *abc*. *acb*. *bac*. *bca*. *cab*. *cba*.

EXAMPLES.

How many changes may be rung on five bells?

RULE.

Multiply 1, 2, 3, 4, 5, one into another, the laſt product is the anſwer.

	1	Admits of no variation.
	2	
So on 5 bells may be rung 120 changes.	2	2 admits of 2 variations.
	3	
And on 6 bells may be rung 720 changes.	6	3 admits of 6 variations.
	4	
	24	4 admits of 24 variations.
	5	
And thus of any other number of bells.	120	5 admits of 120 variations.

EXAM-

EXAMPLE II.

A young ſcholar, but an arithmetician, coming into a town for the conveniency of a good library, demands of a gentleman with whom he lodged, What his diet would coſt for a year? The gentleman aſks him 10*l.* The ſcholar anſwered, he was not certain what time he might ſtay, and would know what he muſt give him for his diet ſo long as he could place his family (conſiſting of 6 perſons beſides himſelf) every day at dinner in a contrary poſition? The gentleman conſidering of it, and thinking it could not be long, tells him, he would allow him his diet ſo long for five pounds; to which the ſcholar aſſents.

The queſtion is, What he gave for his table, *per annum.*

The changes on ſeven quantities will be found to be 5040, which divided by 365, the days in a year, give 13 years, 808, or 10 months and 15 days: and ſo long muſt he board the ſcholar, according to the former conditions, which will hardly amount to ſeven ſhillings and three pence *per annum.*

EXAMPLE.

How many ſeveral combinations, with their variations, are there of three letters of the *Engliſh* alphabet?

RULE.

Multiply all its ſeveral combinations 2024 by its changes 6, and the product 12144 is the number of placing three letters of the alphabet, with all its variations.

From this mutability of variations and combinations, it is no marvel that by 24 letters there ariſeth, and is made ſuch variety of languages in the world, and ſuch infinite number of words in each language, ſeeing the diverſity of ſyllables produceth that effect, and

and alſo by interchanging and placing of letters amongſt the vowels and among themſelves, make thoſe ſyllables; for the alphabet of 24 letters may be varied thus many times, (*viz.*) 620448401733239439360000.

Now if you take in the combinations with the ſeveral variations of 2, 3, 4, 5, 6, *&c.* letters, there may be made and compoſed ſuch a vaſt number of words, that if a man could read 50 thouſand words in an hour, which is more than the *Pſalms of David* contain, (a taſk too great for any man to perform), and if there were a hundred thouſand millions of men, they would not ſpeak theſe words, according to the hourly proportion before mentioned, in an hundred thouſand years; a thing ſeeming moſt impoſſible and incredible, yet moſt certain and infallible in computation.

Hence likewiſe it may appear how many ways the letters of a name or word may be varied, and differently diſpoſed by way of anagram; out of which thoſe of uſe may be gathered, neglecting the reſt; as for example, the word *Roma*, conſiſting of four different letters, may admit of 24 changes, as hereafter.

Roma	*Orma*	*Mroa*	*Arom*
Roam	*Oram*	*Mrao*	*Armo*
Rmoa	*Omra*	*Mora*	*Aorm*
Rmao	*Omar*	*Moar*	*Aomr*
Raom	*Oarm*	*Maro*	*Amro*
Ramo	*Oamr*	*Maor*	*Amor*

Of which theſe, to wit, *Roma*, *Ramo*, *Oram*, *Mora*, *Maro*, *Armo*, *Amor*, are only uſeful, and all the reſt uſeleſs.

But if there be two or more letters of a ſort, divide the whole number of changes by the changes of the number of thoſe letters, and the quotient is the number of changes deſired.

So if the word *Philippa* were given, which conſiſteth of eight letters, of which (without conſidering thoſe which are of the ſame ſort) the changes will be 40320; but becauſe *i* is twice repeated, I divide

40320 by 2, the changes on two letters, the quote is 20160; and this divided again by six, the changes on three, because *p* is thrice repeated, gives in the quotient 3360, which are the changes in the word *Philippa.*

Or if I had divided 40320 by 12, (because 2 times 6 is 12), the quotient will give in the *answer,* at one operation, the same as before.

After this manner may be found the variations or changes which may be made of some particular *Latin* verses, so as to keep the rules of a true verse, and the sense grammatically the same.

So if the two following verses were chosen, (*viz.*)

Lex, rex, grex, res, spes, jus, thus, sal, sol, (*bona*) *lux, laus.*

Mars, mors, sors, fraus, fex, Styx, nox, crux, pus, (*mala*) *vis, lis.*

It is very remarkable how many sundry ways the same may be varied, and yet the sense remain good, and the verse grammatically true; for if we suppose the words *bona* and *mala* continually to keep the same (to wit, the 10th) place, the rest being 11 in number, indifferently changing place with any other in the same verse, the number of variations of 11 places will be 39916800, which doubled for the number of changes in both verses, makes 79833600, which would compose above 249 *folio* volumes, each volume to contain 2000 pages, every page divided into two parts, and every part to contain 80 verses, which at a penny the sheet, would amount to 518 pounds, 15 shillings; and supposing them bound for five shillings a volume, which is not dear, the binding would cost 62 pounds 5 shillings; and the worth of the whole would be 581 pounds.

Of

Of Composition of QUANTITIES.

By *Composition of Quantities* is meant, when in any number of given quantities, as in letters or figures, one row is joined with another row of the same, or with 2, 3, or more other rows, as in placing, and the chances of the dice.

This differs from *combination* and *election*, in that there one quantity is taken but once, here as oft as there are quantities to be taken.

Though this be the most composed way, yet it is not difficult to be performed; for if the compositions of two quantities in 10 (or any other number) were sought, it is but squaring the given number; if of three quantities in 10, it is cubing the given number; if of four quantities, its biquadratic will fit, and so increase the powers, according as your number of quantities increase.

EXAMPLE I.

What number of compositions of three letters in 20? *Facit* 8000, the cube of 20.

EXAMPLE II.

What number of compositions of 6 letters in 24, or in the whole alphabet? *Answer*, 191102976, the squared cube of 24.

EXAMPLE III.

What number of compositions of 24 letters of the alphabet, accounting them by 1 and 1, by 2 and 2, by 3 and 3, and so on to 24? If we account each time 24, the answer would be 1333735776850284124449-081472843776; but since we are to find all the numbers preceding in geometrical progression under it; to perform which, observe the following *rule:*

As the *ratio minus* unity: is to the first term:: So

 the

the whole power of the *ratio minus* unity: To the sum of all the terms.

Thus stated.

As 23 : to 24 : : so 1333735776850284124449081-472843776 : to 1391724288887252999425128493-401200, which is the number of compositions sought.

How many several chances are there on 2, 3, 4, 5, and 6 dice?

Answer, On 2 dice are 36, on 3 dice 216, on 4 dice 1296, on 5 dice 7776, and on 6 dice 46656 chances.

Casts.	*Points.*	*Chances.*	*Sum.*
2 . 12	1 . 1	1	1
3 . 11	1 . 2	2	2
4 . 10	1 . 3 2 . 2	2 1	3
5 . 9	1 . 4 2 . 3	2 2	4
6 . 8	1 . 5 2 . 4 3 . 3	2 2 1	5
7 .	1 . 6 2 . 5 3 . 4	2 2 2	6

The foregoing table shews the several particular chances on two dice. The sum of the chances of 2 . 3 . 4 . 5 6 casts are 15, and of 12 . 11 . 10 . 9 . 8, are 15. Then add the chances on 7 (*viz.*) 6, gives 36, the chances on 2 dice.

The particular chances on 2 dice, otherwise formed, whereby the following observations may be made.

1 . 1

1 . 1	1 . 2	1 . 3	1 . 4	1 . 5	1 . 6
2 . 1	2 . 2	2 . 3	2 . 4	2 . 5	2 . 6
3 . 1	3 . 2	3 . 3	3 . 4	3 . 5	3 . 6
4 . 1	4 . 2	4 . 3	4 . 4	4 . 5	4 . 6
5 . 1	5 . 2	5 . 3	5 . 4	5 . 5	5 . 6
6 . 1	6 . 2	6 . 3	6 . 4	6 . 5	6 . 6

OBSERVATIONS.

Here are 36 chances, the square of 6, as before; all the chances of 6 are placed in the lowermost and furthermost row: whence observe, that in the square of 5, (*viz.*) 25, there is no 6: so there are 25 chances without 6, and 11 more where there is a 6: so that it is above 2 to 1 whoever throws 2 dice, throws not a 6. In the square of 4 (*viz.*) 16. there is neither 5 nor 6. In the square of three (*viz.*) 9, there is neither 4, 5, nor 6. In the square of 2 (*viz.*) 4, there is neither 3, 4, 5, nor 6. This may be applied to other chances.

A TABLE *of the powers of* 6, *and the numbers under it.*

1	2	3	4	5	6
Lat.	*q.*	*c.*	*bq.*	*ſ. ſ.*	*ſ. c.*
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	4096
5	25	125	625	3125	15625
6	36	216	1296	7776	46656

By this table may be accounted, how many several chances there are on 2, 3, 4, 5, or 6 dice, and how many there are where there is no 6, or neither 5 nor 6, or neither 4, 5, nor 6, or neither 3, 4, 5, nor 6.

EXAMPLE.

On 4 dice there are 1296 chances, 625 where there is no 6, 256 where there is neither 5 nor 6, 81 where there is neither 4, 5, nor 6, and 16 where there is neither 3, 4, 5, nor 6: but if it were demanded, on how many chances on 4 dice where there is a 6; I answer, 671? for 1296 *minus* 625, is equal to 671.

And 1040 changes have either a 5 or 6; for 1296 *minus* 256, is equal to 1040.

Likewise 1215 chances have either a 4, 5, or 6; for 1296 *minus* 81, is equal to 1215.

And lastly, 1280 chances have either a 3, 4, 5, or 6; for 1296 *minus* 16, is equal to 1280. And so of any other in the table.

To conclude, I shall here mention a small treatise written not long since, entitled, *Artificial versifying*, shewing any one, though of ordinary capacity, that can write and read, though he understand not a word of *Latin*, how to make thousands of *Hexameter* and *Pentameter* verses, which will be good *Latin*, true verse, and perfect sense, and that in two hours time.

Which is performed by a select number of *Latin* words, artfully digested into tables, the more to amuse the reader.

The words are these which follow.

In the table of *Hexameters* the words are,

1. *Turbida, ignea, pessima, horrida, aspera, martia, barbara, laurida, effera.*

2. *Fata, signa, damna, bella, vincula, sistra, castra, scorta, tela.*

3. *Sequi, fori, pati, tuis, domi, patet, puto, palam, ferant.*

4. *Præmonstrant,*

4. *Præmonstrant, proritunt, promittunt, protendunt, producunt, monstrabant, causabant, puranarrant, promulgant.*

5. *Tempora, pocula, prælia, verbera, lumina, fœdera, agmina, crimina, sidera.*

6. *Dura, sæpe, quædam, acerba, prava, multa, dira, nigra, sæva.*

Here if you take one word out of each line, you will have a true *Hexameter* verse,

In the tables of *Pentameters* the words are,

1. *Tetrica, ardua, perfida, improba, sordida, impia, tristia, turpia, noxia.*

2. *Præstabant, præscribunt, concludunt, prædicunt, perficiunt, consummant, conglomerant, significant, procurant.*

3. *Dura, acta, vina, verba, dicta, facta, labra, arma, astra.*

4. *Dolosa, pudenda, proterva, nefanda, cruenta, superba, molesta, sinistra, maligna.*

5. *Nova, aliis, tibi, viris, scio, mera, malis, vides, mihi.*

Here likewise if you take a word out of every line, you will have a true *Pentameter* verse.

Now if it were required to find how many verses may be composed out of the aforegoing words; seeing every line hath 9 words, I find the squared cube of 9, because there are 6 lines, which will be 531441; and so many verses may be made out of the tables of *Hexameters*, without taking notice of the permutation of places; for you may change most of the 1st and the 6th line into the 5th and 2d line; which verses will compose 30 volumes as big, or bigger, than *Virgil.*

Likewise in the table of *Pentameters* may be composed 59049 verses, being the sursolid or 5th power of 9; because there are 5 lines, and nine words in every line.

But

But if the combinations of 6 in 54 for the *Hexameters*, or 5 in 45 the *Pentameters*, were required, the anſwer will be in the firſt 25827165, in the latter 1221756; but becauſe we are not to take two words in a line, theſe numbers cannot be admitted.

And, in the laſt place, becauſe the contrivance may pleaſe and divert the reader, we will here annex the tables themſelves, with the manner of their uſe, and ſo conclude this rule.

The Tables of Hexameters.

TABLE I.

T	r	p	h	a	m	b	l	e	u
g	e	o	s	a	a	u	f	r	n
ſ	r	p	r	r	r	f	b	e	s
r	e	t	b	i	e	i	a	i	i
r	i	a	d	r	d		m	d	a
a	r	a	a	a		a	a		
a									

TABLE II.

F	s	d	b	v	s	c	s	t	a
i	a	e	i	i	a	c	e	t	g
m	l	n	s	s	o	l	a	n	n
l	c	t	t	r	a		a	a	a
l	r	r	t						a
a	a	a							

TABLE III.

S	f	p	t	d	p	p	p	f	e
o	a	u	o	a	u	a	e	q	r
t	i	m	t	t	l	r	u	i	i
s	i	e	o	a	u	i			
	t		m	n					
			t						

TABLE IV.

p	p	p	p	p	m	c	p	p	r
r	r	o	r	o	a	r	r	æ	o
o	r	o	n	u	æ	o	m	r	m
t	d	s	s	n	m	o	i	i	e
u	t	a	a	u	n	t	t	n	c
r	b	r	l	s	a	t	d	u	a
u	r	g	t	n	u	u	n	b	n
a	a	r	t	n	n	t	u	t	n
n	a		t	t		n		t	t
n					t				t

TABLE V.

T	p	p	v	l	f	a	c	s	e
o	r	e	u	œ	g	r	i	m	c
æ	r	m	d	m	i	d	p	u	l
b	i	e	i	m	e	o	l	i	e
n	r	n	i	r	r	a	a	r	a
a	a	n	a	a			a		
	a								

TABLE

TABLE VI.

D	s	q	a	p	m	d	n	s	u
æ	u	c	r	u	i	i	æ	r	p
æ	e	a	l	r	g	v	a	e	d
r	v	t	a	r	a			a	b
a	a		a				m	a	

The Tables of PENTAMETERS.

TABLE I.

T	a	p	i	s	i	t	t	n	e
r	e	m	o	m	r	u	o	t	d
r	p	r	p	i	r	x	r	u	f
r	d	i	s	p	i	i	a	i	o
i	a	t	i	a	c		d	b	d
	i	a		a		a	a	a	
a									

TABLE II.

P	p	c	p	p	c	c	ſ	p	r
r	o	r	e	o	o	i	r	æ	æ
n	æ	r	n	n	g	o	l	ſ	c
d	t	l	g	n	c	t	c	l	l
i	u	l	i	u	a	r	u	c	c
n		f	r	b	i	d	u	i	m
n		a	u	b	u	n	u	a	e
c		n	u	n	t	n	n	r	a
t		n	t		t	t	a	n	
	t					n	t		
					t				

TABLE

TABLE III.

D	a	v	v	d	f	l	a	a	u
c	i	c	i	a	a	r	f	r	t
n	r	c	c	b	m	t	a	a	a
b	t	t	r	a	r				a
a	a	a		a					

TABLE. IV.

D	p	p	n	c	ſ	m	ſ	m	o
u	r	e	r	u	o	i	a	l	d
o	ſ	u	p	l	n	l	o	e	t
a	e	e	e	i	i	ſ	n	e	n
n	r	ſ	ſ	g	a	d	r	d	t
b	t	t	n		a	v	a	a	a
a	r	a			a				
a									

TABLE V.

N	a	t	v	ſ	m	m	v	m	o
l	i	i	c	e	a	i	i	v	i
b	r	i	r	l	d	h	a	i	i
i	o	a	i	e	i		s		s
		s	s						

The uſe of the tables is very eaſy :

For ſuppoſe we would compoſe an hexameter and pentameter verſe.

For the hexameter, chuſe any ſix of the nine digits, as ſuppoſe 345648; and for the pentameter, any five of the ſaid nine digits, as ſuppoſe 23479.

Firſt, for the hexameter take three, the firſt figure thereof towards the left hand, and looking in the firſt table, count till you come to the third ſquare, where you will find the letter *P* for the firſt letter of the firſt word;

word; then counting forward till you come to the ninth place, which falls in the second column, and second square, where you will find the letter *e* for the second letter of the first word; and counting forward nine places more, which falls in the third column and first square, where you may find the letter *ſ*; and so counting every time nine places from the letter last found, you will have the word *Peſſima*, which is the first word. Then taking the second figure 4 in the said number, and proceeding with it in the second table, according to the former directions, you will find the second word to be *bella*, and thus running through the first six tables with the said six figures, according to the former directions, you will find the first verse to be this that follows;

Peſſima bella domi monſtrabunt verbera nigra.

And if after this manner you proceed with the other five figures in the table of pentameters, you will find the other verse to be,

Ardua concludunt verba moleſta mihi.

Note, If the ninth place chance to be blank, you may know your work is finiſhed.

[illegible] *also*, Your verſes may more readily be found [illegible] Find the first letter as before, and run the ſquare [illegible]lly downward towards the left hand for the remaining letters: if the diagonals be too few, the remainder may be found, by counting nine forward, and running down diagonally as before. So in the first word before found, I find *Peſ.* in the three first diagonals, and counting nine forward, and looking diagonally, *ſima*, which put together makes *Peſſima*. And thus of any other.

Decimal

Decimal ARITHMETIC.

NUMERATION.

WHAT a decimal fraction is was shewed in the introduction; but for the learner's benefit we shall again repeat it.

A decimal fraction is such, whose denominator is not expressed, but understood, and is an unit with as many ciphers annexed as there are places in the numerator. So $\frac{5}{10}$ will be expressed thus, .5; and $\frac{25}{100}$ thus, .25; and $\frac{125}{1000}$ thus, .125; *&c.*

And they have commonly a point or comma prefixed, to distinguish them from an integer.

Note, A cipher placed to the left-hand of an integer, or to the right-hand of a decimal, neither increaseth nor decreaseth the value; but placed to the right-hand of an integer, increaseth the value; and to the left-hand of a decimal, decreaseth it. *Observe the following table.*

The table of NUMERATION.

Hund. of mill.	Tens of mill.	Millions.	Hund. of thous.	Tens of thous.	Thousands.	Hundreds.	Tens.	Units.
9	8	7	6	5	4	3	2	1

Integers.

Tenth parts.	Hund. parts.	Thousand parts.	Ten thous. parts.	Hund. thous. parts.	Millions of parts.	Ten mill. of parts.	Hund. mill. of parts.
2	3	4	5	6	7	8	9

Decimals.

In this table you may observe, that as integers increase in a tenfold proportion to the left hand, so decimal fractions decrease in a tenfold proportion to the right-hand.

So 5, 50, 500, 5000 is Five, Fifty, Five hundred, Five thousand.

And .5, .05, .005, .0005 is 5 Tenth, 5 Hundred, 5 Thousand, 5 Ten thous. Parts.

Reduction in DECIMALS.

By Reduction we find the decimal of any fractional part of coin, weight, measure, *&c.* and, on the contrary, reduce any decimal fraction given into its equivalent fractional parts of coin, weight, measure, *&c.*

PROP. I.

Any vulgar fraction given, to reduce the same into a decimal fraction of equal value. To perform which, the proportion is,

As the denominator of the vulgar fraction given : is to the numerator thereof : :

So is an unit, with ciphers annexed at pleasure : to the decimal fraction required.

Or thus;

Add a competent number of ciphers to the numerator, and divide by the denominator, the quotient is the decimal fraction required.

EXAMPLE I.

Let it be required to find the decimal fraction of $\frac{3}{4}$.

See

See the work.

4) 3000 (.75 *facit* .75

28

20
20

00

Note, The ciphers added, are to be distinguished by a point or comma, and you may annex what number you please; but you must take notice what ciphers you make use of; for so many must be cut off in the quotient; and if at any time it happens, as sometimes it will, there be not a sufficient number in the quotient, they must be supplied by adding ciphers to the left-hand.

EXAMPLE II.

Reduce $\frac{1}{8}$ into a decimal.

8) 1.0000 (.125 *facit* .125

8

20
16

40
40

0

EXAMPLE III.

Reduce $\frac{35}{721}$ into a decimal.

```
721) 35.000000 (.048543
     .....
     2884
     ----
      6160
      5768
      ----
       3920
       3605
       ----
        3150
        2884
        ----
         2660
         2163
         ----
          497 Remainder.
```

In many cafes, as in this example, though you fhould annex a thoufand ciphers, yet your decimal fraction will not come up, but there will ftill be a remainder; but if we bring it five or fix places after the feparatrix, it will be exact enough in moft cafes, and the remainder may be thrown away as of no value; for if you fuppofe this was the fraction of a pound fterling, this decimal will be fo near the truth, as if a pound were divided into a million of parts, it would not err from the truth fo much as one of thofe parts.

And feeing I made ufe of 6 ciphers in the operation, and but 5 figures in the quotient, I add a cipher to the left-hand before I made my feparatrix.

EXAMPLE IV.

Reduce 9 pence into a decimal fraction.

Seeing alfo 240 pence make a pound fterling, 9 pence is equal to $\frac{9}{240}$, which reduce as before.

240)

240) 9.0000 (.0375 the dec. of 9 *d.*

720

1800
1680

1200
1200

00

EXAMPLE V.

Reduce 11 shillings into a decimal.

11 shill. is $\frac{11}{20}$. 20) 11.000 (.55=to 11 shill.

100

100.
100

00

But the decimal answering any number of shillings, may more quickly be found, by halving the number of shillings given.

So for 11 shill. $\frac{1}{2}$ of 11 is 5, and 1 remains, 11 shill. to which suppose a cipher annexed, makes——— 10, whose half is 5, as you see, .55 dec.

So the decimal of 12 shillings is .6; of 15 shillings is .75; of one shilling is .05; and so of any other.

EXAMPLE VI.

What is the decimal of 7 shillings and 6 pence?

The decimal of 7 shillings by the last is .35.

Then 6 is $\frac{6}{240}$, which reduced, is .025, to which adding .75, gives .375.

.35 =7 shillings.
.025=6 pence.

.375=7 shillings and 6 pence.

But if you consider this, the decimal of any number of pence may be found by taking parts of the shilling of which they consist.

So .05 being the decimal of 1 shilling,
$\frac{1}{2}$=.025 is the decimal of 6 pence.

And $\frac{1}{2}$ of the decimal of 6 pence, is the decimal of 3 *d.*

.025 is the decimal of 6 pence.
$\frac{1}{2}$=.0125 is the decimal of 3 pence.

The 3d part of the decimal of 3 pence, is the decimal of a penny, to wit, .004166—and the 4th part of that is the decimal of a farthing, to wit, .0010416.

.35 =7 shillings.
.025=6 pence.
So the dec. of 7 *s.* 7 *d.* $\frac{1}{2}$ is .38125. .00625=1 *d.* $\frac{1}{2}$

.38125=7 *s.* 7 *d.* $\frac{1}{2}$

In the former examples of money, we have supposed the integer to be a pound sterling; but if the decimal of 6 pence were required, and the integer to be a shilling, then the decimal would be .5; which if the integer had been a pound, would have been the decimal of 10 shillings; whereby you may see the decimal alters according as we take our integer; so if we account a penny to be the integer, the decimal of 1 farthing is .25; of 2 farthings is .5; of 3 farthings is .75.

And you may see and take notice, that these three last decimal fractions are general fractions in any case; for .25 is equal to $\frac{1}{4}$ of any thing, .5 is equal to $\frac{1}{2}$ of any thing, .75 is equal to $\frac{3}{4}$ of any thing, as of a pound, shilling, penny, yard, hundred, &c.

For if a pound sterling be the integer, .5 is 10 shillings; if a shilling be the integer, .5 is 6 pence; if a penny be the integer, .5 is 2 farthings; if a yard be the inte-

integer, .5 is $\frac{1}{2}$ yard; and so of any of the rest, which being considered, may be of good use to the learner.

EXAMPLE VII.

Let it be required to find the decimal answering 16 penny-weights, one pound *Troy* being the integer. Seeing there is 240 penny-weight in a pound, 16 penny-weight is $\frac{16}{240}$, which reduced by the examples aforegoing, will be .0666—And so infinitely.

$$
\begin{array}{r}
240)\ 16.00000\ (.666 \\
1440 \\
\hline
1600 \\
1440 \\
\hline
160
\end{array}
$$

EXAMPLE VIII.

What is the decimal of 11 penny-wt. 16 grains?

Bring 11 pen.-wt. 16 grains into grains, which are 280 grains; then, because there are 5760 grains in a pound, 280 grains will be $\frac{280}{5760}$; but cutting off a cipher in each, which, reduced as before, will be .048611.

$$
\begin{array}{r}
5760)\ 280.000000\ (.048611 \\
23040 \\
\hline
49600 \\
46080 \\
\hline
35200 \\
34560 \\
\hline
6400 \\
5760 \\
\hline
6400 \\
5760
\end{array}
$$

EXAMPLE IX.

What is the decimal of 3 quarters and 14 pounds, one hundred, or 112 pounds being the integer? *Ans.* .875.

The decimal of $\frac{3}{4}$ is, as was said before, .75, and 14 pounds is $\frac{14}{112}$, which reduced, is .125.

112) 14,000 (.125

```
 112

 280      Unto .75
 224      Add  .125
 ---      ---------
  560     Facit .875
  560
  ---
   00
```

So the decimal of 2 pints, 1 gallon the integer, will be .25, 45 minutes of an hour is .75.

These *examples* being understood and considered, are sufficient to reduce any other weights and measures into decimals; so we will conclude this *proposition*.

PROP. II.

To find the value of any decimal fraction in the known parts of the integer, as of *coin*, *weight*, *measure*; to perform which, observe the following *rule*.

RULE.

Multiply the decimal given by the number of parts of the next inferiour denomination, cutting of as many figures from the product as the decimal given consists of; the remainder, if any, multiplied by the parts of the next inferiour denomination, cutting of as before. Thus must you do till the decimal given be brought into its least parts, the parts signified by the decimal, will be thrown over the separatrix.

EXAM.

EXAMPLE I.

What is the value of .725 of a pound ſterling?

20 ſhillings in a pound.

ſhill. 14.500

12 pence in a ſhill.

1000

500

6.000

EXAMPLE. II.

What is the value of .696875 of a pound ſterling?

20 ſhillings in a pound.

ſhill. 13.937500

12 pence in a ſhilling.

	s.	*d.*	*q.*	
				1875000
Facit	13	11	1	937500

pence 11.250000

4 farth. in a penny.

Farth. 1.000000

EXAMPLE III.

What is .72065 of a pound ſterling?

20 ſhillings in a pound.

ſhill. 14.41300

12 pence in a ſhilling.

82600

41300

pence 4.95600

4 farthings in a penny.

Farthings 3.82400 remains leſs than a farthing, and ſo not to be accounted of.

Note,

Note, But the value of any decimal fraction of a pound sterling may be more easily found thus: For the first figure after the separatrix, is a double number of shillings; if the second figure be 5, or above, for the 5 account one shilling more; then the 2d figure, if under 5, or the excess, if above 5, added to the 3d, is so many farthings, remembering to abate 1 from the farthings, if the sum be above 13, and 2 from the farthings, if the sum be above 40.

EXAMPLE.

In the decimal .76565, the first figure 7 doubled is 14, which are shillings; and because the 2d figure is above 5, subtract 5 from it, and account one shilling more: and the 15 farthings are 3 pence and 3 farthings: So the value of it is 15 shillings and 3 pence 3 farthings; as for the rest of the figures, they being but the fraction of a farthing, are inconsiderable in practice.

So the value of .6666 of a pound will be 13 shillings and 4 pence. Of .2065 will be 4 shillings and 3 halfpence: and so of any other.

EXAMPLE IV.

What is the value of .625 of a pound *Troy* weight?

```
                    12
                   ----
                   1250
Facit 7 oz.         625
10 pen. wt.        ----
          Ounces 7.500
                    20 penny-wt. in an ounce.
               ------
       Penny-wt. 10.000
```

EXAM-

EXAMPLE V.

What is the value of .6725 of a hundred weight?

```
           4 quarters in a hundred.
        ------
Quarters 2.6900
          28 pounds in a quarter.
        ------
         55200
        13800
        -------
Pounds 19.3200
          16 ounces in a pound.
        -------
         19200
         3200
        -------
Ounces 5.1200 Facit 2 q. 19 lb. 5 oz.
```

Ounces 5.1200 *Facit* 2 *q.* 19 *lb.* 5 *oz.*

And the value of .6125 of a yard, is 1 foot and 10 inches. Or .725 of a gallon, is almost 6 pints.

But lest these ways of finding the decimals of any parts, as likewise the value of any decimal should seem tedious, we have annexed tables of the most eminent known parts of *money, weight, measure,* &c.

The tables follow.

Decimal

Decimal TABLES of Coin, Weight, and Measure.

TABLE I.

English *coin, one pound the integer.*

Shillings	Decimals	Shillings	Decimals
19	.95	9	.45
18	.9	8	.4
17	.85	7	.35
16	.8	6	.3
15	.75	5	.25
14	.7	4	.2
13	.65	3	.15
12	.6	2	.1
11	.55	1	.05
10	.5		

Pence	*Decimals*
11	.045835
10	.041666
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	016666
3	.0125
2	.008333
1	.004166

Farth.	*Decimals*
3	.003125
2	0020833
1	.0010416
½	.0005208
¼	.0002604

TABLE II.

English *coin, one noble the integer.*

Shill.	*Decimals*
6	.9
5	.75
4	.6
3	.45
2	.3
1	.15

Pence	*Decimals*
11	.1375
10	.1250
9	.1125
8	.1
7	.0857
6	.075
5	.0625
4	.05
3	.0375
2	.025
1	.0125

Farth.	*Decimals*
3	.009375
2	.00625
1	.003125
½	.0015625
¼	.00078175

TABLE III.

English *coin, one shill. long measure, one foot the integer.*

Pence / *Inches*	*Decim.*
11	.916666
10	.833333
9	.75
8	.666666
7	.583333
6	.5
5	.416666
4	.333333
3	.25
2	.166666
1	.083333

Farthings. / *Q. Inches.*	*Dec.*
3	.0625
2	.041666
1	.020833
½	.010416
¼	.0052083

TABLE IV.

Troy *weight, one pound the integer.*

Ounces the same as pence in the last Table.

Pen-wt.	*Decim.*
19	.079166
18	.075
17	.070833
16	.066666
15	.0635

The

The rest of the table.

14	.058333
13	.054166
12	.05
11	.045833
10	.041666
9	.0375
8	.033333
7	.029166
6	.025
5	.020833
4	.016666
3	.0125
2	.008333
1	.004166

Grains.	*Decimals.*
23	.003993
22	.003819
21	.003646
20	.003472
19	.003298
18	.003125
17	.002951
16	.002778
15	.002604
14	.002431
13	.002257
12	.002083
11	.001910
10	.001736
9	.001562
8	.001389
7	.001215
6	.001042
5	.000868
4	.000694
3	.000521
2	.000347
1	.000173
½	.000086

Troy weight, *one ounce the integer.*

Penny weight, the same as shillings in the first Table.

Grains	*Decimals.*
23	.047916
22	.045833
21	.04375
20	.041666
19	.039583
18	.0375
17	.035416
16	.033333
15	.03125
14	.029166
13	.027083
12	.025
11	.021916
10	.020833
9	.01875
8	.016666
7	.014583
6	.0125
5	.010416
4	.008333
3	.00625
2	.004166
1	.002083

TABLE V.

Averdupois *wt.* 112 *lb. the integer.*

Quart.	*Hundred. Decimals.*
3	.75
2	.5
1	.25

The rest of the table.

Pounds.	*Decim.*
27	.241071
26	.232143
25	.223214
24	.214286
23	.205357
22	.196428
21	.1875
20	.178571
19	.169643
18	.160714
17	.151785
16	.142857
15	.138928
14	.125
13	.116071
12	.107143
11	.098214
10	.089286
9	.080357
8	.071428
7	.0625
6	.053571
5	.044643
4	.035714
3	.026786
2	.017857
1	.008928

Ounces.	*Decimals.*
15	.008370
14	.007812
13	.007254
12	.006696
11	.006138
10	.005580
9	.005022
8	.004464
7	.003906

The rest of the table.

6	.003348
5	.002790
4	.002032
3	.001674
2	.001116
1	.000558

Quar. oz.	*Dec.*
3	.000418
2	.000279
1	.000139

Averdupois *wt.* 1 *lb. the integer.*

Ounces.	*Decimals.*
15	.9375
14	.875
13	.8125
12	.75
11	.6875
10	.625
9	.5625
8	.5
7	.4375
6	.375
5	.3125
4	.25
3	.1875
2	.125
1	.0625

Drams.	*Decimals.*
15	.058593
14	.054687
13	.050781
12	.046875
11	.042968

The rest of the table.

10	.039062
9	.035156
8	.03125
7	.027343
6	.023437
5	.019531
4	.015625
3	.011718
2	.007812
1	.003906

TABLE VI.

Liquid *measure, one gallon; dry measure, one qr. the integer.*

Pints.	*dec.*	*bush.*
7	.875	7
6	.75	6
5	.625	5
4	.5	4
3	.375	3
2	.25	2
1	.125	1

Qr. pts.	*d.*	*pecks.*
3	.09375	3
2	.0625	2
1	.03125	1

Decim.	*qr. Peck.*
.0234375	3
.015625	2
.0078125	1

The rest of the table.

Decimals.	*Pints.*
.005859	3
.003906	2
.001953	1

TABLE VII.

Time; *One year the integer.*

Months, the same as pence in the third Table.

Days.	*Decimals.*
30	.082191
29	.079452
28	.076712
27	.073972
26	.071233
25	.068493
24	.065753
23	.063014
22	.060274
21	.057534
20	.054794
19	.052055
18	.049315
17	.046575
16	.043836
15	.041096
14	.038356
13	.035616
12	.032876
11	.030137
10	.027397

The

The rest of the table.

9	.024657
8	.021917
7	.019178
6	.016438
5	.013698
4	.010959
3	.008219
2	.005479
1	.002739

Time; one day the integer.

Hours.	*Decim.*
23	.958333
22	.916666
21	.875
20	.833333
19	.791666
18	.75
17	.708333
16	.66666
15	.625
14	.583333
13	.541666
12	.5
11	.458333
10	.416666
9	.375
8	.333333
7	.219666
6	.25
5	.208333
4	.166666
3	.125
2	.083333
1	.041666

The rest of the table.

59	.040972
58	.040277
57	.039583
56	.038888
55	.038194
54	.0375
53	.036805
52	.036111
51	.035416
50	.034722
49	.034027
48	.033333
47	.032638
46	.031944
45	.03125
44	.030555
43	.029861
42	.029166
41	.028472
40	.027777
39	.027083
38	.026388
37	.025694
36	.025
35	.024305
34	.023611
33	.022916
32	.022222
31	.021527
30	.020833
29	.021138
28	.019444
27	.01875
26	.018055
25	.017361
24	.016666

The rest of the table.

23	.015972
22	.015277
21	.014583
20	.013888
19	.013194
18	.0125
17	.011805
16	.011111
15	.010416
14	.009722
13	.009027
12	.008333
11	.007638
10	.006942
9	.00625
8	.005555
7	.004861
6	.004166
5	.003472
4	.002777
3	.002083
2	.001388
1	.000694

TABLE VIII.

Cloth *measure.*

One yard the integ

Quart.	*Decim*
3	.75
2	.5
1	.25

Nails.	*Decim*
3	.1875
2	.125
1	.0625

Concerning the construction of these tables, we need to say nothing, that being sufficiently shewn in the first nine examples of this rule, but proceed to their use in a proposition or two, and so conclude this rule.

PROP. I.

To find the decimal answering any fractional part of coin, weight, measure, *&c.*

This is for the most part given by inspection, or, at most, by single addition of two or three numbers.

EXAMPLE I.

What is the decimal answering 17 *s.* 1 *l.* being the integer?

Seek in the first table for 17 shillings, in the table of shillings, and against it, in the column adjoining, is .85, the decimal required. So the decimal of 5 shillings, one noble being the integer by the second table is found to be .75; and the decimal of 8, one shilling being the integer, by the third table, is .666666; which is likewise the decimal of 8 inches, or 8 months, a foot being the integer in the one, and a year in the other; likewise the decimal of 21 pounds by the fifth table, 112 *lb.* being the integer, will be 1875; and so of any other.

EXAMPLE II.

What is the decimal answering 7 *s.* 9 *d.* 1 *q.* a pound *Sterling* being the integer?

By the first table	7 Shillings is	.35
	9 Pence is	.0375
	1 Farthing is	.001041

The decimal of 7 *s.* 9 *d.* 1 *q.* is=388541

So

So the decimal of 3 *qr.* 7 *lb.* 8 *oz.* by the fifth table, will be found to be 816964.

For the decimal of 3 quarters is .75
of 7 pound is .0625
of 8 ounces is .004464

Of 3 *qr.* 7 *lb.* 8 *oz.* is .816964

After the same manner the decimal of 3 *s.* 9 *d.* a noble the integer, by the second table, will be found to be .5625; and so of any other.

The second table was added, because some may dislike the first table, in respect some parts of coin will not be exactly expressed in decimals by it, and though infinitely near the truth, yet will never equal it.

But if you make a noble the integer, you may express any parts exactly by it, as may be seen by the table itself.

PROP. II.

Any decimal fraction of coin, weight, measure, being given, to find the value thereof.

This is but the converse of the last proposition; and he that understands that, cannot be ignorant of this; however, take an example or two.

EXAMPLE I.

Let .65 be the decimal fraction of a pound *Sterling*, and let the value thereof be required.

Seek in the first table of *English* coin for .65, and in the column of shillings over-against it I find 13 shillings, the value of the decimal given: so .75 parts of a pound *Sterling* will be found to be 15 *s.*; but if the integer had been a hundred weight, the value thereof, by the fifth table, will be found to be $\frac{3}{4}$, or 3 quarters.

Likewise .049315, being the decimal of a year, by table the 7th, will be found to be 18 days.

EXAMPLE II.

But sometimes your decimal given cannot be found at one time, then use the following method.

So if the value of .46725 of a pound *Troy* were required, seek in the 4th table; and because therein the ounces are not expressed, because they are the same as pence in the 3d table find the ounces there, and the penny-weight and grains in the 4th table, and the value will be found to be 5 ounces, 12 penny-weight and 3 grains.

See the following work.

The remainder, being less than a grain, is inconsiderable, and so not taken notice of.

Decimal given	.46725
Nearest	.41666 = 5 ounces.
Remainder	.05059
Nearest	.05 = 12 pen.-wt.
Remainder	.00059
Nearest	.00052 = 3 grains.

So if the value of .77777 of a hundred weight were sought, it would be found to be 3 *qr.* 3 *lb.* 1 *oz.* $\frac{3}{4}$.

See

See the work.

Decimal given	.77777	
Nearest less	.75	=3 quarters.
Remainder	.02777	
Nearest less	.02678	=3 pound.
Remainder	.00099	
Nearest less	.00055	=1 ounce.
Remainder	.00044	
Nearest less	.00041	=3 qr. an ounce.
		3 neglect.

And so of any other.

It remains only we should say something of the 3d table, and so conclude.

This table is of excellent use, being not only a decimal table of pence, 1 shilling being the integer, but likewise of inches, months, dozens, or any other weight or measure where the integer is divided into 12 parts: it will likewise give the decimal of any pence or farthings, supposing a pound *Sterling* were the integer; for if to the decimal of any number of pence in the table, you prefix a cipher, and take $\frac{1}{2}$ that number, it shall be the decimal of the same number of pence, a pound being the integer.

So if the decimal of 7 pence were required, and a pound the integer, it would be found to be .029166.

The decimal of 7 pence in the table, 1 *s.* the integer. } .583333

The same with a cipher prefixed is .0583333

One half of the last is the answer=.02961666=7 *d.*

And that this is the decimal of 7 *d.* may be proved by the first table.

The use of this table being so excellent, it ought by every learner to be got by heart, which is easy to do, by reading the numbers as they are expressed underneath.

For

For 11*d*. read nine, one and all ſixes.
10——Eight and all threes.
9——Seven, five.
8——All ſixes.
7——Five, eight, and all threes.
6——Five.
5——Four, one, and all ſixes.
4——All threes.
3——Two, five.
2——One, and all ſixes.
1——Nought, eight, and all threes.

Addition of DECIMALS.

ADdition of Decimals is not much different from *Addition of integers;* only you muſt take care to keep units under units in integers, and tenths under tenths in decimal parts.

EXAMPLE I.

Let it be required to add .7125 of a pound to .42 of a pound. The ſum is 1.1325, or 1*l*. 2*s*. 7*d*. 3*q*. $\frac{2}{10}$.

Place your numbers thus	.7125,	not thus .7125
	.42	.42
And the ſum will be	1.1325	Not .7167

Note, When you have added your decimals together, ſo many muſt be cut off with a daſh of your pen, as that decimal number conſiſts of, which in your example contains the moſt places; the reſt, if any, are integers; as may be ſeen in the examples.

So

	l.		
So the sum of	.7426 .846 7.612 5.5	is	l. 14.7006
	Sum 14.7006		

	l.
And the sum of	421.625 46.1625 5.21925 .416786 74.8 6618.1 24424.712
	Will be 31591.035536

But if your numbers given to be added are not all of the same denomination, they must be brought into fractions of like denominations, as in the following example is done.

Let it be required to add .725 of a pound, and .625 of a shilling, into one sum.

First, find what decimal of a pound .625 will represent, which is easily done if you prefix a cipher; for then half the number is the decimal of a pound.

The number with a cipher prefixed, is .0625, $\frac{1}{2}$ is .03125.

Then add $\left\{\begin{array}{l}.725\\.03125\end{array}\right\}$ *Sic de cæteris.*

The sum is .75625

Subtraction

Subtraction in DECIMALS.

Subtraction in Decimals differs but little from *Subtraction in Integers;* only in placing your numbers you must, as in Addition, keep units under units in integers, and tenths under tenths in decimal parts.

EXAMPLE.

Let it be required to subtract .617 from .84125, which are to be placed thus:

From	.84125
Subt.	.617
The remainder is=	.22425

	l.
So if from	25.75
You subtract	6.9845
There will remain	18.7655

If the decimal parts in either number have fewer places than the other, the vacancy is to be supplied by annexing so many ciphers as will make them equal, or supposing them to be annexed. As here,

Ciphers annexed		Ciphers supposed annexed.	
From	426.4500	From	426.45
Subt.	129.6925	The same number Subt.	129.6925
Rest	296.7575	The remainder	296.7575

But if your numbers given to be subtracted are not of the same denomination, you must, as in Addition, bring them into one denomination, as in the following example.

Let it be required to subtract .03125 of an ounce *Troy,* from .0625 of a pound *Troy.*

Seeing

Seeing one is the decimal of an ounce, and the other the decimal of a pound, bring them both into the decimal of a pound, by dividing .03125 the decimal of an ounce, by 12 of the ounces in a pound, and it will give .002604.

.03125
$\frac{1}{12}$.002604

Then { From .0625
Subt. .002604

pw. gr.
Reft .059896=14 : 8

Or you may bring them both into the decimal of an ounce, by multiplying .0625 the decimal of a pound, by 12 the ounces in a pound, which is the converſe of the laſt, and it will give .7500 or .75, both being the ſame.

.0625
12

.7500

Then { From .75
Subt. .03125

pw. gr.
Reſt .71875=14 : 8

And ſo of any other.

Multiplication in DECIMALS.

1. *Multiplication in Decimals*, both in placing your figures, and in the work itſelf, differs nothing at all from Multiplication of integers; only, when your work is finiſhed, you muſt take care that with a daſh of your pen you make as many places of decimals in your product, as there are places of decimals both in your multiplier and multiplicand; but in caſe of want in your product, annex ciphers to the left-hand.

2. In multiplication of decimals, it will be convenient to make that number the multiplicand which contains moſt places, though ſometimes it may be leſs in quantity. And *note*, That if both terms to be multiplied be decimals, the product will be a decimal; or if both be mixed, that is, if both terms conſiſt both

of

of integers and decimals, the product will be mixed: but if one be mixed and the other a decimal, the product will sometimes be mixed, sometimes a decimal.

EXAMPLE I.

Let it be required to multiply .75 by .425, the product will be found to be .31875.

Mul.	.425	multiplicand.
By	.75	multiplier.
	.2125	
	2975	
Facit	.31875	

EXAMPLE II.

The length of a board is seven foot .615 parts.
The breadth of a board is one foot .15 parts.
What is the superficial content?

Facit 8.75725; or 8 foot $\frac{3}{4}$.

Mul.	7.615	multiplicand.
By	1.15	multiplier.
	38075	
	7615	
	7615	
Facit	8.75725	the product.

EXAMPLE III.

Let it be required to multiply 2 shill. 6 pence by 2 shill. and 6 pence, 1 pound being supposed to be the integer.

The decimal answering 2 *s.* 6 *d.* or $\frac{1}{8}$ of a pound, is .125.

The

Then I multiply .125 multiplicand.
By the same .125 multiplier.

625
250
125

		l.	*s.*	*d.*	*q.*
Facit	.015625 =	00	00	3	3

The same question performed by vulgar fractions.

Multip. $\frac{1}{8}$ of a pound
By $\frac{1}{8}$ of a pound.

	d.	*q.*
Facit $\frac{1}{64}$ of a pound =	3	3

The value of $\frac{1}{64}$ of a pound, by the 4th note, in *reduction of vulgar fractions* is equal to 3 *d.* 3 *q.* as before; or if you reduce $\frac{1}{64}$ into a decimal, by proposition the first, in *reduction of decimals*, the decimal will be the same as in the former work, which may be a proof of the question.

Think it not strange that 2 *s.* 6 *d.* multiplied by 2 *s.* 6 *d.* produceth but 3 *d.* 3 *q.*; but you must understand that fractions multiplied together become less in the same proportion as integers by multiplying become greater.

Some of our pretenders to art deny this; but for the benefit of the young learner, and to stop the others mouths, we shall give you a demonstration thereof by the first proposition of the second book of *Euclid*.

Any two numbers being to be multiplied together, if you divide either or both into as many parts as you please; if then you multiply those parts one by another, the sum of those products will be equal to the product of one number multiplied by another.

Let us divide the former numbers, one into two parts, and the other into three parts.

First, Let us divide one into 1 *s.* and 1 *s.* 6 *d.* and the other into 2 *s.* and into 6 *d.* then multiply those parts one by another, as followeth.

Firſt, 6 *d.* by 6 *d.* or .025 by .025 is = .000625
Secondly, 1 *s.* by 6 *d.* or .05 by .025 is = .00125
Thirdly, 1 *s.* by 6 *d.* or .05 by .025 is = .00125
Fourthly, 6 *d.* by 2 *s.* or .025 by .1 is = .0025
Fifthly, 1 *s.* by 2 *s.* or .05 by .1 is = .005
Sixthly, 1 *s.* by 2 *s.* or .05 by .1 is = .005

Products.

Sum of the products = .015625

Which product is the ſame as was found by the multiplication of the two numbers before, which ſhews the work to be right.

But ſuppoſe the former queſtion were propounded, and a ſhilling to be the integer, then the work would have been as underneath, and the product would be 6.25, or 6 *s.* 3 *d.*

See the work.

```
 2.5
 2.5
----
 125
 50
----
Facit 6.25
```

Thus you may ſee your product will alter in value, according as you alter your integer.

EXAMPLE IV.

Let it be required to multiply 5 *l.* 12 *s.* 9 *d.* by 3 *l.* 6 *s.* 3 *d.*

Decimal

	s.	d.	
Decimal anſwering	12	9	is .6375
And of	6	3	is .3125

Then multiply 5.6375
by 3.3125

```
          281875
         112750
        56375
       169125
      169125
                          l.   s.   d.  q.
The product 18.67421875=18 : 13 : 5 : 3 3/4
```

But ſeeing we have, for the moſt part, but occaſion for three or four figures after the ſeparatrix, and ſometimes the multiplications are tedious and long, we will therefore give you a rule how you may contract your work, and yet ſecure what places of decimals you pleaſe. To do which, obſerve the following method.

Having ſet down your multiplicand as uſual, ſet the unit's place of your multiplier under that figure in your multiplicand, which ſtands as far from unity as the laſt figure of your product is deſired to ſtand, and write the reſt in the inverſe order; then multiply by your multiplier, as uſual; only *note*, That you need only begin in your multiplicand with that figure that ſtands over the figure you multiply by; having a reſpect to the increaſe that would come from the following figures of the multiplicand, placing every ſingle product exactly even at the right-hand, contrary to the common way, and adding them as they ſtand, you muſt cut off ſo many figures in your product as was deſigned; which you may better underſtand by the work of the following example.

See the last question wrought, and but three figures cut off.

Multiplicand	5.6375
Multiplier transverse	5213.3
	16913
Here you may see the work contracted much, and the product to three places, as was before.	1691
	56
	11
	3
	18.674

So if .125 were to be multiplied by .125, as in the third example before-going, and to have four figures of decimals after the separatrix.

See the work.

Multiplicand	.125	
Multiplier inverse	521.	
Here because there was but three places, I prefix a cipher, and the product to four places is the same as before.	125	
	25	
	6	
	d.	*q.*
	.0156=3	3

When a decimal fraction, or mixed number is to be multiplied by an unit with ciphers, (as 10, 100, 1000, *&c.*), you need only to remove the separatrix so many places towards the right-hand, as there are ciphers annexed to the unit. So if .1278 were to be multiplied.

$$\text{By}\begin{Bmatrix}10\\100\\1000\\10000\end{Bmatrix}\text{The product will be}\begin{Bmatrix}1.278\\12.78\\127.8\\1278.\end{Bmatrix}$$

Division

Division in DECIMALS.

Division in Decimals differs nothing from division of integers, either in placing the numbers, or in the work itself: all the difficulty being in discovering the true value of the quotient; which to do, observe the following rule, which is but the converse of that in multiplication, and is this that follows.

As many figures as are cut off in the dividend, so many must be cut off in the divisor and quotient; or thus, so many figures must be cut off in the quotient, as will make those cut off in the divisor equal to those in the dividend; taking notice if there be not so many in the quotient, you must add ciphers to the left-hand. *Note also*, If your dividend be an integer, or have less cut off than is in the divisor, it is convenient you add ciphers to the dividend, till they be equal or more; then the work will be easy.

The following examples will make all plain.

EXAMPLE I.

Where the dividend is a mixed number, and the divisor an integer, divide 742.651 by 41.

See the work. *l. s. d.*

```
41) 742.651 (18.113=18 : 2 : 3 : 1/4
    41
    ---
    332
    328
    ---
      46
      41
      ---
       55
       41
       ---
       141
       123
       ---
The quotient   18 remains.
 is 18.113.
```

 EXAM-

EXAMPLE II.

Where both are mixed numbers,
Divide 4672.565, by 25.635.

See the work.

```
25.635) 4672.565 (182
         25635
         ------
         210906
         205080
         -------
          58265
          51270
          -----
           6995 remainder.
```

In this example, because there is alike cut off in both, the quotient is an integer. And with adding of ciphers, you must bring it as far after the separatrix as you please.

EXAMPLE III.

Where both numbers are decimals, divide .75 by .0125.

Seeing I cannot divide, I add ciphers to the dividend, to wit, two, and there will be alike cut off in both; then, as in the last example, the quotient will be an integer.

See the work.

```
.0125) .7500 (60
        750
        ---
         00
```

Facit 60 in the quotient.

By

By which you may obſerve, that as multiplication of fractions decreaſeth their value, ſo diviſion of fractions increaſes the value, contrary in both to the nature of integers.

This laſt example is the ſame as if it were demanded to divide 15 ſhill. by 3 pence, the quotient will be found to be 60 pounds. The proof is eaſy by multiplication.

For if we multiply 3 *d.* or .0125 by 60 *l.* the quotient will be .75, or 15 ſhill. as you may ſee by the work.

$$\begin{array}{r} .0125 \\ 60 \\ \hline .7500 = 15 \text{ ſhill.} \end{array}$$

Suppoſing ſtill a pound *Sterling* to be the integer.

EXAMPLE IV.

Where the dividend is an integer, and the diviſor a decimal, let it be required to divide 1425, by .6252.

Here before diviſion can be well made, it will be convenient to add a competent number of ciphers.

If you only require the integral part of the quotient, add ſo many ciphers to the dividend as there are decimal parts in your diviſor, then your quotient will be wholly integral; but if you require decimal parts, ſo many ciphers more muſt be added, (beſides the number to make them equal), as you deſign to have decimal parts in your quotient.

Let us in this queſtion have three places of decimals after the integral part of the quotient, which will be 2279.270.

See

See the work.

```
.6252) 1425.0000000 (2279.270
              ......
       12504
       -----
        17460
        12504
        -----
         49560
         43764
         -----
          57960
          56268
          -----
           16920
           12504
           -----
            44160
            43764
            -----
              396 remainder.
```

EXAMPLE V.

Where the dividend is a decimal, and the divisor an integer. Let us divide .13975, by 43.

See the work.

When the division was finished, there were but three figures in my quotient; and seeing there should be 5 cut off, I therefore annex two ciphers to the left-hand, as may be seen in the example.

```
43) .13975 (.00325
        ..
     129
     ---
      107
       86
      ---
       215
       215
       ---
         0
```

And

And if 5.29125 were divided by 42.5, the quotient would be .1245.

See the work.

```
42.5) 5.29125 (.1245
       ...
      425
      ----
      1041
       850
      ----
      1912
      1700
      ----
       2125
       2125
       ----
         0
```

When any decimal fraction, or mixed number, is to be divided by an unit with any number of ciphers annexed, it is but removing the separatrix so many places towards the left-hand, as there were ciphers annexed to the unit.

So if 17.28 were given to be divided

By	10	The quotient will be	1.728
	100		.1728
	1000		.01728
	10000		.001728

By what goeth before, it may be observed, That if the dividend be greater than the divisor, the quotient will either be an integer or a mixed number; but if the divisor be greater, the quotient will be a decimal.

Multiplication and division in decimals (as in integers) interchangeably prove each other.

To prove multiplication, divide the product by the multiplier, quotes the multiplicand; or by the multiplicand, quotes the multiplier.

To prove division, multiply the quotient by the divisor, produceth the dividend.

Before

Before we leave division in decimals, we will give the learner the resolution of two excellent problems, which will be of good use.

The first is, having a multiplicator, to find a divisor.

Divide an unit with ciphers by the multiplicator, the quotient will be the divisor sought.

EXAMPLE.

What divisor is that, by which dividing 7315, shall give a quotient equal to the product of the same number, multiplied by 125? *Facit* .008.

See the work.

```
125) 1.000 (.008.
     1.000
     -----
        0
```

The PROOF.

```
  7315             .008) 7315.000 (914375
   125                   72
 -----                   --
 36575                    11
14630                      8
7315                      --
------                     35
914375                     32
                           --
                            30
                            24
                            --
                             60
                             56
                             --
                              40
                              40
                              --
                               0
```

Here you may see the product and quotient are the same.

The ſecond is, having a diviſor, to find a multiplicator.

This is but the converſe of the former; for if you divide unity with ciphers annexed by the given diviſor, the quotient will be the multiplicator ſought.

EXAMPLE.

What multiplicator is that by which multiplying 7315, ſhall give a product equal to the quotient of the ſame number, divided by .008? *Facit* 125.

See the work.

```
.008) 1.000 (125
       8
      ---
       20
       16
       ---
        40
        40
        ---
         0
```

The proof is in the laſt.

Golden Rule in DECIMALS.

We ſhall not in this place need to give a definition of the *Rule of Three*, that being ſufficiently done in *Vulgar Arithmetic*.

For ſeeing the *Rule of Three* in decimals is the ſame, both in the ſtating and working of the queſtion, as in the *Rule of Three* before taught, reſpect being had to the rules in decimals aforegoing, which if well underſtood, any queſtion of the *Golden Rule*, though conſiſting of never ſo croſs fractional parts, will receive its reſolution as eaſily as if the queſtion were compoſed of integers only; which ſhall be made plain in the following examples.

QUEST.

QUEST. I.

If 7 yards and three quarters of cloth cost 2 *l.* 12 *s.* 9 *d.* what will 140 yards and a half of the same cloth cost?

The fractional parts reduced to decimals by the rules aforegoing, and stated as taught in the *Rule of Three* in *Vulgar Arithmetic*, the work will stand as follows.

```
     Yds.      l.        Yds.
If 7.75 : 2.6375 : : 140.5
            140.5
          -------
           131875
          105500
         26375
         --------
7.75) 275.61875 (35.563
      2325 ....
      -----
       4311
       3875
       -----
        4368
        3875
        -----                l.    s.   d.
         4937  Answer 35 : : 11 : 3.170
         4650
         -----
          2875
          2325
          -----
           550
```

QUEST. II.

If a chest of sugar weighing 7 *C.* 2 *qr.* 14 *lb.* cost 36 *l.* 12 *s.* 9 *d.* what will 2 *C.* 1 *qr.* 21 *lb.* of the same sugar cost?

The fractional parts reduced into decimals, and stated as before taught, the work will stand as underneath.

	C.	lb.		C.
If	7.626 :	36.6375	::	2.4375

```
                2.4375
              --------
              1831875
             2564625
            1099125        Ans. 11 l. 14 s. 2 d. 1 q.
           1465500
           732750
          ----------
7.626) 89.30390625 (11.71045
          ----------
Remainder  1455
```

QUEST. III.

A grocer buys 24 tuns, 12 hundred, 2 quarters, 14 pounds, and 12 ounces of tobacco, for 3678 pounds, 6 shillings and 4 pence; What will 1 ounce of this tobacco cost?

The question reduced and stated, will stand thus, as beneath.

	C.	lb.		C.
If	492.6316 :	3678.3166	::	.000558

```
                    .000558
                   ---------
492.6316) 2.0525007 (.00416
```

The work at large is neglected, only the product and quotient. } *Ans.* 1 *d.* the ounce *fere.*

QUEST. IV.

If 16 pioneers make a trench in 1 month and 14 days, how many pioneers will make the same trench in 12 days? *Ans.* 56 pioneers, 28 days to the month.

See the work.

M. *P.* *M.*

If 1.5 : 16 : : .42857

1.5

80

16

.42857) 24.00000 (56

214285

257150

257142

8

In my product, because I had but one decimal place, I annexed 4 ciphers, to equal the number of decimal places in my divisor, that so my quotient might be an integer.

Note, This and the two next questions are done by the reverse rule.

QUEST. V.

If when wheat is sold for 12 shillings the quarter, the half-penny white-loaf ought to weigh 1 pound, 1 ounce, and 12 penny-weight; what must the half-penny white-loaf weigh, when wheat is sold for 1 pound 16 shillings and 3 pence the quarter? *Answer*, The half-penny white-loaf ought to weigh 4 ounces and 10 penny-weight.

l. *lb.* *l.*

If .6 : 1.1333 : : 1.8125

.6

oz. *pwt.*

1.8125) .68000 (.3751 = 4 : 10

54375

136250

126875

93750

90625

31250

QUEST.

QUEST. VI.

What length of a board 9 inches broad will make a ſquare foot, when 12 times 12, or 144 inches make 1 foot? Say, If 12 in breadth require 12 in length, what will 9 in breadth require? *Anſ.* 16 inches in length.

See the work.

```
     B.   L.   B.
If 12 : 12 :: 9
          12
        ----
      9) 144 (16
          9
        ----
          54
          54
        ----
           0
```

Double Golden Rule in DECIMALS.

We ſhall here give you an example or two in the *Double Rule of Three in Decimals*, or *Rule of Plural Proportion*, and ſo conclude this rule.

QUEST. I.

If three labourers in two months and twelve days thraſh 221 quarters, three buſhels, and two pecks of corn, how much will nine labourers thraſh in one month, two weeks, and five days.

Lab. *Quar.* *Lab.*

Firſt ſay, If 3 thraſh 221.4375, what will 9 thraſh?

Quar.

Facit 664.3125

Say again, If 2.42857 months thraſh 664.3125 quarters, what will 1.67857 months thraſh? *Facit* 459.156 equal to 459 quarters, one buſhel, and one peck.

QUEST. II.

Mr *Bridges*, in his *Lex Mercatoria*, page 222, hath the following question.

If 100 *l.* in 12 months gain 8 *l.* what will 7390 *l.* 13 *s.* 11 *d.* 3 *q.* gain in 9 months? And further saith, The most methodical way of working this question will require about 300 figures more than the practical way he shews in the work of the said question, *p.* 188 of the said book; which we will examine.

First, I say, If 100 : 8 : : 7390.699

8

Facit 591.25592 = 1 year's gain.

M. *l.* *M.*

Say again, If 12 : 591.25592 : : 9

9

5321.30328

$\frac{1}{12}$ 443.4419 = 443 *l.* 8 *s.* 10 *d.* the *ans.*

Here is but about 40 figures besides the *answer*, taking in the stating of the question and all, which we might have contracted into much fewer; and his practical way, before-mentioned, have above 60 figures besides the *Answer*; and how he will find a more methodical way I know not.

Mr *Bridges* had no reason to undervalue decimal arithmetic so much, if he had notice thereof; it and the logarathims being two of the most famous inventions the preceding ages have been masters of; and if we use the practical way, it is convenient the fractional parts be reduced into decimals, as in the annexed operation is manifest.

First,

First, I multiply by 8, cutting off 5 fig. *viz.* 3. for the decimal, and 2 instead of dividing by 100; that done, I took half for six months, and half of five months for 3 months, which two numbers added, make 443*l.* 8*s.* 10*d.* as before.	7390.699 8 ——— 591.25592 ——— 295.6279 147.8139 ——— 443.4418=443*l.* 8*s.* 10*d.*

QUEST. III.

If 2 angels be equal to 20 shillings, and 15 shillings equal to 3 crowns, and 60 crowns equal to 15 pounds, and 13 pounds equal to 12 guineas; how many angels will countervail 650 guineas?

s. *Ang.* *s.*
First, I say, If 20 : 2 : : 15 : *Facit* 1.5 angels.
Cr. *Ang.* *Cr.*
Secondly, If 3 : 1.5 : : 60 : *Facit* 30 angels.
l. *Ang.* *l.*
Thirdly, If 15 : 30 : : 13 : *Facit* 26 angels.
Guin. *Ang.* *Guin.*
Lastly, If 12 : 26 : : 650 : *Facit* 1408.333 angels.

Ans. 1408.333 *ang.* or, 1408 *ang.* 3 *shill.* and 4 *pence.*

The last question may be wrought by division only, by placing your numbers as underneath.

THUS,

If 2 *angels* equal 20 *shillings*,
And 15 *shillings* equal 3 *crowns*,
And 60 *crowns* equal 15 *pounds*,
And 13 *pounds* equal 12 *guineas*,
What will 650 *guineas* equal?
Answer, 1408 *angels* $\frac{1}{3}$.

Here if you multiply the 1st, 3d, 5th, 7th, and 9th for a dividend; and the 2d, 4th, 6th, and 8th, for a divisor, the quotient is the *answer*, which you may try at your leisure.

QUEST. VII.

If wine worth 15 *l.* 12 *s.* 9 *d.* be sufficient for the ordinary of 100 men, when it is worth 25 *l.* 15 *s.* *per* tun: how many men will 3 pounds worth satisfy, when wine is worth 50 *l.* *per* tun?

	l.	*Men.*		*Men.*
First say, If	15.6375 :	100 : :	3 : *Facit*	19.184.

Say again, If 25.75 *l.* *per* tun suffice 19.184, what number of men will 50 *l.* *per* tun require? *Facit* 10 men *fere*.

Rule of Practice in DECIMALS.

ALthough we have treated before of the *Rule of Practice*, yet because many times your question may consist of integers and fractional parts, and seeing many times questions may be more easily wrought by reducing the parts into decimals, whereby the operation will be the same as in integers; we thought fit to give the learner a touch thereof, no one having done it before.

First, If at any time your question consists of the aliquot parts of two shillings, your question may be wrought at one operation, without reducing the answer into pounds; for two shillings being the decimal part of a pound, the answer at two shillings is found by inspection, by separating the last figure towards the right-hand from the rest, for a decimal fraction; the rest is pounds.

EXAM.

EXAMPLES.

(1.)
At 6*s.* the yard, what will 64.3 yards cost?
Ans. l. 19 : 5 : 9½.

(2.)
At 8*d.* the yard, what will 144 yards cost?
Ans. at 2*s.* is 14.4.
⅓ for 8*d.* is 4.8, or 4 *l.* 16*s.*
Ans. 4 *l.* 16*s.*

More EXAMPLES.

(3.)
At 4 *d.* the yard, what will 144 yards cost?

At 2*s.*	14.4
⅙ part is	2.4

Ans. 2 *l.* 8*s.*

(4.)
At 3 *d.* the yard, what will 145 yards cost?

At 2*s.*	14.5
⅛ part is	1.8125

Ans. 1 *l.* 16*s.* 3 *d.*

And if your question consist not of aliquot parts of two shillings, you may easily divide it into aliquot parts, as in the *examples* following.

(1.)
At 9 *d.* the yard, what will 724 yards cost?

$\frac{1}{10}$ yard is	72.4
¼ for 6 *d.* is	18.1
½ of 6*d.* for 3 *d.* is	9.05
Sum	27.15

Ans. 27 *l.* 3*s.*

(2.)
At 16 *d.* the yard, what will 721 yards cost?

$\frac{1}{10}$ is	72.1
⅓ for 8 *d.* is	24.0333
the same	24.0333
Sum	48.0666

Ans. 48 *l.* 1 *s.* 4 *d.*

And so of any other.

Now, seeing half the number of shillings is the decimal thereof, any question of practice, consisting of any number of shillings, may be answered by an easy multiplication, and the answer given in pounds, and parts of a pound; the value of which fraction, by the compendious method of valuing a fraction of a pound *Sterling*, taught in *Reduction* aforegoing, may be known by inspection.

EXAM-

EXAMPLES.

(1.)

At 12 *s.* the yard, what will 72 yards cost?

Mult. 72
By .6

Prod. 43.2
Ans. 43 *l.* 4 *s.*

(2.)

At 15 *s.* the yard, what will 65 yards cost?

Mult. 65
By .75

325
455

Prod. 48.75
Ans. 48 *l.* 15 *s.*

(3.)

At 1 *l.* 13 *s.* 9 *d.* the yard, what will 48 yards cost?

For one pound is 48 *l.*
Prod. by 65 for 13 *s.* is 31.2
$\frac{1}{4}$ of 4.8 for 6 *d.* is 1.2
$\frac{1}{2}$ of that for 3 *d.* is .6

Ans. 81 *l.* 0.

(4.)

At 14 *s.* 8 *d.* the yard, what will 144 yards cost?

Mult. 144
By .7

100.8
4.8 $\frac{1}{3}$ of 14.4 for 8 *d.*

Prod. 105.6 *Ans.* 105 *l.* 12 *s.*

But if at any time, as often it will happen, that your given number hath a fraction annexed, such fraction is to be reduced into a decimal: so will the operation be as easy and facil, as if it consisted of integers only.

EXAMPLES.

I. At 8 *d.* the yard, what will 672 $\frac{3}{5}$ yards cost?

Your number after reduction stands thus,	672.6
$\frac{1}{10}$ for 2 shillings is	67.26
$\frac{1}{3}$ of 2 shillings for 8 *d.* is	22.42

Ans. is 22 *l.* 8 *s.* 4 *d.* 3 *q.* $\frac{2}{10}$ or $\frac{1}{5}$.

II. At

II. At 3 *s.* 6 *d.* the groſs, what will $25\frac{3}{4}$ groſs coſt?
Your number after reduction ſtands thus, 25.75 groſs.
Mult. by the decimal of 3 *s.* 6 *d.* to wit, .175

```
   12875
  18025
  2575
 4.50625
```

Anſw. 4 *l.* 10 *s.* 1 *d.* $\frac{1}{2}$.

Take a queſtion or two in *Averdupois weight.*

QUEST. I.

At 5 *l.* 15 *s.* 7 *d.* $\frac{3}{4}$ the hundred, what will 218 *C.* 3 *qr.* and 24 *lb.* coſt?

See the work at large.

The weight reduced ſtands thus, 218.96428
Multiply by 5.78229

```
          197067852
         43792856
        43792856
       175171424
      153274996
     109482140
    1266.1149666012
```

See Kerſey *upon* Wingate, page 332.

Anſ. 1266 *l.* 2 *s.* 3 *d.* 3 *q. ferè.*

The work by contraction in multiplication of decimals.

Multiplicand 218.964
Multiplicator inverſe 92287.5

```
1094821
 153275
  17517
    438
     44
     19
1266.114
```

Anſwer, 126 *l.* 2 *s.* 3 *d.* 3 *q. ferè,* as before.

QUEST.

QUEST. II.

At 1 *l.* 17 *s.* 5 *d.* $\frac{1}{4}$ the hundred, what will 57 *C.* 2 *qrs.* 17 *lb.* 11 *oz.* cost?

The multiplicand	57.6579
Multiplicator inverse	78178.1
	576579
	461263
	40360
	577
	461
	40
Answer 107 *l.* 18 *s.* 6 *d.* $\frac{3}{4}$	107.9280

Take a question or two in *lead-weight*.

QUEST. I.

At 9 *l.* 17 *s.* 5 *d.* the fodder of lead, what will 17 fodder, 14 hundred, 1 quarter, and 17 pound cost? 19 hundred and half, one fodder,

The multiplicand,	17.73855
Multiplicator inverse	38078.9
	1596470
	141908
	12416
	142
	5
	175.0941

See Spiedel's *Arithmetical Extraction*, pages 95 *and* 96, *where you may find the work of this question, whereby you may see the facility of this operation from that.*

Answer, 175 *l.* 1 *s.* 10 *d.* $\frac{1}{2}$ +

But lest the learner should stumble in reducing these sorts of weight into decimals, and seeing they are useful operations, I thought it convenient, for the learner's advantage, to annex a *decimal table* for that purpose; and it is as followeth.

A

A TABLE of *Lead Weight*.

Lead Weight in decimals, one fodder the integer.

Hund.	*Decimals.*
19	.974358
18	.923076
17	.871794
16	.820512
15	.769230
14	.717948
13	.666666
12	.615384
11	.564102
10	.512810
9	.461538
8	.410256
7	.358974
6	.307692
5	.256410
4	.205128
3	.153846
2	.102564
1	.051282

Quart.	*Hundred. Decimals.*
3	.038461
2	.025641
1	.012820

Pounds.	*Decim.*
27	.0123626
26	.0119047
25	.0114469
24	.0109890
23	.0105311
22	.0100732
21	.0096153
20	.0091575
19	.0086996
18	.0082417
17	.0077838
16	.0073260
15	.0068681
14	.0064102
13	.0059523
12	.0054945
11	.0050366
10	.0045787
9	.0041208
8	.0036630
7	.0032051
6	.0027472
5	.0022893
4	.0018315
3	.0013736
2	.0009157
1	.0004578

Ounces	*Decimals.*
15	.0004292
14	.0004006
13	.0003720
12	.0003434
11	.0003148
10	.0002861
9	.0002575
8	.0002289
7	.0002003
6	.0001717
5	.0001430
4	.0001144
3	.0000858
2	.0000572
1	.0000286

Quar. oz.	*Dec.*
3	.0000214
2	.0000143
1	.0000071

QUEST.

QUEST. II.

At 9 *l.* 11 *s.* 1 *d.* ½. the fodder, what will 18 fodder, 13 hundred, 3 quarters, 17 pound, 13 ounces cost?

The weight reduced by the foregoing table is	18.7133
Multiplicator inverse	52655.9
	1684197
	93567
	9357
	1122
	37
Answer, 178 *l.* 16 *s.* 7 *d.* *fere.*	9
Product	178.8289

Note, Though some of the foregoing questions be wrought by *Multiplication in Decimals,* and not by the practical way of aliquot parts before taught, the reason thereof is, that in those, and the like particular questions, the practical way is more intricate, and tedious, and requires more figures than the method here used, as may be observed, if the reader peruse Mr *Spiedel's Arithmetical Extraction,* pages 95, 96, and 97, in the practical way of the said questions.

Here follows a question or two in exchange, and so we will conclude this rule.

QUEST. I.

A merchant having received a bill of exchange for 593 ½ pieces, at 7 shill. 9 ¾. pence *per* piece, what Sterling money will they amount to? *Answ.* 231 *l.* 16 *s.* 8 *d.* 2 *q.* ½.

The work.

Number given	593.5
$\frac{1}{4}$ for 5 shillings is	148.375
$\frac{1}{10}$ for 2 shillings is	59.35
$\frac{1}{10}$ of 5 shillings for 6 *d.* is	14.8375
$\frac{1}{2}$ of 6 *d.* for 3 *d.* is	7.41875
$\frac{1}{4}$ of 3 *d.* for 3 *q.* is	1.85468
Sum	231.84593

QUEST. II.

One hath changed 759 double pistolets at 11 *s.* 8 *d.* $\frac{1}{2}$. *per* piece, what will they come to? *Answ.* 444 *l.* 6 *s.* 7 *d.* $\frac{1}{2}$.

Given number,	759.
$\frac{1}{2}$ for 10 shillings	379.5
$\frac{1}{10}$ of 10 *s.* for 1 *s.*	37.95
$\frac{1}{2}$ of 1 *s.* for 6 *d.*	18.975
$\frac{1}{3}$ of 6 *d.* for 2 *d.*	6.325
$\frac{1}{4}$ of 2 *d.* for 2 *q.*	1.58125
The sum,	444.33125

Hereunto let us annex a compendious method of buying or selling, by the hundred neat, or hundred *Averdupois*; as oft as your question is but of a small price.

For the little, or true hundred, for as many farthings as the pound costs, account twice so many shillings, and once so many pence.

For the great hundred, or 112 pound, as many farthings as the pound costs, twice so many shillings, and once so many groats the hundred gross will cost.

EXAMPLES in both.

I. At 3 *d.* ½ the pound, what will 100 pound cost?

	l.	*s.*	*d.*
3 *d.* ½ is 14 far. twice so many shill. are 28, or	1	8	0
Once so many pence are 14, or	0	1	2
Answer, 1 *l.* 9 *s.* 2 *d.* Sum	1	9	2

II. At 2 *d.* 1 *q.* the pound, what will the hundred *Averdupois*, or 112 pound, cost?

	l.	*s.*	*d.*
2 *d.* ¼ is 9 farth. twice so many shill. are	0	18	0
Once so many groats is	0	03	0
Answer, 1 *l.* 1 *s.* Sum	1	01	0

I shall in this place annex one question, to make the learner a good husband if possible.

The question is, that if one square yard of land cost a penny, what will buy an acre, 160 perches being an acre and seven yards a perch?

Yards in a perch	7	Perches in an acre	160
Multip. by itself	7	Square yards in a perch	49
Product	49		1440
Square yds in a perch.			640
		Yards in an acre	7840

At 1 *d.* the yard, what will 7840 yards cost?

653|3=4 *d.*

Facit 32 *l.* 13 *s.* 4 *d.*

And the yearly rent which 32 *l.* 13 *s.* 4 *d.* will purchase at 6 *per cent* compound interest, or the annual rent of an acre, will, by the rules in compound interest following, be found to be 1.96 *l.* or 1 *l.* 19 *s.* 2 *d.* 1 *q.* $\frac{6}{10}$. near 40 shillings.

Whereby

Whereby it is evident, that he that spends one penny, spends or makes away a square yard of as good land as most in *England*, from him and his heirs for ever.

And it is a question whether *England* be worth 20 shillings an acre annually, taking one acre with another. How much good land we make away, it is easy to judge.

And he that spends a penny a day, spends one pound, one half-pound, one groat, and one penny; and so by consequence, two pence a-day will be two pounds, two half-pounds, two groats, and two pence; and three pence, three pounds, three half-pounds, three groats, and three pence, *&c. per* year.

Extraction of the SQUARE ROOT.

A Square number is that which is contained under two equal numbers, or which is equally equal.

So 4 is a square number, contained under two equal numbers; to wit, 2 and 2; for two times 2 is 4: and the square number 9 is contained under 3 and 3; for 3 times 3 makes 9; and of the rest as in the following table.

A table of squares with their genitive equal numbers, as far as the first 9 digits.

Equal Numbers.	Squares
1 —— into —— 1 —— is	1
2 —— into —— 2 —— is	4
3 —— into —— 3 —— is	9
4 —— into —— 4 —— is	16
5 —— into —— 5 —— is	25
6 —— into —— 6 —— is	36
7 —— into —— 7 —— is	49
8 —— into —— 8 —— is	64
9 —— into —— 9 —— is	81

And when it is required to extract the ſquare root of any given number, we have nothing to do but to find that equal number of which it is compoſed: ſo if the root of 16 were required, it would be found to be 4, as in the ſaid table.

Here four is the root, called by ſome *the firſt power*, and 16 is the ſquare, called *the ſecond power*.

Of numbers to be extracted, are three ſorts.

Firſt, Single.

Secondly, Compound.

Thirdly, Irrational.

Single, are ſuch ſquares as are compoſed or made up of any of the 9 digits; of which ſort are thoſe in the aforegoing table.

Compound, are all ſuch ſquares as are compoſed of more figures than one, as 100, whoſe root is 10; 121, whoſe root is 11; or 144, whoſe root is 12, *&c.*

Irrational, are all ſuch ſquares, whoſe roots cannot be diſcovered by art exactly, neither in whole numbers or fractions, but ſomething will ſtill remain, there being no proportion yet found betwixt an irrational number and its root; ſuch numbers are 3. 7. 19. 74. 156. 751. *&c.*

The extraction of the ſquare root is not much unlike diviſion, only there our diviſor is fixed, here we are to ſeek a new one for each operation.

The root of any ſingle ſquare number is found by inſpection, as in the foregoing table may be ſeen.

But if it be a compound ſquare number, it muſt be prepared by pointing thus: Make a point under your unit's place, and omitting one, point every other figure. And as many points as your number contains, ſo many figures will your root conſiſt of.

Then proceed by the following directions.

A

A Rule to be got by heart.

The root of your first period you
Must place in quote, if you work true:
Whose square from your said period then
You must subduct; and to th' remain.
Another period being brought,
You must divide as here is taught.
By the double of your quote, but see
Your unit's place you do leave free;
Which place will be suppli'd by th' square
Of your next quoted figure there:
Next multiply, subduct, and then
Repeat your work unto the end;
And if your numbers be irrational
Add pairs of ciphers for a decimal.

EXAMPLE.

Let it be required to find the square root of 451584.

Having pointed it as in the work, shews the root will have three places.

1. Seek the greatest root of your first period 45, which by your table you will find to be 6, which place in your quotient, and the square thereof under 45 your first period, subtract 36 from 45, rest 9. This is your first work, and is no more to be repeated.

2. To the remainder bring down your next period 15, makes 915 for a dividend, or, as some call it, a resolvend as you may see in the work.

The work.

451584 (6
36
—
9

451584 (6
36
—
) 915

3. Double

3. Double your quote 6 makes 12 for a divisor, then seek how oft 12 in 91, or how oft 1 in 9, (reserving the unit's place for the square of my sought figure), which I find to be 7, which I place in my quotient, and, to save trouble of addition to the right-hand of my divisor as a part thereof, making it 127; then multiplying 127 by 7, the product I place under my dividend, or resolvend, as you see.

```
     451584 (67
      . . .
     36
     ——
127) 915
     889
     ——
```

This work is every time to be repeated.

4. Subtract 889 from 915, rest 26, to which I bring down my third and last period 84; then shall I have 2684 for a new dividend, to resolvend, as you may see in the work itself.

```
     451584 (67
      . . .
     36
     ——
127) 915
     889
     ——
   ) 2684
```

5. Double your quotient 67, *facit* 134 for a new divisor, then I ask how oft 134 in 268, (still reserving my unit's place in the dividend), or, which is the same, how oft 1 in 2?

```
      451584 (672
       . . .
      36
      ——
 127) 915
      889
      ——
1342) 2684
      2684
      ——
      0
```

Answ. 2 times, which I place in my quotient, and likewise on the right-hand of my divisor, making it 1342; then multiplying 1342 by 2, the product, to wit 2684, I place under my dividend; and seeing they are equal, and that nothing remains, I find my number

ber was a square rational number, and the root is 672.

To prove your work, multiply 672=root
By 672

```
                              1344
                             4704
                            4032
                            ------
                            451584=Given number.
```

After the like manner the square root of 2985984 would be found to be 1728.

But if your number to be extracted have a remainder, then you may know it is irrational, and the root cannot be got exact: although, by adding ciphers, you may come as near the truth as you please.

EXAMPLE.

Let it be required to extract the square root of 160, or, which is the same, to find the length of one side of a square acre.

Having pointed my number, and wrought as before, I find 12 for my nearest root, and 16 to remain, to which adding two ciphers, I find my next figure to be 6, which I cut off from the rest, as part of a decimal fraction, which by continually adding pairs of ciphers to each remainder, I increase to 5 places, which is exact enough, not wanting 2 parts, if unity were divided into a hundred thousand parts; for if I square 12.64911, it will produce 159.9998379921.

See the work.

```
  160(12.64911
  . .
  1
  ------
22)060
    44
  ------
246)1600
    1476
  ------
2524) 12400
      10096
  ------
25289) 230400
```

Thus

Thus the ſquare root of any mixed number, may be found, the fractional part firſt reduced into even places of decimals, or ſupplied, if need be; ſo if the ſquare root of $17\frac{1}{2}$ were required to 3 places of decimals, the work would ſtand as here, and the ſquare root would be 4.183.

```
25289) 230400
       227601
       ------
252981) 279900
        252981
        -------
2529821) 2691900
         2529821
         -------
          162079
```

See the work.

```
17.500000 (4.83
   .  .  .
16
--
81) 150
     81
    ---
828) 6900
     6624
     ----
8363) 27600
      25089
      -----
       2511
```

The ſquare root of a vulgar fraction, that is commenſurable to its root, may eaſily be found, by extracting the ſquare root of the numerator for the numerator of the root, and likewiſe the ſquare root of the denominator for the denominator of the ſaid root, which fraction is the root ſought. So if the ſquare root of $\frac{9}{49}$ were required, it would be found to be $\frac{3}{7}$, for the ſquare root of 9 is 3, and 49 is 7, equal to $\frac{3}{7}$: and ſo of any other.

After this manner may the ſquare root of a mixed number,

number, which is commensurable to its root, be easily found.

But if your fraction be incommensurable to its root, then the best way will be to reduce it into a decimal, and extract the root as before taught.

So if the square root of $\frac{3}{80}$ were required unto 4 places of decimals, it would be .1936, as you see in the work.

$\frac{3}{80}$ is equal to .0375

Then .0375 (.1936

```
         1
       -----
  29) 275
      261
      -----
 383) 1400
      1149
      ------
3866) 25100
      23196
      ------
       1904
```

1904 And so farther if you please.

But if you have it to fall in some operation, you may prefix its radical sign before it thus, $\sqrt{\frac{3}{80}}$; and so of any other.

In the last place, I will shew how to find the square root of an irrational number nearly, without the help of decimals, being a useful notion for such as understand not those fractions; and it is thus.

After you have found the integral part of your root to its quadruple, add unity for the denominator of the fractional part, and the remainder doubled is numerator: so the root of 160 in this method will be $12\frac{32}{49}$; and thus of any other.

Extraction of the CUBE ROOT.

A Cube number is that which is contained under 3 equal numbers, or which is equally equal.

So 8 is a cube number, contained under 3 equal numbers, to wit, 2 2 and 2, for 2 times 2 is 4, and 2 times 4 is 8; and the cube number 27, is contained under 3 3 and 3, for 3 times 3 is 9, and 3 times 9 is 27; and of the rest as in the following table.

A TABLE of cubes, with their genitive equal numbers, as far as 9 the digits.

Equal numbers.	Cubes.
1 —— into 1 —— into 1 —— is——	1
2 —— into 2 —— into 2 —— is——	8
3 —— into 3 —— into 3 —— is——	27
4 —— into 4 —— into 4 —— is——	64
5 —— into 5 —— into 5 —— is——	125
6 —— into 6 —— into 6 —— is——	216
7 —— into 7 —— into 7 —— is——	343
8 —— into 8 —— into 8 —— is——	512
9 —— into 9 —— into 9 —— is——	729

And when it is required to extract the cube root of any given number, we have nothing to do but to find that equal number of which it is composed; so if the root of 64 were required, it would be found to be 4, as in the table.

Here four is the root, or first power, and 4 times 4 is 16, the second power, and 4 times 16 is 64, or the third power, which is the cube.

Of cube numbers to be extracted, are three sorts.

First, Single.

Secondly, Compound.

Thirdly, Irrational.

Single,

Single, are all ſuch cubes as are compoſed or made up of any of the 9 digits, of which ſort are thoſe in the foregoing table.

Compound, are all ſuch cubes as are compoſed of more figures than one, as 1000, whoſe root is 10, or 1331, whoſe root is 11, or 1728, whoſe root is 12, *&c.*

Irrational, are all ſuch cubes, whoſe root cannot be diſcovered by art exactly, neither in whole numbers, nor fractions, but ſomething will ſtill remain, there being no proportion yet found betwixt an irrational or ſurd number, and its root: ſuch numbers are 5. 7. 36. 160. 1526. *&c.*

The extraction of the cube root participates ſomething of the nature of diviſion, yet a deal more difficult. The root of any ſingle cube number is found by inſpection; as in the foregoing table may be ſeen.

But if it be a compound cube number, it muſt be prepared by pointing thus: Make a point under your unit's place, and omitting two, point every third figure; and as many points as your number contains, ſo many figures will your root conſiſt of. Then proceed by the following directions.

A rule to get by heart.

The cube of your firſt period take,
And of its root a quotient make;
Which root into a cube muſt grow,
And from your period taken fro:
To the remainder, then you muſt
Bring down another period juſt;
Which being done, then you muſt ſee
Your numbers ſtreight divided be
By juſt three hundred times the ſquare
Of what your quotient figures bear;
Which do ſo that you in may take
The fact your quotient figures make;

Last, squar'd and multiply'd by th' rest
And product thirty times exprest.
The cube of your last found figure too
You must put in, if right you do;
Repeat your work, and so descend
From point to point unto the end;
That done, if ought remain you shall
Add trebled ciphers for a decimal.

EXAMPLE I.

Let it be required to extract the cube root of 46656.

1. First, point your number as directed, whereby you may see the root will have but two places.

2. Seek the greatest root of your first period 46, which, by the aforegoing table, you will find to be 3: which place in your quotient, and the cube thereof 27 place under 46. Subtract 27 from 46, and there will rest 19, as you see, if you observe the work: this is your first work, and no more to be repeated.

```
46656 (3
27
——
19
```

3. To your remainder 19, bring down your next and last period 656, and it will make 19656 for a dividend; then square your quotient, 3 makes 9, which multiply by 300, produceth 2700 for a divisor. Seek how oft 3 in 19? *Answer*, But 6 times, because of the increase that will come from my quotient. Then I multiply my divisor by 6, and the product 16200 I place orderly under my dividend, having separated them with a small line; then proceed to find the increase coming from my quotient; thus square your last figure 6, *facit* 36, which multiply by the rest of your quotient here by 3, *facit* 108, and this by 30, *facit* 3240, which place orderly under my last number 16200; then cube the figure last placed in your quotient here 6, *facit* 216, which place orderly under your last number 3240, and add your three subducends

cends (for ſo may you have in every operation after the firſt) into one ſum, *facit* 19656; and ſeeing it is equal to my dividend, and no more periods to bring down, I ſee my work is finiſhed, and my number a right cube number, and the root is 36.

Note, As many operations or periods as you have, except the firſt, ſo oft this laſt work is to be repeated.

See the work.

	46656	(36 quote, equal the root.
	27	
2700)	19656	dividend.
	16200	ſubducends.
	3240	
	216	
Sum	19656	from dividend ſub.
Reſt	00	

P R O O F.

Root	36	Square	1296
	36	Root	36
	216		7776
	108		3888
Square	1296		46656

E X A M P L E II.

Let it be required to find the cube root of this number 673373097125.

1. Firſt, I point my number, by which I ſee my root will have four places.

2. Next, seek the greatest root of your first period 673, which by the table is 8, which place in your quote, and the cube thereof 512 place under 673, and subtract, rest 161; this is the first work, and no more to be repeated.

673373097125 (8
. . . .

512

161

3. To the remainder 161, bring down your next period 373, makes 161373 for a dividend, to which 19200 (being 300 times the square of 8 your quotient) is a divisor; and considering how oft my divisor is contained in my dividend, (so as to allow place for my subducends), I find it 7 times; place 7 in the quotient, by which multiplying my divisor, the product I place under my dividend for my first subducend. Next, I square my last figure 7, which multiplied by 8, and then by 30, gives 11760 for my second subducend, which I place under my last, and the cube of 7, my last quoted figure, is my third subducend, which I place under the other two, and adding them, the sum is 146503, which I subtract from my dividend, and the remainder is 14870; then will the work appear thus:

673373097125 (87
. . . .

512

1 Divisor 19200) 161373 dividend 1.

134400 }
11760 } subducends.
343 }

Sum 146503 from dividend subtract.

Rest 14870

3. To

3. To this remainder bring down your next period 097; then will your second dividend be 14870097, to which 2270700 (being 300 times the square of your quotient 87) is the divisor, and dividing by the caution before given, I find the next figure of my root to be 6, and my first subducend is 13624200 square 6, *facit* 36, which multiplied by 87, makes 3132, and this by 30, gives 93960 for my second subducend, and the cube of 6, which is 216, is my third subducend; which placed as before taught, and as you see in the work, and then added, the sum is 13718376, which I subtract from my last dividend, and the remainder is 1151721.

Then will the work appear as in the following operation.

673373097125 (876

512

(1.) Divisor 19200) 161373 dividend (1.)

$$\left.\begin{array}{r}134400\\11760\\343\end{array}\right\}\text{ subducends.}$$

Sum 146503 from dividend sub.

(2.) Divisor 2270700) 14870097 dividend (2.)

$$\left.\begin{array}{r}13624200\\93960\\216\end{array}\right\}\text{ subducends.}$$

Sum 13718376 from dividend sub.

Rest 1151721

4. To this remainder bring down the last period 125, and your third and last dividend will be 1151721125, to which 23021 2800 is divisor, which is 300 times the

ſquare of 876 your quotient; and dividing as before, I find my fourth figure to be 5, and my firſt ſubducend is 1151064000, and mult. 876 by the ſquare of 5, and that by 30, gives 657000 for my ſecond ſubducend, and the cube of 5, to wit, 125, is my third ſubducend, which added into one ſum makes 1151721125: and ſeeing it is equal to my laſt dividend, and no more to bring down, I ſee my work is finiſhed, and the number given a right cube number; and my root ſought is 8765. And the whole work appears as here.

673373097125 (8765

512

(1.) Diviſor 19200) 161373 dividend (1.)

134400
11760 } ſubducends.
343

Sum=146503 from dividend ſub.

(2.) Diviſor 2270700) 14870097 dividend (2.)

13624200
93960 } ſubducends.
216

Sum 13718376 from dividend ſub.

(3.) Diviſor 230212800) 1151721125 dividend (3.)

1151064000
657000 } ſubducends.
125

Sum 1151721125 from divid. ſub.

Reſt 00

PROOF.

PROOF.

Root 8765 } Mul.	Multiply 76825225 square
8765 }	By 8765
43825	384126125
52590	460951350
61355	537776575
70120	614601800
76825225 square	673373097125 cube.

But if your number to be extracted have a remainder, it is then an irrational number, and the exact root cannot by art be discovered, though you may find it near enough for practice; if to the remainder in every operation you add 3 ciphers, and so work as far as you will.

EXAMPLE.

Let it be required to extract the cube root of 282, or, which is the same, to find the side of that cubical vessel which shall just contain a gallon of ale, being 282 solid inches.

Seeing

Seeing there will be but one point in the given number, the integral part of the root is found by inspection of your table, and is six inches; then adding three ciphers to every remainder throughout the whole operation, I find the fractional part, to three places of decimals, to be .557; so that the side of the cubical vessel is 6 inches and 557 parts of a thousand. And thus of any other.

The work.

```
                281 (6.557
                216
             ---------
        10800) 66000
             ---------
               54000 }
                4500 }
                 125 }
             ---------
               58625
             ---------
     1267500) 7375000
             ---------
              6337500 }
                48750 }
                  125 }
             ---------
              6386375
             ---------
  128707500) 988625000
             ---------
             900952500 }
                962850 }
                   343 }
             ---------
             901915693
             ---------
   Remainder  86709307
```

And after this manner may the cube of any fraction, or mixed number, be found, by reducing the fractional part into decimals, either of 3, 6, 9, or 12 places, as you desire your root to be more or less exact.

So if the cube root of $\frac{3}{4}$ were required, the work to 3 places of decimals would stand thus, and the root will be .908.

See

See the work.

$$
\begin{array}{r|l}
 & .750000000\ (.908 \\
 & 729 \\
\hline
2430000) & 21000000 \\
\hline
 & 19840000 \\
 & 172800 \\
 & 512 \\
\hline
 & 20013312 \\
\hline
 & \text{Rest}\ \ 986688
\end{array}
$$

So the cube root of $25\frac{5}{8}$ will be 2.9481, and so of any other.

But if your mixed number or fraction be commensurable to its root, then you may extract the cube root of the numerator for the numerator of the root, and the cube root of the denominator for the denominator of the said root; so the cube root of $\frac{27}{64}$ will be $\frac{3}{4}$; for the cube root of 27 is 3, and of 64 is 4, which is $\frac{3}{4}$; and so of any other.

But if your fraction or mixed number be incommensurable to its root, you must work as before; or if you have no present occasion for it, you may prefix its radical sign. So the cube root of $\frac{11}{19}$ would be expressed thus $\sqrt[3]{\frac{11}{19}}$; and so of any other.

As in the square root, so here, I will shew you how to find the cube root of an irrational number near, without the use of decimal fractions, and it is thus;

After you have found the integral part of your root, to the treble thereof add unity, and that sum added to the square of the said root tripled, is the denominator, to which the remainder is numerator; so the cube root of 282 will be found to be $6\frac{66}{127}$, which is near enough for ordinary practice; or, which is the same,

if

if you find the difference betwixt the cube of the root, and the cube of the root *plus unity*, you have the denominator as before.

For the cube of the root 6 is 216, and the cube of 7, *viz.* the root *plus unity*, is 343; their difference is 127, which is the denominator as before.

The Use of the Square and Cube Roots.

HERE follow some uses of the *Square and Cube Roots*, both in arithmetic and geometry.

PROBLEM I.

To find a mean proportional between any two numbers given.

RULE.

The square root of the product of the given numbers is the mean proportional sought.

So a mean proportional between 16 and 64 will be 32.

This problem is of excellent use in finding the side of a square equal to any parallelogram, *rhombus*, *rhomboides*, triangle or regular polygon.

For if in a parallelogram you suppose the two sides, or in a *rhombus*, or *rhomboides*, the side and perpendicular falling thereon; in a triangle, the base and $\frac{1}{2}$ the perpendicular, or perpendicular and $\frac{1}{2}$ the base, and in a regular polygon, the $\frac{1}{2}$ piremeter and perpendicular, or $\frac{1}{2}$ perpendicular and piremeter; I say, if you suppose them as two numbers given, and by the foregoing problem find a mean proportional given, is the side of a square equal sought.

From this problem by consequence follows *Prob.* 2.

PROB.

PROB. II.

To find the side of a square equal in *area* to any given superficies whatsoever.

RULE.

The square root of the content of any given superficies is the side of the square equal sought.

So if the content of a given circle be 160, the side of the square equal will be 12 $\frac{32}{49}$ *ferè*, or more exact in decimals 12.64911.

Here if you suppose the content to be the product of two numbers, as in many cases it is, it will be the same as to find a mean proportional betwixt those two numbers.

PROB. III.

The area of a circle given, to find the diameter.

RULE.

As 355 : To 452, or as 1 to 1.273239 : : So the area : To the square of the diameter.

What length of cord will fit to tie to a cow-tail, the other end fixed in the ground, to let her have liberty of eating an acre of grass, and no more, supposing the cow and tail to be five yards and a half.

Say, As 355 : To 452 : : So 160, being the area of a circle, whose content is an acre : To 203.7183, whose square root is the diameter (*viz.*) 14.273 perches. The semi-diameter is 7.136; from which subtract one perch for the cow and tail, rest 6.136 perch for the length of the cord.

PROB. IV.

The area of a circle given, to find the periphery.

RULE.

Say, As 113 : To 1420, or as 1 to 12.56637 : : So the area : To the square of the periphery.

So

So if the area of a circle be 160, the periphery will be found to be 44.84 *feré*.

PROB. V.

The sum of the squares of two numbers together, with the square of the $\frac{1}{2}$ sum being given, to find the numbers.

RULE.

From the sum of the squares subtract the doubled square of the $\frac{1}{2}$ sum, half the remainder is the square of $\frac{1}{2}$ their difference; and if to the $\frac{1}{2}$ sum you add their half difference, you have the greater number, and by subtraction the less.

Let the sum of the squares of 2 numbers be 3161, and the square of their $\frac{1}{2}$ sum 1560.25, and let the 2 numbers be sought.

The doubled square of the $\frac{1}{2}$ sum is 3120.5, which subtract from the sum of the squares 3161, there will rest 40.5, half of which is 20.25, whose square root is 4.5, and is the $\frac{1}{2}$ diff. which add to the square root of 1560.25 (*viz.*) 39.5, and it will give 44, the greater number; and if you subtract 4.5 from 39.5, you have the less, to wit 35.

PROB. VI.

The sum of the squares of two numbers, together with the square of the $\frac{1}{2}$ diff. being given, to find the numbers themselves.

RULE.

From the $\frac{1}{2}$ sum subtract the square of $\frac{1}{2}$ their diff. the remainder is the square of their $\frac{1}{2}$ sum of those two numbers; then work by the last.

Let the sum of the squares of two numbers be 3161, and the square of their half diff. is 20.25; I demand the two numbers.

Half the sum of the squares is 1580.5, from which subt. 20.25, the square of their half diff. rest 1560.25, whose

whose square root is 39.5, which is the $\frac{1}{2}$ sum; and the square root of 20.25, is 4.5; then the sum of 4.5 and 39.5 is 44, the greater number; and their difference is 35, the lesser number.

PROB. VII.

The sum of the squares of the half sum, and half the difference of two numbers, with one of them, being given, to find the other. The rule follows.

From the doubled sum of the said squares subtract the square of the given number, the remainder is the square of the number required.

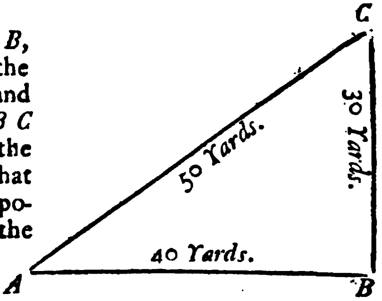
Let the sum of the squares of $\frac{1}{2}$ the sum, and $\frac{1}{2}$ the diff. of 2 numbers be 1580.5; and let the lesser number be 35, from 3161, the double sum of the squares, subtract 1225, the square of 35, the remainder is 1936, whose square root is 44, which is the other number.

PROB. VIII.

Any two sides of a right-angled triangle being given, to find the third side.

In this useful problem lies hid a great part of the mathematics, the invention whereof is fathered upon *Pythagoras*. The demonstration thereof *Euclid* hath in the 47th proposition of the first book of his *Elements of Geometry*; where it is proved that the square of the hypothenuse, or longest side of a right-angled triangle, is equal to the sum of the squares of the base and perpendicular, or the other two sides.

In the annexed triangle *A B C*, let the base or ground *A B* represent the breadth of a mote or ditch, and let the perpendicular *B C* represent the height of a castle, tower, or city wall, and let the hypothenuse, or longest side, represent the length of a scaling ladder.



Let the base *A B*, or the breadth of the ditch be 40 yards, and the perpendicular *B C* or the height of the wall, be 30 yards; what length will the hypothenuse *A C*, or the scaling-ladder be?

RULE.

The square root of the sum of the squares of the base and perpendicular, is the length of the hypothenuse.

Answ. 50 yards the length of the ladder.
For the square of the base 40 is 1600
And the square of the perpend. 30 is 900

The sum is 2500 (50 the root.
25
000

But if the breadth of the ditch were required, and the perpend. and hypothenuse were given, then this is

The RULE.

The square root of the difference of the squares of the hypothenuse and perpendicular, is the length of the base, or, breadth of the ditch.
For the square of the hypoth. *A C* is 2500
And the square of the perpend. *B C* is 900

Diff. 1600 (40 Root.
Here you see the base is 40.
16
000

And

And if *B C* were required from the given ſides *A B*, and *A C*, then the ſquare root of the difference of the diſtance of the ſquares of the hypothenuſe and baſe is the height of the perpendicular, or *B C*.

P R O B. IX.

To divide a number given by extreme and mean proportion.

R U L E.

Multiply the ſquare of your given number by 5, and divide the product by 4, and from the ſquare-root of the quotient ſubtract $\frac{1}{2}$, your given number, the remainder is the greater portion, which ſubtracted from the whole, gives the leſs.

Let the given number be 12, whoſe ſquare is 144, which multiplied by 5 produceth 720; which product divided by 4, gives 180, whoſe ſquare root is $13\frac{22}{53}$, from which ſubtract 6, $\frac{1}{2}$ your given number, reſt $7\frac{22}{53}$ for your greater part, which ſubtracted from the whole number 12, gives $4\frac{35}{53}$ for the leſs.

P R O B. X.

Any number of men being given, to form them into a ſquare battle, or to find the number of ranks and files.

R U L E.

The ſquare root of the number of men given, is the number of men either in rank or file.

Let there be an army of men of 32400, and let us form them into a ſquare battle: extract the ſquare root of 32400, and it will be found to be 180; which ſhews there will be 180 men in rank, and as many in file.

See the work.

```
      32400 (180 Root.
      . . .
      1
      ----
  28) 224
      224
      ----
      000
```

Thus much ſhall ſuffice for the uſe of the ſquare root; we will now proceed to ſome uſes of the cube root.

The chief uſe of the cube root, is to find out a proportion between the ſolids, as globes, cylinders, cubes, &c.

PROB. I.

If a bullet of braſs of eight inches diameter weigh 72 pound, what will a bullet of braſs weigh, whoſe diameter is 4 inches?

RULE.

Since like ſolids are in triple proportion to their homologous ſides, diameters, lines, &c. it holds,

As the cube of the diameter given :
To the weight thereof : :
So the cube of the diameter ſought :
To the weight thereof.

See the work.

```
     C.D.  lb. C.D.
   If 512 : 72 : : 64
                64
               ----
               288      Facit 9 pound.
              432
              ----
     512) 4608 (9
          4608
          ----
             0
```

PROB.

P R O B. II.

If a ſhip of 100 tun be 44 foot long at the keel, of what length will the keel be of a ſhip of 220 tun?

Say, As 100 : to the cube of 44 (*viz.*) 85184 : : So 22 : to 187404.80, whoſe cube root is 57.225, the length of the keel ſought.

P R O B. III.

The ſide of the cube being given, to find the ſide of that cube that ſhall be double, trible, *&c.* in quantity to the given cube.

R U L E.

Cube your ſide given, and multiply it by 2, 3, *&c.* The cube root of the product is the ſide ſought.

There is a cubical veſſel, whoſe ſide is 12 inches, and it is required to find the ſide of that veſſel which ſhall contain 3 times as much.

The cube of 12 is 1728
Multiply by 3

5184 product.

The cube root of which product is 17.306, the ſide ſought.

After the ſame method may you find a ſide that ſhall contain $\frac{1}{2}$ as much, $\frac{1}{4}$ as much, or any other given quantity.

P R O B. IV.

To find the ſide of a cube that ſhall be equal in ſolidity to any given ſolid, as a globe, cylinder, priſm, cone, or ſuch like.

R U L E.

The cube root of the ſolid content of any ſolid body given, is the ſide of the cube of equal ſolidity.

So, if the content of a globe were found to be 15625 solid inches, seek the cube root of 15625, which is 25, which is the side of a cube of equal capacity.

PROB. V.

Between two numbers given, to find two mean proportionals.

RULE.

Multiply the less extreme by the cube root of the quotient of the greater extreme divided by the less, the product is the lesser of the two mean proportionals; which multiplied by the said cube root, gives the greater mean sought. So if two mean proportionals betwixt 6 and 162 were sought, they would be found to be 18 and 54; for 162 divided by 6, quotes 27, whose cube root is 3; by which multiplying 6, the less extreme, gives 18 for the less mean; and 18 multiplied by the same root 3, gives 54 for the greater; or if you divide the greater mean by the same root, it quotes the lesser mean as before.

PROB. VI.

The concave diameter of two guns being known, together with the quantity of gun-powder sufficient to charge one, to find what will be sufficient to charge the other. The capacities are one to another, as the cubes of their diameters.

EXAMPLE.

If .45 pound of gun-powder be sufficient to charge a gun, whose concave diameter is one inch $\frac{1}{2}$, or 1.5, how much gun-powder will suffice to charge a gun whose concave diameter is 7 inches? *Ans.* 43 pound and $\frac{7}{10}$.

Say, As 3.375 : to .45 : : So is 343 : to 43.7 pound.

But

But if the gun-powder given and required, be of different ſtrengths, the queſtion requires two operations; the firſt of which finds the quantity either of a ſtronger or weaker ſort, and the proportion is inverſe; the ſecond is as in this example. Many more uſes might be named, but let this ſuffice in this place.

Simple INTEREST.

WE ſhall proceed now to intereſt of money, wherein the greateſt and moſt uſeful practice of *decimal arithmetic* conſiſts.

When a ſum of money is lent by one to another for any time agreed on, and an allowance granted for the loan of the ſame; here the money ſo lent is called *the principal* or *ſtock*, and the allowance, or gain, is called *the intereſt*.

Intereſt formerly was very high, *viz.* 12, 10, or 8, *per cent. per annum*; but by an act of parliament made in *Auguſt* 1660, it was brought down to 6 *per cent. per annum*, and is now at 5, ſince 12th of Q. *Anne*; above which no perſon dare pretend to take, nor are any obliged to give.

And a ſufficient perſon may have money in moſt places at 5 *per cent. per annum*, and in ſome places for leſs.

Intereſt is either ſimple or compound.

When a ſum of money is lent, and the intereſt thereof when due, is not paid, but kept in the borrower's hands, and yet becomes not a part of the principal, then it is called *Simple Intereſt*.

The buſineſs of *Simple Intereſt* is performed by a rank of numbers arithmetically proportional, from which naturally will ariſe this theorem.

If

If a pair of ranks of numbers ſhall be ſo poſited, as to have the ſame common *ratio* betwixt every pair of correſpondents; then it follows, that the numbers themſelves, the correſpondent ſums, and correſpondent differences have the ſame common *ratio*.

ILLUSTRATION.

2 .	8	.6 .	3.0
3 .	12	.7 .	3.5
4 .	16	.8 .	4.0
5 .	20	.9 .	4.5
14 .	56	3.0 .	15.0

In the firſt pair of ranks the *ratio* is 4; then you may take any number in the firſt rank of the firſt pair, ſuppoſe 4; then it holds,

As 4 : to 16 : : So is 14 : to 56, *& contra* underſtand the ſame in the ſecond pair of ranks.

In the ſolution of queſtions of ſimple intereſt, four things are to be conſidered.

Firſt, The principal or money lent. Secondly, The time for which it is lent. Thirdly, The rate or gain, ſuppoſe of one pound in a year. And, fourthly, The amount.

Any three of theſe being given, to find the fourth, as in theſe four *Propoſitions* following.

PROP. I.

Principal, rate, and time given, to find the amount.

RULE.

To the product of the rate, multiplied by the time, add unity; that ſum multiplied by the principal, gives the amount.

EXAM.

EXAMPLE I.

What will 20 *l.* amount to, forborne seven years, at 6 *per cent.* simple interest?

Principal 20 *l.* rate .06 *l.* time 7 years.

The work.

```
 .06
   7
----
1.42
  20
----
28.40        Answer, 28 l. 8 s.
```

EXAMPLE II.

What will 36 *l.* amount to, lent from *May* the 9th 1753, until *November* the 17th next following, simple interest being computed at 5 *per cent.*? Principal 36, rate .05, time .526. *Ans.* 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$.

```
                             .526
                              .05
                         --------
13 Months = 1 Year        1.02630
 4 Weeks  = 1 Month            36
 7 Days   = 1 Week       --------
                           615780
                          307890
                         --------
                         36.94680
```

And

And ſeeing the time is both given and required in years and parts, we have annexed a *decimal table* thereof, ſuppoſing a year the integer, and divided, as noted before: the uſe is the ſame as other decimal tables, and needs no explication.

Betwixt *May* the 9th, and *November* the 17th, are 192 days, gathered as in the annexed work; and by the annexed decimal table is found to be 6 months, 3 weeks, and 3 days: the decimal of which is .526027. And becauſe the fourth figure is a cipher, we have only uſed three places, they being ſufficient. To find the days in every month, obſerve this old rule:

May	22
June	30
July	31
Aug.	31
Sept.	30
Oct.	31
Nov.	17
Sum	192

A decimal table of time, one year the integer.

Mon.	*Dec.*	*Days*
12	.920548	336
11	.843835	308
10	.767123	280
9	.690411	252
8	.613698	224
7	.536986	190
6	.460274	168
5	.383561	140
4	.306849	111
3	.230137	84
2	.153424	56
1	.076712	28
Weeks.	*dec.*	*days*
3	.057534	21
2	.038356	14
1	.019178	7
Days.	*Decimals.*	
6	.016438	6
5	.013698	5
4	.010959	4
3	.008219	3
2	.005479	2
1	.002739	1

Thirty days hath September,
April, June, *and* November,
All the reſt have thirty-one,
Excepting February *alone,*
Which hath but 28 *days clear,*
And 29 *every* Leap *year.*

And

And that nothing may be wanting, we have added a small table, which gives the number of days betwixt any two given times with much ease: as for *example*.

The TABLE.

13	334	*January*	00	31
59	306	*February*	31	28
90	275	*March*	59	31
120	245	*April*	90	30
151	214	*May*	120	31
181	184	*June*	151	30
212	153	*July*	181	31
243	122	*August*	212	31
273	92	*September*	243	30
304	61	*October*	273	31
334	31	*November*	304	30
365	00	*December*	334	31

1. From the beginning of the year, to the 11th of *July*, what number of days?

Over-against *July*, on the right-hand, I find 181, and 11 days more make 192, the *answer*.

2. Betwixt *May* the 9th, and *September* the 17th next following, how many days? Subtract *May* 120 *plus* 9, equal to 129; from *September* 243 *plus* 17, equal 260, rest 131, the number of days sought.

3. From the 5th of *November* 1752, to the 16th of *May* 1753, how many days?

Add 25, the complement of 5, to 30, (the days in *November*), to 31 found on the left-hand *November*, and to that sum add 120, for *May plus* 16, the sum is 192; and those are the number of days sought.

But to proceed.

PROP. II.

The amount, rate, and time being given, to find the principal.

RULE.

To the rate multiplied by the time, add unity; by which dividing the amount, quotes the principal.

EXAMPLE I.

What present money will pay a debt of 28*l.* 8*s.* due

due ſeven years hence, at 6 *per cent.* ſimple intereſt? Amount 28.4*l.* rate .06, time 7 years. *Anſ.* 20*l.*

The work.

```
          .06
           7
        -----
    1.42) 28.4 (20l.
          284
          ---
            0
```

EXAMPLE II.

What preſent money will pay a debt of 36.9468, or 36*l.* 18*s.* 11*d.* $\frac{1}{4}$. due 192 days, or 6 months, 3 weeks, and 3 days hence, rebate being allowed at 5 *per cent.* ſimple intereſt? Amount 36,9468, rate .05, time .256. *Anſwer* 36*l.*

The work.

```
          .526
          .05
     -------------
     1.02630) 36.9468 (36l.
              307890
              ------
               615780
               615780
               ------
                    0
```

PROP. III.

Principal, amount, and rate given, to find the time.

RULE.

From the amount ſubtract the principal, the difference divided by the product of the principal and rate, gives the time.

EXAMPLE I.

In what time will 20*l.* raiſe a ſtock of 28*l.* 8*s.* or 28.4 at 6 *per cent.* ſimple intereſt? Principal 20*l.* amount 28.4, rate .06. *Anſwer,* In 7 years.

The

The work.

```
  20            28.4
 .06            20
 ----           ----
 1.20     1.20) 8.40 (7
                840
                ---
                  0
```

EXAMPLE II.

In what time will 36 *l.* amount to 36.9468, or, 36 *l.* 18 *s.* 11 *d.* $\frac{1}{4}$. at 5 *per cent.* simple interest? Principal 36, amount 36.9468, rate .05. *Answer,* In 6 months, 3 weeks, and 3 days.

The work.

```
                36.9468
                36
                -------
         1.80)   .9468 (.526=191 days, or 6
                        months, 3 weeks, and 3
  36             900    days.
  05             ---
 ----            468
 1.80            360
 ----            ----
                 1080
                 1080
                 ----
                    0
```

PROP. IV.

The principal, time, and amount given, to find the rate.

RULE.

From the amount subtract the principal, the remainder divided by the product of the principal and time, gives the rate.

EXAMPLE I.

At what rate of ſimple intereſt will 20 *l.* amount to 28.4 in 7 years? *Anſwer*, 6 *per cent.*

The work.

20	28.4
7	20
140)	8.40 (06
	840
	0

EXAMPLE II.

If 36 *l.* amount to 36.9468, or 36 *l.* 18 *s.* 11 *d*, $\frac{1}{4}$. in 6 months, 3 weeks, and 3 days, what rate of ſimple intereſt is implied in this bargain? Principal 36, time .526, amount 36.9468. *Anſwer*, 5 *per cent.*

The work.

.526		36 9468
36		36.0
3156	18.936)	.94680 (.05
1578		94680
18.936		0

The two firſt propoſitions being of moſt common uſe, I have annexed two tables, which give the amount and rebeat, or the preſent worth of any ſum of money for any time under 32 years: the conſtruction of them lies in the *propoſitions* themſelves, by making one pound the principal in the firſt, and one pound the amount in the ſecond; or, if you divide unity by the number in the firſt table, the quotient is the number in the ſecond.

TABLE

TABLE I.

Shewing the amount of one pound for 31 *years, at* 5 *and* 6 per cent. *Simple Interest.*

Years	5	6
1	1.05	1.06
2	1.10	1.12
3	1.15	1.18
4	1.20	1.24
5	1.25	1.30
6	1.30	1.36
7	1.35	1.42
8	1.40	1.48
9	1.45	1.54
10	1.50	1.60
11	1.55	1.66
12	1.60	1.72
13	1.65	1.78
14	1.70	1.84
15	1.75	1.90
16	1.80	1.96
17	1.85	2.02
18	1.90	2.08
19	1.95	2.14
20	2.00	2.20
21	2.05	2.26
22	2.10	2.32
23	2.15	2.38
24	2.20	2.44
25	2.25	2.50
26	2.30	2.56
27	2.35	2.62
28	2.40	2.68
29	2.45	2.74
30	2.50	2.80
31	2.55	2.86

TABLE II.

Shewing the rebate of one pound for 31 *years, at* 5 *and* 6 per cent. *Simple Interest.*

5	6
.952380	.943396
.909091	.892857
.869565	.847457
.833333	.806451
.800000	.769230
.769230	.735294
.740740	.704225
.714286	.675675
.689655	.649350
.666666	.625000
.645161	.602409
.625000	.581395
.606060	.561797
.588235	.543478
.571428	.526315
.555555	.510204
.540540	.495049
.526315	.480769
.512820	.467289
.500000	.454545
.487804	.442477
.476190	.431034
.465116	.420168
.454545	.409836
.444444	.400000
.434781	.390625
.425532	.381679
.416666	.373134
.408163	.364963
.400000	.357143
.393157	.349650

In the use of these tables, the proportion runs; As 1 : To the tabular number : : So the sum of money given : To the answer sought. And seeing the first number is an unit, the whole work requires only a single multiplication; as in the examples following may be seen.

EXAMPLE I.

In the use of the first table.

What will 20 *l.* amount to, forborne 7 years, at 6 *per cent.* simple interest? *Answer*, 28 *l.* 8 *s.*

Tabular number answering 7 years, and under 6 *per cent.* in the first table, is 1.42

Multiply by 20

28.40

EXAMPLE II.

What will 3 *l.* 17 *s.* 6 *d.* amount to, forborne 21 years, at 5 *per cent.* simple interest? *Ans.* 7 *l.* 18 *s.* 10 *d.* $\frac{1}{2}$.

Tabular number under 5 *per cent.* and answering 21 years, is 2.05

Multiply by the principal, 3.875

```
     1025
    1435
   1640
   615
  -------
  7.94375
```

EXAMPLE III.

In the use of the second table.

What ready money will pay a debt of 28 *l.* 8 *s.* due 7 years hence, at 6 *per cent.* simple interest? *Answer*, 20 *l.*

Tabular

Tabular number against 7 years, and under 6 *per cent.* is .704225
Multiply by the debt, 28.4

```
 2816900
 5633800
1408450
19.9999900
```

EXAMPLE IV.

What ready money will pay a debt of 12 *l.* 12 *s.* 6 *d.* due 20 years hence, rebate being allowed, at 5 *per cent.* simple interest? *Answer*, 6 *l.* 6 *s.* 3 *d.*

Tabular number against 20 years, and under 5 *per cent.* is .500000
Multiply by the debt, 12.625

Product=6.3125

And thus may any question of this nature be resolved, if the time given be even years, but if not, use the propositions themselves, as was shewn before.

SECT. II.

But if your question be concerning annualrents, payments, or annuities to be bought or sold for some time, then you are to consider it under these four particulars.

First, The annuity or pension.

Secondly, The time of continuance.

Thirdly, The rate of interest.

Fourthly, The present worth.

Any three of these being given, the fourth thence may be found, as in the four following propositions may be seen.

PROP. I.

Annuity, rate, and time given, to find the present worth.

RULE.

To the ſquare of the time multiplied by the rate, add the double of the time; from which ſubtract the rate multiplied by the time, the remainder multiplied by the annuity, and that product divided by the double of the rate, multiplied by the time *plus* 2, the quotient is the preſent worth.

EXAMPLE I.

What is an annuity of 20 *l. per annum* clear value, to be ſold for 7 years, worth in ready money, ſimple intereſt being computed at 6 *per cent.*? *Anſwer*, 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$ *ferè.*

Annuity 20 *l.* time 7 years, rate .06.

The work.

```
                                   16.52           .12
                                      20             7
                                  ------          ----
49=ſquare time   2.84) 330.40 (116.338           2.84
.06=the rate.          284
-----                  ----
 2.94      .06          464
14           7          284
-----      ----         ----
16.94      .42          1800
   42                   1704
-----                   ----
16.52                     960
                          852
                          ----
                          1080
                           852
                          ----
                          2280
                          2272
                          ----
                             8
```

EXAMPLE II.

There is an annuity of 12 *l.* 10 *s. per annum*, to continue

continue 21 years; what is its worth at 5 *per cent.* simple interest? *Answer,* 192 *l.* 1 *s.* 5 *d.*. $\frac{1}{2}$.

Annuity 12.5 *l.* time 21 years, rate .05.

The work.

```
 21
 21
 ——
 21
42
————
441
 .05
——————
22.05
42
——————
64.05
 1.05
——————
63
```

```
21
 2
———
42
———
21
.05
———
1.05
```

```
      12.5 annuity
       63
      ———
      375
     750
     ————
4.1) 787.5 (192.0731
     41..
     ———
     377
     369
     ———
      85
      82
      ———
      300
      287
      ———
       130
       123
       ———
         70
         41
        ———
         29
```

P R O P. II.

Present worth, time, and rate given, to find the annuity.

R U L E.

Multiply the double of the rate multiplied by the time *plus* 2, by the present worth; that product divided by the square of the time multiplied by the rate, *plus* the double of the time, *minus* the rate multiplied by the time, the quotient is the annuity.

E X A M-

EXAMPLE I.

What annuity, to continue 7 years at 6 *per cent.* simple interest, will 20 *l.* purchase? *Answer*, 3 *l.* 8 *s.* 9 *d.* ¼ *ferè.*

Present worth 20 *l.* time 7 years, rate .06.

The work.

```
                .12
                  7
             ------
             2.84
                 20
             ------
16.52) 56.80 (3.4382
       4956
       ------
        7240
        6608
        ------
         6320
         4956
         ------
         13640
         13216
         ------
          4240
```

```
   49
  .06
 ----
 29.4
  14.
 ----
16.94
  .42
 ----
16.52
```

EXAMPLE II.

What annuity, to continue 21 years, will 192.0731 *l.* or 192 *l.* 1 *s.* 5 *d.* ½ purchase, at 5 *per cent.* simple interest?

Purchase-money 192.0731 *l.* time 21 years, rate .05.

RULE.

R U L E.

Purchase money	192.0731	multi-
Product of the double of the rate, multiplied by the time *plus* 2, is	=4.10	ply.

```
63) 7875 (12.5
    63
    ---
    157
    126
    ---
     315
     315
     ---
       0
```

Answer, 12*l*. 10*s*.

This is the converse of *question* the second, and *proposition* the first, aforegoing.

P R O P. III.

The annuity, present worth, and time being given, to find the rate of interest.

R U L E.

The product of the annuity and time, *minus* the present worth, being multiplied by 2, and divided by the sum of double the present worth, multiplied by the time, *plus* the annuity multiplied by the time, *minus* the annuity multiplied by the square of the time, the quotient is the rate.

E X A M P L E I.

At what rate of simple interest will 116.338, or 116*l*. 6*s*. 9*d*. $\frac{1}{4}$. purchase an annuity of 20*l*. to continue for 7 years? *Answer*, 6 *per cent*. the rate.

Annuity 20*l*. present worth 116.338, time 7 years.

The

The work.

116.338	20	20	20
2	7	7	49
232.676	140	140	180
7		116.338	80
1628.732		23.662	980
140		2	
1768.732		788.732) 47.3240 (.06	
980			
788.732			

EXAMPLE II.

At what rate of simple interest will 250 *l.* purchase an annuity of 30 *l.* *per annum*, to continue 10 years? *Ans.* 4 *l.* 6 *s.* 11 *d.* $\frac{1}{2}$. *ferè.* the interest *per cent.* sought.

Annuity 30 *l.* present worth 250, time 10 years.

The work.

2300) 1000.0000 (.043478 *per L.* 1.
——————or *L.* 4.347 *per cent.*

PROP. IV.

Annuity, rate, and present worth being given, to find the time of continuance.

RULE.

To the doubled product of the present worth, multiplied by the rate, add the product of the annuity multiplied by the rate, the square of the difference betwixt this last sum, (which we may call *A*), and double the annuity (which may be called *B*), added to the octuple product of the present worth; annuity and rate multiplied one into another, the square root of this last sum added to, or subtracted from, the former difference, according as *A* was either greater or less than *B*; this last sum or difference, divided by *double the* annuity, multiplied by the rate, quotes the time sought.

EXAMPLE.

In what time will 7 pound *per annum* pay a debt of 120.4 or 120.4 8 *s.* at 6 *per cent.* simple interest?

Annuity

Annuity 7 *l.* rate .06 *l.* present worth 120 *l.* 8 *s.*

Answer. In 25 years.

The work.

Present worth	120.4	Annuity	7
Rate	.06	Rate	.06
	7.224		.42

Mult. by 2

14.448

Add 42

14.868 = *A*

14.000 = *B* double the annuity.

.868 difference.

Doub. of ann. 14
Rate .06
.84

.868
.868

6944
5208
6944

.753424
404.544

405.297424 (20.132 = root

Present worth 120.4
Rate .06
7.224
Annuity 7
50.568
Mult. by 8
Prod. 404.544

4

401) 00529
401

4023) 12874
12069

40262) 80524
80524

0

.868 = former difference, = *A*. greater than *B*:

21. sum

.84) 21.00 (25 years.
168

420
420

0

PROP.

PROP. V.

To the aforegoing four *propoſitions* we may add a fifth, which is by having the annuity, time and rate given, to find the amount.

RULE.

From the given number of years ſubtract an unit, half of the remainder, multiplied by the product of the rate and time, and to this product adding the number of years given, the ſum multiplied by the annuity gives the amount.

EXAMPLE I.

An annuity of 20 *l. per annum* is forborne 7 years, what will then be due at 5 *per cent.* ſimple intereſt? Annuity 20 *l.* time 7 years, rate .05. *Anſw.* 161 *l.*

The work.

$\frac{1}{2}$ 6=time—1	.35	.05
	3	7
3	1.05	.35
	7	
	8.05	
	20	
	161.00	

EXAMPLE II.

A tradeſman binds his ſon an apprentice for 7 years, and at the ſame time lets an annuity of 36 *l.*-15 *s.* run to the expiration of the ſaid term, that it may be a ſtock for his ſon. The queſtion is, What this ſtock will be, accounting ſimple intereſt at 6 *per cent.*? *Anſwer,* 303 *l.* 11 *s.* 1 *d.* $\frac{1}{4}$ *ferè.*

The

The work.

7	7	36.75
.06	1	8.26
———	——	———
.42	6	22050
3	——	7350
———	3	29400
1.26		———
7		303.5550
———		
8.26		

The first, second and fifth propositions being of most common use, we have annexed tables fitted thereto, at 5 and 6 *per cent.* as in the first section, whereby the answer may be found at one single operation, as in the use of them may be seen.

We need not say any thing of their composition, the propositions themselves being the fountains whence they were drawn.

	The present worth of one pound annuities for 31 years, at 5 and 6 per cent. Simple Interest.		*The annuity that one pound will purchase for 31 years, at 5 and 6 per cent. Simple Interest.*		*Amount of 1 pound Ann. for 31 y. at 5 and 6 per c. Simp. Int.*	
Years.	TABLE I.		TABLE II.		TAB. III.	
	5	6	5	6	5	6
1	0.952386	0.943396	1.05	1.06	1.00	1.00
2	1.863636	1.839285	.536585	.543689	2.05	2.06
3	2.739130	2.694915	.365079	.371069	3.15	3.18
4	3.583333	3.516129	.279069	.284403	4.30	4.36
5	4.400000	4.307692	.227272	.232142	5.50	5.60
6	5.192307	5.073529	.192607	.197101	6.75	6.90
7	5.962962	5.816901	.167702	.171913	8.05	8.26
8	6.714285	6.540540	.148936	.152892	9.40	9.68
9	7.448275	7.246753	.134259	.137992	10.80	11.16
10	8.166666	7.937500	.122041	.125984	12.25	12.70
11	8.870967	8.614457	.114545	.116084	13.75	14.30
12	8.562500	9.279069	.104575	.107738	15.30	15.96
13	10.242424	9.932584	.097633	.100678	16.90	17.68
14	10.911764	10.853260	.091644	.092184	18.55	19.46
15	11.571428	11.210526	.086420	.089207	19.25	21.30
16	11.222222	11.836734	.081818	.084482	22.00	23.20
17	12.864864	12.435445	.077731	.080286	23.80	25.16
18	13.500000	13.067067	.074074	.076527	25.65	27.18
19	14.128205	13.672897	.070780	.073137	27.55	29.26
20	14.750000	14.272727	.068644	.070063	29.50	31.40
21	15.365853	14.865044	.065079	.067262	31.50	33.60
22	15.976190	15.456896	.062593	.064697	33.55	35.86
23	16.581393	16.042016	.060308	.062336	35.65	38.18
24	17.181818	16.602459	.058201	.060158	37.80	40.56
25	17.777777	17.200000	.056255	.958139	40.00	43.00
26	18.369565	17.173437	.054438	.056263	42.25	45.50
27	18.957447	18.343511	.052749	.054515	44.55	48.06
28	19.541666	18.910447	.051172	.052880	46.90	50.68
29	20.122449	19.472627	.049695	.051349	49.30	53.36
30	20.700000	20.035714	.048309	.049911	51.75	56.10
31	21.274510	20.594404	.047005	.048556	54.25	58.90

The

The use of these tables are the same with those aforegoing, as in the following examples may be seen.

EXAMPLE I.

In the use of the first table.

There is an annuity of 20 *l. per annum*, to continue for 7 years, to be sold for ready money; what is the value thereof, allowing simple interest at 6 *per cent.*?

Tabular number, answering 7 years, and under 6 *per cent.* In the first table is 5.816901

Multiply by 20

Answer, 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$. *ferè.* 116.338020

This agrees with example the first, in the first proposition of this section.

Note, But if this were wrought by the common tables of simple interest, shewing the present worth of annuities, printed in several books of Arithmetic, the answer will be found to be 113 *l.* 19 *s.* 6 *d.* 3 *q. ferè.* So, if their tables be true, he that gives 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$, is cheated of 2 *l.* 7 *s.* 2 *d.* $\frac{1}{4}$.; but this we will try.

A lends *B* 113 *l.* 19 *s.* 6 *d.* $\frac{3}{4}$. for 7 years, which at the end thereof amounts to 161 *l.* 16 *s.* 11 *d.* $\frac{1}{2}$; this is not denied. At the same time *B* delivers up to *A* an annualrent of 20 *l. per annum*, to continue the same term, and accordingly received it annually. Now, the question is, whether the reception of these 7 annual payments quit the scores betwixt *A* and *B*, both being obliged under the same rate of interest, for by their tables it must: but it may appear to any considerate man it doth not; which I prove thus:

	paid before it be due *Years.*				
If 100 *l.*	6	be worth	136	what 20 *l.*? *facit.*	27.20
	5		130		26.00
	4		124		24.80
	3		118		23.60
	2		112		22.40
	1		106		21.20
	0		1[illegible]		
				Sum is=	165.20

Thus

Thus you may see the 6 annual payments are worth, at the end of 7 years, 165 *l.* 4 *s.* and the whole amount that *A* could claim of *B* was but 161 *l.* 16 *s.* 11 *d.* $\frac{1}{4}$. ; therefore *A* is in *B*'s debt 3 *l.* 7 *s.* 0 *d.* $\frac{1}{2}$.

But by this table of ours, if *A* had lent to *B* 116.338, or 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$. it would, upon the condition aforesaid, have amounted to 165.2, or 165 *l.* 4 *s.* whereby it is evident our table is founded on a firm foundation. *See* Dary *in his* Interest Epitomized.

EXAMPLE II.

An annuity of 51 *l.* 15 *s.* to continue 21 years, is to be sold for ready money, what is it worth at 5 *per cent.* simple interest? *Answer,* 795 *l.* 3 *s.* 8 *d. ferè.*

Tabular number answering 21 years, and under 5 *per cent.* in the first table, is 15.365853

Multiply by 51.75

```
   15.365853
       51.75
  ----------
    76829265
   107560971
    15365853
   76829265
  ----------
795.18289275
```

EXAMPLE III.

In the use of the second table.

What annuity, to continue 7 years, will 20 *l.* purchase, at 5 *per cent.* simple interest? *Answ.* 3 *l.* 7 *s.* 1 *d. ferè.*

Tabular number answering 7 years, and under 5 *per cent.* in table the second, is .167702

Multiply by 20

```
 .167702
      20
--------
3.354040
```

QUEST.

QUEST. II.

What annuity, to continue 21 years, will 65 *l.* 10 *s.* purchase, at 6 *per cent.* simple interest?

Tabular number answering 21, and under 6 *per cent.* in the second table, is .067262

Multiply by 65.5

336310
336310
403572

Answ. 4 *l.* 8 *s.* 1 *d.* ¼. 4.4056610

EXAMPLE. I.

In the use of the third table.

An annuity of 20 *l. per annum* is forborne for 7 years; what will then be due at 5 *per cent.* simple interest? *Answer,* 161 *l.*

Tabular number answering 7 years, and under 5 *per cent.* in the third table, is 8.05

20

161.00

EXAMPLE II.

What will an annuity of 36 *l.* 11 *s.* 3 *d.* amount to, forborne 21 years, at 6 *per cent.* simple interest? *Answ.* 1228 *l.* 10 *s.*

Tabular number answering 21 years, and under 6 *per cent.* in table the third, is, 33.60

Multiply 36.5625

By 33.6

2193750
1096875
1096875

1228.50000

Thus may any other question be resolved, if the time given be even years, not exceeding 31 years; and the rate either 5 or 6 *per cent.* If otherwise, you must use the propositions themselves.

Here follow some more questions for the use of the said tables, wherein appear some more variety.

QUEST. I.

A hath an annuity of 20 *l.* *per annum*, to continue 7 years. *B* hath an annuity of 5 *l.* 10 *s.* to continue 21 years. These two persons would change annuities, and allow each other simple interest at 6 *per cent.* The question is, who must pay money, and how much? *Answer*, *A* must receive from *B* 34 *l.* 11 *s.* 7 *d.* $\frac{1}{2}$. on the condition aforesaid.

Seek by table the first, in the second or last head of tables, the present worth of 20 *l.* *per annum*, to continue 7 years, which you may see by the work will be 116 *l.* 6 *s.* 9 *d.* $\frac{1}{4}$.

$$\begin{array}{r} 5.816901 \\ 20 \\ \hline A=116.338020 \end{array}$$

Seek likewise by the same table the present worth of 5 *l.* 10 *s.* *per annum*, to continue 21 years, which by the work you may see will be 81 *l.* 15 *s.* 1 *d.* $\frac{3}{4}$.

$$\begin{array}{r} 14.865044 \\ 5.5 \\ \hline 74325220 \\ 74325220 \\ \hline B=81.7577420 \end{array}$$

	l.	*s.*	*d.*	*q.*
From	116	06	9	1
Sub.	81	15	1	3
Rest	34	11	7	2

QUEST.

QUEST. II.

A lends *B* 360 *l.* upon a mortgage of land, whose rent is 75 *l. per ann.* *B* keeps his money five years, during which time *A* receives the said rent; at the end of which time they come to account. The question is, whether this rent hath cleared the mortgage? if not, what must *B* pay to *A* simple interest, at 6 *per cent.* being accounted on both sides.

First, Seek by the first table in the former head of tables, the amount of 360 *l.* to continue 5 years at 6 *per cent.* simple interest, which you will find to be 468 *l.*

$$
\begin{array}{r}
1.30 \\
360 \\
\hline
78 \\
39 \\
\hline
468.00
\end{array}
$$

Secondly, Seek by table the third in the second head of tables, what an annuity of 75 *l. per annum* will amount to, forborne 5 years, at 6 *per cent.* simple interest, which you will find to be 420 *l.* which *B* must pay unto *A* before he have discharged his mortgage.

$$
\begin{array}{r}
5.60 \\
75 \\
\hline
2800 \\
3920 \\
\hline
420,00
\end{array}
$$

QUEST. III.

A merchant is indebted 360 *l.* the creditor is contented

tented to receive the same at 10 equal yearly payments, the debtor allowing for the forbearance of the same money after the rate of 6 *per cent.* simple interest. The question is, what those payments ought to be? *Answer*, 45 *l.* 7 *s.* 1 *d.*

Seek in the second table of the last head of tables for the numbers over-against 10 years, and under 6 *per cent.* which number multiplied by 360, gives the answer.

See the work:

```
.125984
    360
-------
7559040
377952
---------
45.354240
```

QUEST. IV.

A gentleman bequeathed 1500 *l.* to his daughter, to be paid her at 14 years end. The executors desire to pay ready money, so they may be abated after the rate of 5 *per cent.* simple interest. The question is, what ready money will pay this legacy? *Answer*, 882 *l.* 7 *s.* 0 *d.* $\frac{1}{2}$.

Seek in the second table in the first head of tables, for the number over-against the 14th year, and under 5 *per cent.* which multiplied by 1500 gives the answer.

The work.

```
.588235
   1500
--------
2941175
588235
----------
882.352500
```

QUEST. V.

A gentleman hath 160 *l.* which he would lay out to purchase 20 *l. per annum.* How many years must the said annuity continue, simple interest being computed at 6 *per cent.*?

First, Divide 160 by 20, quotes 8, which shews he gives 8 years purchase for the said annuity.

Secondly, Seek in the first table of the last head of tables, and under 6 *per cent.* for the number 8; but not finding it, I look the next less, which is 7.937500, and over-against it is 10 years, which shews it must continue somewhat above 10 years; and to find what, say, as the difference betwixt this and the next bigger number .676757 : to 1 : : So the difference of the said next less and 8, (*viz.*) .0625 : to .092 parts of a year, equal to 1 month and 5 days, accounting 13 to the year. So his annuity must continue 10 years, 1 month, and 5 days.

Let these suffice in this place.

We should now proceed to *Compound Interest* and *Annuities*; but these things being best performed by logarithims, we will therefore treat of them when we come into *Logarithmetical Arithmetic*.

We shall now proceed to other parts of Arithmetic, as *Tret* and *Tare*, *Barter*, *Fellowship*, &c.

Rules in TARE *and* TRET, *&c.*

THESE are allowances commonly used among merchants, in such commodities as are sold by weight.

Tare, is the weight of the bag, chest, hogshead, *&c.* wherein the goods are carried or put.

Tret,

Tret, is an allowance of 4 *lb.* in 100, or 104 *lb.* for goods wherein is loſs, as *treacle*, *ſugar*, &c.

Cloff, is an allowance of 2 pound upon every draught which exceedeth 300 groſs weight.

Subtile, is the weight of the goods when the tare is ſubtracted, but not the tret.

Neat weight, is the remainder when both (if both be allowed) are taken away.

QUEST. I.

If 6 bags of raiſins, marked with the groſs weight, as followeth, tare 20 pound *per* bag, what neat weight?

Firſt, Multiply 20, the weight of each bag, by 6, the number of bags, produceth 120, which divided by 112, gives 1 *C.* 0 *q.* 8 *lb.* which ſubtracted from the groſs, gives the neat wt.

	C.	*q.*	*lb.*	
A	7	2	11	
B	8	1	17	
C	6	3	14	
D	9	0	10	
E	8	2	17	
F	6	1	20	
	47	0	05	groſs wt.
Sub.	1	0	8	tare.
Anſw.	45	3	25	neat wt.

```
      20
       6
    ——— C. q. lb.
112) 120 (1  0  8
     112
     ———
       8
```

QUEST. II.

In 346 *C.* 3 *q.* 12 *lb.* groſs, tare 12 *lb.* *per* *C.* how many *C.* neat?

Firſt, find the pounds groſs.

Thus

	C.	*q.*	*l.*	To find the tare.
Thus	346	3	12	*lb.* *lb.* *lb.*
	4			Say, if 112 : 12 :: 38848

```
 1387 quarters.          12
   28                  -----
 -----                 77696
 11098                38848
 2775                 -----
 -----           112) 466176 (4162 = the tare.
 38848 lb. gross.      448
  4162 tare sub.       ---
 -----   C. q. lb.     181
112)34686 (309 2 22 neat wt.   112
 336                   ---
 ----                  697
  1086                 672
  1008                 ---
  ----                  256
Rest 78 lb. = 2 qrs. 22 lb.   224
                        ---
                   Rem. 32 Inconsid.
```

QUEST. III.

In 674 *C.* 2 *q.* 16 *lb.* gross weight, tare 14 *lb.* *per* *C.* what neat weight?

	C.	*q.*	*lb.*
$\frac{1}{8}$	674	2	16 gross wt.
	84	1	9 tare.
	590	1	7 neat wt.

When tare, as here, is an aliquot part of 112, take such part of the gross weight; which subtracted from the gross, leaves the neat weight, as in the example I take $\frac{1}{8}$ part for 14 *lb.*

Any aliquot part is shewn in the annexed table.

Aliquot parts 112.
lb.
7 is = $\frac{1}{16}$
14 is = $\frac{1}{8}$
16 is = $\frac{1}{7}$
28 is = $\frac{1}{4}$
56 is = $\frac{1}{2}$
84 is = $\frac{3}{4}$

The

The tare in the second question might have been found more quickly, as in the following question, where the tare is no aliquot part of 112.

QUEST. IV.

In 246 *C.* 2 *q.* 16 *lb.* gross, tare 21 *lb. per C.* how many hundred neat? *Answ.* 200 *C.* 1 *q.* 16 *lb.*

Reduce the gross into pounds thus,	To find the tare.
C. q. lb.	*C. q. lb.*
246 2 16	246 2 16
4	21
986	246
28	492
7894	5166
1973	$10\frac{1}{2}$
	3
27624 *lb.* gross.	
$5179\frac{1}{2}$ tare	$5179\frac{1}{2}$ tare

Here to find the tare I multiply the gross *C.* by 21, and for the 2 quarters I took $\frac{1}{2}$ of the 21 *lb.* and because 16 is $\frac{1}{7}$ of 112, I took $\frac{1}{7}$ of 21, which is 3, which added is the tare, as in the work.

```
                        C.   q.   lb.
112) 22444½ (200   1    16½ neat wt.
     224
     ----- q.  lb.
       044=1   16
```

The last question may be answered as in the annexed work; for though 21 *lb.* is not an aliquot part of 112 *lb.* it may be parted into aliquot parts (*viz.*) 14 and 7. For 14 *lb* take $\frac{1}{8}$ part, and for 7 *lb.* take $\frac{1}{2}$ of 14; those two added together give the tare, which subtracted from the gross, leaves the neat weight, as before. Thus you may do in many cases, which the learner should observe.

C.

	C.	q.	lb.	
$\frac{1}{8}$	246	2	16	groſs wt.
$\frac{1}{2}$	30	3	9	tare.
	15	1	$18\frac{1}{2}$	
Sub.	46	0	$27\frac{1}{2}$	
	200	1	$16\frac{1}{2}$	neat wt.

QUEST. V.

In 72 *C.* 3 *q.* 12 *lb.* groſs, tare 16 *lb. per C.* tret 4 *lb. per* 104, how many *C.* neat?

The tare is found as before, which ſubtracted from the groſs, leaves the ſubtile, which I divide by 26, which is $\frac{1}{4}$ of 104, quotes the tret, which taken from the ſubtile, leaves the neat weight.

	C.	q.	lb.	
$\frac{1}{7}$	72	3	12	groſs wt.
	10	1	18	tare.
$\frac{1}{26}$	62	1	22	ſubtile.
	2	1	17	tret.
	60	0	05	neat wt.

QUEST. VI.

In 6 hogſheads of tobacco, containing 56 *C.* 2 *q.* 20 *lb.* groſs, tare 30 *lb. per* hogſhead, 4 *lb. per* 104 tret, and 2 *lb.* upon every 336 groſs weight for cloff; what will the neat weight coſt at 6 *d.* $\frac{1}{2}$. *per* pound; 51 *l.* 11 *s.* 8 *d.* being deducted for cuſtom and other charges? *Anſwer,* 108 *l.* 11 *s.* 9 *d.*

See the following work.

The work.

lb.	Mul. 30	336) 5931 (17=the [cloff.
112	By 6	336
56	180 tare	2571
672	26) 6168 (237$\frac{3}{13}$ tret.	2352
560	52..	219
56	96	At 6 *d.* $\frac{1}{2}$ wt. 5914
20	78	2957
6348 grofs.	188	246.5
180 tare.	182	*s.* 320|3.5 *d.*
6168 fubtile.	6	*L.* 160. 3.5
237 tret.		51.11.8
5931		*Anf. L.* 108.11.9
17 cloff.		
5914 neat.		

The Rule of BARTER.

BARTER is a rule by which merchants or others exchange goods of feveral prices and quantities, fo as to receive no lofs by fuch truck or change.

Obferve the nature and work of the following queftions.

QUEST. I.

How many pounds of fugar at 4 *d.* $\frac{1}{2}$ *per* pound, muft be given in barter for 60 grofs of incle at 8 *s.* 8 *d.* *per* grofs?

First,

First, by practice find the value of the 60 grofs of incle at 8 *s*. 8 *d*. *per* grofs, which will be 26 *l*. which divided by the decimal of 4 *d*. $\frac{1}{2}$, which is .01875, quotes $1386\frac{2}{3}$, the number of pounds fought.

	s.	*d.*		
at	8 :	8	grofs wt.	60 gro.
				24
				2
			Facit	26 *l.*

```
.01875) 26.00000 (1386.666
         ...
        1875
        ----
         7250
         5625
         -----
         16250
         15000
         -----
          12500
          11250
          -----
           1250
```

At 4 *d*. 1 *lb*. wt. $1386\frac{2}{3}$

Proof,	17	6	6
	8	13	3
			3
	26	0	0

Or the laft queftion may be wrought thus: bring the price of a grofs (*viz*. 8 *s*. 8 *d*.) into pence, which will be 104; then fay,

If 4 *d*. $\frac{1}{2}$, or 4.5, become 104 *d*. what will 60 become? *Facit* $1386\frac{2}{3}$, as before.

But you may obferve the terms are not methodically ftated; but becaufe the 3d multiplied by the 2d, and divided by the 1ft, gives the true anfwer, we have fo placed the numbers: the reader therefore is to confider fuch cafes, that he be at no lofs; for the reafon is evident.

QUEST. II.

[illegible] packs of cotton-wool at 240 *lb*. *per* pack, and

and at 15 *d. per* pound ready money; in barter he will have 16 *d.* ½. *B* hath broad beds at 5 *s. per* yard ready money. The question is, to know how *B* must raise his beds in barter *per* yard, that he be no loser; and how many yards will be equivalent to the two packs of cotton?

Find first how much he must raise his beds in barter; thus, if 15 *d.* becomes 16 *d.* ½ what will 5 *s.*? *Facit* 5 *s.* 6 *d.*; this done, at 16 *d.* ½ *per lb.* what will 2 packs or 480 *lb.* cost? And *note*, That as many pence as the pound costs, so many pounds will the pack cost. So the 2 packs will cost 33 pounds: then, as in the last question, divide 33 by the decimal of 5 *s.* 6 *d.* (*viz.*) 275, quotes 120, the number of yards sought.

See the work.

```
          d.    d.       s.
First, If 15 : 16.5 :: 5.
                  5
               ----
          15) 82.5 (5.5      Facit, 5 s. 6 d.
              75
              ---
               75
               75.
               ---
                0

Secondly, .275) 33.000 (120 yards.
                275
                ---   At 5 s. 6 d. Yd. Wt.  ┌ 120
                 550                        │ ---
                 550               Proof,  <   30
                 ---                        │   3
                   0                        └ ---
                                          Facit 33
```

QUEST. III.

A hath 100 yards of kersey at 3 *s.* *per* yard ready money, which he barters with *B* at 3 *s.* 6 *d.* taking small hair-buttons at 15 *d.* *per* gross, which are but worth 12 *d.* How many gross of buttons will pay for the kersey; and whether doth *A* or *B* get the better bargain; and by how much?

First, at 3 *s.* 6 *d.* *per* yard, what 100 yards?

12.5
5.0

Facit, 17.5 or 17 *l.* 10 *s.*

Secondly, ,0625) 17.5000 (280 numb. of gr. sought.

1250

5000
5000

0

Thirdly, At 3 *s.* Yd. Wt. 100

10
5

Facit 15 *A.*

Fourthly, At 12 *d.* gr. what 280

14 *B.*

A's goods are worth 15 *l.* and *B*'s goods worth 14 *l.* by which it is evident *B* gets the better bargain by 1 pound, or 20 shillings.

But if it were required to know how much *per cent.* *B* hath the better bargain, then say, If 14*l.* become 15*l.* what will 100*l.* become? *Ans.* 107.142 *l.* or 7.142, the gain *per cent.*

14) 1500 (107.142
14
—
100
98
—
20
14
—
60
56
—
40

QUEST. IV.

A hath linen-cloth at 10*d.* the ell ready money, in barter 12*d.* *B* hath 3610 pound of sugar at 7*d.* the pound ready money, and would have of *A* 35*l.* in ready money, the rest in linen-cloth. The question is, What rate the sugar bears in barter, and how much linen cloth *A* must give unto *B*?

It is evident that as much sugar as *B* receives ready money for, which is 35 pounds worth, he ought but to have 7*d. per* pound for it.

However, first find the barter thus; If 10*d.* : 12*d.* : : 7*d.* : *facit* 8 $\frac{4}{10}$. Then find the quantity of sugar *A* must have for his 35*l.* Thus if $\frac{7}{240}$: $\frac{1}{2}$: : $\frac{35}{1}$ *facit* 1200 *lb.* of sugar, which subtract from 3610, resteth 2410, which at 8*d.* $\frac{4}{10}$. the pound will amount to 84*l.* 7*s.* or 84.35, which divided by the decimal of 1 shilling, *viz.* .05, gives 1687, the quantity of linen-cloth in ells, which was required.

The

The rate the sugar bears in barter is 8.4 *l.* *B* must receive 1687 ells of linen-cloth, and 35 *l.* in ready money.

QUEST. V.

Two merchants barter: *A* hath 20 hundred of cheese at 21 *s.* 6 *d.* the hundred; *B* hath 8 pieces of *Irish* cloth, at 3 *l.* 14 *s.* *per* piece. The question is, Whether must receive money, and how much? *Ans.* *A* must pay to *B* 8 *l.* 2 *s.*

First, Say, at 21 *s.* 6 *d.* the *C.* what 20 *C.*?

Ans. 21 *l.* 10 *s.* *A*'s goods.	1	
	0	10
Facit	21	10

Secondly, Say, at 3 *l.* 14 *s.* *per* piece, what 8 pieces?

					24	
Ans.	29	12	*B*'s goods.		4	
Sub.	21	10			1	12
Rest	8	2		*Facit*	29	12

QUEST. VI.

A barters with *B* silk-stockings at 30 *s.* *per* pair, which are vendible but for 26 *s.* and would have $\frac{1}{3}$ ready money, and again 10 *l.* *per cent.* for stuff at 4 *s.* *per* yard ready money: how must the yard of stuff be valued to equal the barter?

First, I say, if 100 *l.* : 110 *l.* : : what 30 *s.*? *Facit* 33 *s.*

30

33.00

Then, because *A* would have $\frac{1}{3}$ in ready money, the barter only resteth in the $\frac{2}{3}$, which is the remainder:

der: wherefore ſubtract $\frac{1}{3}$ of 33, which is 11, from the true price 26, reſteth 15, and from itſelf, to wit 33, leaves 22. Then ſay,

$$\text{If } 15 : 22 :: 4$$

$$\begin{array}{r} 4 \\ \hline 15)\ 88\ (5\frac{13}{15} \\ 75 \\ \hline 13 \end{array}$$

Anſ. at 5 *s.* $\frac{13}{15}$.

QUEST. VII.

A barters with *B* cloves at 6 *s.* for 7 *s.* 6 *d.* *per* pound, and is willing to loſe 10 *l.* *per cent.* to have $\frac{1}{3}$ ready money: What is the juſt price of a yard of velvet delivered at 22 *s.* to equal the barter?

$$\begin{array}{lrrr} & l. & l. & s. \\ \textit{First}, \text{ Say if} & 100 : & 90 :: & 7.5 \end{array}$$

$$\begin{array}{r} 7.5 \\ \hline 450 \\ 630 \\ \hline 6.750 \end{array}$$

Facit, 6 *s.* 9 *d.*

Next, for $\frac{1}{3}$ ready money, which he deſires, abate from 7 *s.* and 6 *d.* the bartering price, $\frac{1}{3}$ thereof, which is 2 *s.* 6 *d.* reſt 5 *s.* Then take 2 *s.* 6 *d.* from 6 *s.* 9 *d.* reſt 4 *s.* 3 *d.*

Then

Then say, If $\overset{s.}{5} : \overset{s.}{4.25} :: \overset{s.}{22}$

```
       22
      ----
       850
      850
     -----      s.
  5) 93.50 (18.7 equal to the just price
                 of a yard of velvet.
     5
     --
     43
     40
     --
      35
      35
     --
      00
```

The Rule of FELLOWSHIP.

THE *Rule of Fellowship* is for merchants, or other traders, where they have joint stocks in company, to distribute unto every one his proportional share of the gain or loss, according to his stock laid out.

It is divided into two parts, commonly called the *Single* and *Double Rule of Fellowship*; of which in their order.

In the *Single Rule*, having the particular stocks, and the whole gain or loss, to find each particular gain or loss, observe this general rule;

As the total sum of the stocks :
To the total gain or loss : :
So each man's particular stock :
To each man's particular gain or loss.

QUEST.

QUEST. I.

Three merchants put in money together, *A*, *B*, and *C*; *A* put in 20 *l.* *B* put in 30 *l.* *C* put in 40 *l.* they gained 180 *l.* what is each man's part of the gain?

A 20
B 30 } The total sum 90 *l.* Gain 180 *l.*
C 40

Sum 90

Then say,

First, For *A*'s share; If 90 *l.* : 180 *l.* : : 20 *l.*

```
        20
    ------
90) 3600 (40 for A.
    360
    ------
     00
```

Secondly, For *B*'s share; If 90 *l.* : 180 *l.* : : 30 *l.*

```
        30
    ------
90) 5400 (60 for B.
    540
    ------
     00
```

Thirdly, For *C*'s share; If 90 *l.* : 180 *l.* : : 40 *l.*

```
        40
    ------
90) 7200 (80 for C.
    720
    ------
     00
```

Part.
A's 40 *l.*
B's 60 *l.*
C's 80 *l.*

Sum 180 for proof.

But if you consider the observation belonging to the question, in the *Rule of Three*, you may contract your work in this and the like questions, as was there intimated.

mated; for if you divide your ſecond number 180 by your firſt number 90, and by the quotient 2, multiplying *A*, *B*, and *C*'s ſtock, (*viz.*) 20, 30, 40, produceth 40, 60, 80, *A*, *B*, and *C*'s gains.

See the work.

```
90) 180 (2      Mult 20 l.        Mult. 30
    180         By      2         By     2
    ———             ———               ———
      0             40=A's gain.      60=B's gain

                                  A's 40 gain.
    Mult. 40                      B's 60 gain.
    By     2                      C's 80 gain.
       ———                            ———
       80=C's gain.                  180 proof.
```

But if you conſider this queſtion, the anſwer may more quickly be found yet; for ſeeing the gain was double to the whole ſtock, each man's gain will be double to his ſtock; and ſuch conſiderations as theſe may be of good uſe in many caſes.

QUEST. II.

A chapman breaking, owes unto four men the following ſums of money, *viz.*

		l.	*s.*	*d.*
To	*A*	21	9	6
	B	72	19	3
	C	114	13	9
	D	264	17	6
Sum=		474	00	0

His whole eſtate is found to be but 148 *l.* 2 *s.* 6 *d.* What muſt each have of the ſame, and what will it be *per* pound?

If you work this queſtion the contracted way, the quotient will be the anſwer *per* pound, to wit, 6 *s.* 3 *d.*

See

See the whole work.

474) .148.125 (.3125=6 *s.* 3 *d.* what each must have *per pound.*

```
474) .148.125 (.3125
      1422
      ----
       592
       474
       ----
       1185
        948
        ----
        2370
        2370
        ----
           0
```

Reduce the broken part of each man's money into decimals, which multiplied by .3125, gives in the several products, what each must have.

```
Mult. 114.6875      Mult. 72.9625      Mult. 21.475
By       .3125      By       .3125     By      .3125
   -----------         -----------        ---------
      5734375            3648125            107375
     2293750            1459250             42950
    1146875             729625              21475
   3440625             2188875              64425
 ------------       ------------        ----------
C=35.83984375      B=22.80078125       A=6.7109375
```

```
Mult. 264.875
By       .3125
   ----------
     1324375
     529750
    264875
   794625
  -----------
D=82.7734375
```

	l.		*l.*	*s.*	*d.*	*q.*
A=	6.7109375	=	6 :	14 :	2 :	2
B=	22.80078125	=	22 :	16 :	0 :	0
C=	35.83984375	=	35 :	16 :	9 :	3
D=	82.7734375	=	82 :	15 :	5 :	3
Proof	148.12500000	=	148 :	02 :	6 :	0

QUEST.

QUEST. III.

A, *B* and *C* put in money together; *A* put in 20 *l.* *B* and *C* put in 85 *l.* they gained 63 *l.* of which *B* took up 21 *l.* What did *A* and *C* gain, and *B* and *C* put in?

First, Find *A*'s gain, thus; If 105, the sum of all stocks, gain 63 *l.* what will 20 *l.* *A*'s stock gain? *Facit* 12 *l.* for *A*'s gain.

Secondly, Find *B*'s stock, thus; If 12 *l.* which is *A*'s gain, come from 20 *l.* *A*'s stock; what will 21 *l.* come from, which is *B*'s gain? *Facit* 35 *l.* which is *B*'s stock; then *C*'s stock must be the remainder to 85, (*viz.*) 50 *l.* And if you subtract *A* and *B*'s gain 33, from the whole gain 63, rests 30 for *C*'s gain.

QUEST. IV.

A, *B* and *C* put in money together; *A* put in 20 *l.* *B* 30 *l.* *C* a sum unknown; they gained 36 *l.* *C* took up 16 *l.* What did *A* and *B* gain, and *C* put in?

Subtract *C*'s gain, 16 *l.* from the whole gain 36 *l.* resteth 20 *l.* Then say, If 50 *l.* *A* and *B*'s stock, gain 20 *l.* what will *A*'s stock, 20 *l.* gain? *Facit* 8 for *A*, then *B* must have 12. To find *C*'s stock, say, If 8 *l.* which is *A*'s gain, come from 20, what will 16 *l.* *C*'s gain, come from? *Facit* 40 *l.* which *C* put in.

QUEST. V.

A, *B* and *C* put in 360 *l.* and gained 270 *l.* of which as oft as *A* took up 3 *l.* *B* took up 5 *l.* and as oft as *B* took up 5 *l.* *C* took up 7 *l.* What did each gain, and put in?

Suppose some number for *A*, that the rest of the parts may be taken without fractions: As suppose *A* had 9 *l.* then *B* must have 15 *l.* and *C* 21 *l.* whose sum is 45.

Then the proportion is,

		9	54
As 45 :	To 270, So, : :	15 :	To 90
		21	126

 But

But you had better find a common multiplicator, as in the contracted way.

45) 360 (8=com. mult.	45) 270 (6=com. mult.
360	270
———	———
0	0
Then 8 times 9 is 72 *A*.	Then 6 times 9 is 54 *A*.
And 8 times 15 is 120 *B*.	And 8 times 15 is 90 *B*.
And 8 times 21 is 168 *C*.	And 6 times 21 is 126 *C*.
———	———
Proof=360	Proof=270

QUEST. VI.

Two merchants company; *A* puts in 36 *l.* and taketh $\frac{3}{5}$ of the gain; what did *B* put in?

If *A* take up $\frac{3}{5}$, *B* must needs have $\frac{2}{5}$.

Then say, If $\frac{3}{5}$: 36 : : $\frac{2}{5}$: *Facit* 24 for *B*.

```
        2
      ———
    3) 72 (24
       6
      ———
       12
       12
      ———
        0
```

Seeing the denominators of the fractions are equal, I neglect them, and work with the numerators.

QUEST. VII.

Two merchants company; *A* put in 20 *l.* and *B* put in 144 *ducats*; they gained 67 *l.* 10 *s.* of which *A* took up 30 *l.* What is the value of a *ducat*? *Ans.* 6 *s.* 3 *d.*

First,

First, find a stock for *B*, equivalent to *A*'s stock, thus:

```
                 l.    l.    l.
              If 30 : 20 : : 67.5
   144) 45000 (.3125=        20
          ·· to 6 s. 3 d.  ------
        432            30) 1350.0 (45 l.=to 144 duc.
        ---                   ··
        180                120
        144                ---
        ---                 150
         360                150
         288                ---
         ---                 00
          720
          720
          ---
           0
```

QUEST. VIII.

Two merchants put in money together, and gained 120 *l.* Their agreement was, that *A* should have 10 *l.* *per cent.* gain, and *B* 8 *l.* *per cent.* What must each have?

Suppose each man's gain *per cent.* to be his stock; so *A*'s stock will be 10 *l.* and *B*'s stock 8 *l.* whose sum is 18 *l.* Then say, If 18 *l.* : 120 *l.* : : 10 *l.* *Facit* $66\frac{2}{3}$, for *A*'s gain, then *B* must have $53\frac{1}{3}$.

```
        18 : 120 : : 10
                     10
               ----
           18) 1200 (66 12/18 = 2/3 A's part.
                ·
               108
               ---
               120    120
               108    66 2/3
               ---    ------
                12    53 1/3 B's part.
```

QUEST. IX.

A, *B*, *C* and *D* put in money together, and gained a sum of money, of which *A*, *B* and *C* took 60 *l.* *B*, *C* and *D* took 90 *l.* *C*, *D* and *A* took 80 *l.* and *D*, *A* and *B* took up 70 *l.* What distinct gain did each take up?

Add these 4 numbers into one sum, which makes 300 *l.* in which each man's money is named 3 times; therefore take $\frac{1}{3}$ of it, makes 100 *l.* for the whole gain; from which subtract *A*, *B* and *C*'s gain 60 *l.* leaves *D*'s gain 40 *l.* and from the same sum 100 *l.* subtract what *B*, *C* and *D* took up, leaves 10 *l.* for *A*; and from the same sum 100 *l.* subtract what *C*, *D* and *A* took up, *viz.* 80 *l.* leaves 20 *l.* for *B*'s gain. And lastly, If from 100 *l.* you subtract what *D*, *B* and *A* took up, 70 *l.* leaves 30 *l.* for *C*'s gain. So *A* had 10 *l.* *B* 20 *l.* *C* 30 *l.* and *D* 40 *l.*

QUEST. X.

Four men bought a hive of bees for 20 shillings, of which *A* must pay $\frac{1}{3}$, *B* $\frac{1}{4}$, *C* $\frac{1}{5}$, *D* $\frac{1}{6}$; what must each pay of the 20 shillings?

Here if you take the natural parts of 20 shillings as they are expressed in the question,

	s.	s. d.	
Then	$\frac{1}{3}$ of 20	= 6 : 8	But the sum to be paid is 20 *s.*
	$\frac{1}{4}$ of 20	= 5 : 0	wherefore 20 *s.* must be di-
	$\frac{1}{5}$ of 20	= 4 : 0	vided into such proportion as
	$\frac{1}{6}$ of 20	= 3 : 4	the parts bear one to another.
	Sum	= 19 : 0	

Wherefore reduce the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, into a common denominator, which will be $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$, and neglecting the denominators, add the numerators into one sum, which will be 57; then the proportion is,

As

$$\text{As } \overset{s.}{57} : \text{To } 20 :: \text{So} \left\{ \begin{array}{l} \overset{s.}{20} : \text{To} : 7\tfrac{1}{57} \text{ for } A\text{'s} \\ 15 : \text{To} : 5\tfrac{15}{57} \text{ for } B\text{'s} \\ 12 : \text{To} : 4\tfrac{12}{57} \text{ for } C\text{'s} \\ 10 : \text{To} : 3\tfrac{29}{57} \text{ for } D\text{'s} \end{array} \right\} \text{payment.}$$

Proof, 20 : 0

But if the agreement had been, that *A* must pay $\frac{1}{2}$, *B* $\frac{1}{3}$, *C* $\frac{1}{4}$, *D* $\frac{1}{5}$, then the parts would have exceeded the whole.

	s.	d.
For $\frac{1}{2}$ of 20 is	10 :	0
And $\frac{1}{3}$ of 20 is	6 :	8
And $\frac{1}{4}$ of 20 is	5 :	0
And $\frac{1}{5}$ of 20 is	4 :	0
Sum is	25 :	8

In this case work as before, by reducing the fractions into common denominators, which will be $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, the sum of the numerators is 77. Then,

$$\text{As } \overset{s.}{77} : \text{To } 20 :: \text{So} \left\{ \begin{array}{l} \overset{s.}{30} : \text{To} : 7\tfrac{61}{77} \text{ for } A\text{'s} \\ 20 : \text{To} : 5\tfrac{15}{77} \text{ for } B\text{'s} \\ 15 : \text{To} : 3\tfrac{69}{77} \text{ for } C\text{'s} \\ 12 : \text{To} : 3\tfrac{9}{77} \text{ for } D\text{'s} \end{array} \right\} \text{payment.}$$

Proof, 20 : 0.

FACTORSHIP.

UNder this head may be reckoned those questions which belong to Factorship, such as are these following.

QUEST. I.

A merchant delivers unto his factor 50 *l.* and if the factor put in 30 *l.* he will allow him half of the gain; What is the factor's person esteemed at?

Subtract 30 *l.* from 50 *l.* resteth 20 *l.* and so much is the factor's person esteemed at. The reason is evident.

QUEST. II.

A merchant delivers unto his factor 60 *l.* and allows him for his gains $\frac{1}{3}$ of the gain: What money must the factor put in that he may have equal gain?

From 60 *l.* take $\frac{1}{3}$, which is 20 *l.* which subtract from 60 *l.* leaves 40 *l.* which the factor must put in to have half the gains.

QUEST. III.

A merchant delivers to his factor 500 *l.* and esteemed his person at 200 *l.* When they made up their accounts, they gained 20 *l.* *per cent.* What is the factor's part?

To 500 *l.* add 200 *l.* the sum is 700: Then say, If 100 *l.* gain 20 *l.* what 700 *l.*? *Facit*, 140 *l.* Then, If 700 *l.* gain 140 *l.* what 200 *l.*? *Facit*, 40 *l.* the factor's part, and the merchant must have 100 *l.*

QUEST. IV.

A factor receives 1000 *l.* from a merchant, to which he adds 300 *l.* of his own; his person is esteemed at 260 *l.* What must the factor have of the gain?

It is evident the merchant's stock is 1000 *l.* and the factor's own stock 300 *l.* together with 260 *l.* his personal value, makes 560 *l.* to which the factor's stock is equivalent.

The merchant's stock	1000
The factor's stock	560
Sum	1560

Then say, If 1560 : 1 : : 560, *Facit* $\frac{14}{39}$, for the factor's share; then the merchant must have $\frac{25}{39}$ for his share.

Double

Double FELLOWSHIP.

IT is called *Double Fellowship*, when their gains are different, not only in respect to their stocks, but in respect of the time of continuance in company. And that you may work any such question, observe this general rule.

As the total sum of the products of each man's money and time : is to the total gain : : So the particular product of each man's money and time : is to each man's particular gain.

QUEST. I.

A and *B* put in money together; *A* put in 20 *l.* and *B* put in 20 *l.* likewise: but *A*'s money was in company 9 months, and *B*'s but 6 months; they gained 60 pounds; what must each have?

A's money multiplied by his time, is	180
B's money multiplied by his time, is	120
The sum	300

Then, as 300 : 60 : : $\begin{cases} 180 : 36 \text{ } A\text{'s gain.} \\ 120 : 24 \text{ } B\text{'s gain.} \end{cases}$

QUEST. II.

A, *B* and *C* put in money together; *A* put in 20 *l.* for 6 months; *B* put in 40 *l.* for 3 months, and *C*'s money was 60 *l.* which continued in company 2 months; their common gain was 36 *l.* What must each have of the gain?

First,

First, Multiply *A*'s money	20	pounds.
By his time	6	months.
The product of *A*'s money and time =	120	
Multiply *B*'s money	40	pounds.
By his time	3	months.
The product of *B*'s money and time =	120	
Multiply *C*'s money	60	pounds.
By his time	2	months.
The product of *C*'s money and time =	120	

Now seeing the three products are equal one with another, it is evident every man must have an equal share of the gain: so each man's part is 12 *l.*

QUEST. III.

A, *B* and *C* put in money together; *A* put in 20 *l.* for 3 months, *B* put in 30 *l.* for 5 months, and *C* put in 40 *l.* for 7 months; they gained 60 *l.* What must each have of the gain?

Multiply every man's money by his time, and the three products will be found to be, for *A* 60 *l.* for *B* 150, and for *C* 280, whose sum is 490 *l.*

$$\text{Then say, If } 490 : 60 :: \left\{\begin{array}{c}60\\150\\280\end{array}\right\} \textit{Facit} \left\{\begin{array}{c}7\frac{17}{49}\\18\frac{18}{49}\\34\frac{14}{49}\end{array}\right\}$$

Proof, 60.00

QUEST. IV.

A and *B* company: *A* put in the first of *January* 50 *l.* but *B* could not put any money in till the first of *May*; What must *B* then put in, to have an equal share with *A* at the year's end?

If I multiply 50 *l.* which is *A*'s money, by 12 months, which is *A*'s time, it will produce 600 *l.* Now it is plain,

plain, that *B*'s money can but be in company 8 months; it is likewiſe evident that ſo much money is required of *B*, which multiplied by 8, ſhall produce 600; divide therefore 600 by 8, quotes 75, which is what *B* muſt put in.

$$
\begin{array}{l}
8)\ 600\ (75 \\
\ \ \ \ 56 \\
\hline
\ \ \ \ \ \ 40 \\
\ \ \ \ \ \ 40 \\
\hline
\ \ \ \ \ \ \ \ 0
\end{array}
$$

QUEST. V.

A, *B* and *C* keep company; *A* put in the firſt of *March* 60 *l.* *B* put in the firſt of *May* 160 yards of broad cloth, and *C* put in the firſt of *June* 240 *ducats*. On the firſt of *January* following, they accounted their gain; of which *A* and *B* took up 456 *l.* *B* and *C* took up 431 *l.* and *C* and *A* took up 375 *l.* The queſtion is, What was gained as well in the whole as a-part; what *B* valued a yard of cloth at, and what was *C*'s ducat *per* piece?

If you add the three numbers together, and take half that ſum, becauſe every man's money is there named twice, you will have the whole gain.

See the work.

To find the ſeveral gains.	
From 631, the whole gain,	*A* and *B*'s gain was 456
Sub. 431, *B* and *C*'s gain,	*B* and *C*'s gain was 431
	C and *A*'s gain was 375
Reſt 200=to *A*'s gain:	
	The ſum is=1262
From 631, the whole gain,	The half or whole ga. 631
Sub. 375, *C* and *A*'s gain,	
	l.
Reſt 256=to *B*'s gain.	Then *C*'s gain muſt be 175

To

To find the value of a yard of cloth, there are ſeveral ways; we ſhall perform it at two operations by the *Rule of Three*, which I conceive may be moſt beneficial to the learner, becauſe one operation will be inverſe.

Firſt, Therefore ſay, If 200 *l.* come from 60, what will 256 *l.* come from? *Facit*, 79.8.

```
         l.    l.     l.
     If 200 : 60 : : 256
                       60
                  -------
              200) 15360 (76.8
                   1400
                   ----
                    1360
                    1200
                    -----
                     1600
                     1600
                     ----
                        0
```

Then ſay, If 10 months come from 76.8; what will 8 months come from? *Facit* 96. For ſeeing the time is leſs, it muſt come from a greater ſtock.

```
     M.    l.     M.
  If 10 : 76.8 : : 8.
                   10
            ---------
          8) 768.0 (96=the value of the
             72          whole cloth.
             --
             48
             48
             --
              0
```

Divide

Divide 96 *l.* by 160, gives the value of a yard, *viz.* 12 shillings.

160) 96.0 (.6=to 12 *s.*
960
—
0

After the same method you must find the value of a *ducat*; for, first, I say, If 200 *l.* which is *A*'s gain, come from 60 *l.* which is *A*'s stock, what will 175 *l.* come from, which is *C*'s gain? *Ans.* from $52\frac{1}{2}$, or 52.5.

l. *l.* *l.*
If 200 : 60 : : 175
60
—
200) 10500 (52.5
1000
—
500
400
—
1000
1000
—
0

Secondly, If 10 months produce 52.5; what will 7 months? *Answer*, 57 *l.*; for seeing the time is less, the money will be more.

If

$$\begin{array}{l} \quad M. \quad l. \quad M. \\ \text{If } 10 : 52.5 :: 7 \\ \qquad\quad 10 \\ \hline 7)\ 525.0\ (75\ l. = \text{to } 240 \text{ ducats.} \\ \quad 49 \\ \hline \quad 35 \\ \quad 35 \\ \hline \quad\ \cdot 0 \end{array}$$

Which if divided by 240, gives in the quotient .3125, which is equal to 6 *s.* 3 *d.* the just price of a ducat.

QUEST. VI.

A, *B* and *C* company, and put in together 3822 *l.* *A*'s money was in 3 months, *B*'s money was in 5 months, and *C*'s money was in 7 months: they gained 234 *l.* which was so divided, as the $\frac{1}{2}$ of *A*'s gain was equal to $\frac{1}{3}$ of *B*'s gain, and $\frac{1}{3}$ of *B*'s gain was equal to $\frac{1}{4}$ of *C*'s gain. What did each merchant gain and put in?

Suppose *A*'s gain was 4 *l.* then *B* must have 6 *l.* and *C* 8 *l.* according to the tenor of the question; which numbers added together make 18: Then I say,

$$\text{If } 18 : 234 :: \left\{\begin{array}{l} 4 \\ 6 \\ 8 \end{array}\right\} \textit{Facit} \left\{\begin{array}{l} 52\ A\text{'s gain.} \\ 78\ B\text{'s gain.} \\ 104\ C\text{'s gain.} \end{array}\right.$$

Next, multiply every man's gain by his time, and the sum of the three products will be 1274; by which dividing the whole stock, you will compute a common multiplicator, by which every man's gain and time multiplied, gives each man's stock sought.

Set

See the work.

1274) 3822 (3 Com. mult.
3822

0

A's gain and time=156
3

468 *A*'s ſtock.

B's gain and time=390
3

1170 *B*'s ſtock.

Then *C*'s ſtock muſt be 2184

QUEST. VII.

A, B and *C* company; *A* put in the firſt of *January* 100 *l.* and the firſt of *May* puts in 150 *l.* more; and on the firſt of *September* takes out 30 *l.* The remainder ſtays in till the year's end.

B put in the firſt of *January* 250 *l.* and on the firſt of *June* 60 *l.* more; and on the firſt of *November* 100 *l.* more, which continues in till the year's end.

C put in the firſt of *January* 300 *l.* and the firſt of *April* takes out 200 *l.* and on the firſt of *Auguſt* takes out 50 *l.* more; the remainder ſtays in till the year's end. What muſt each have of the gain, which was 133 pounds?

It is plain that *A* hath 100 *l.* in for 4 months, and 250 *l.* for 4 months, and 220 for 4 months, which 3 products will be 2280 for *A*'s whole money and time; and it is evident that *B* hath 250 *l.* in for 5 months; and 310 *l.* for other 5 months, and 410 *l.* for 2 months, which 3 products will be 3620 for *B*'s whole money and time. It is likewiſe evident that *C* hath 300 *l.* in for 3 months, and 100 *l.* for 4 months, and 50 *l.* in for

5 months; so the 3 products will be 1550 for *C*'s whole money and time: and by the work of the second question the parts of the gain will be found,

For $\left\{\begin{array}{ll} A\text{'s part} & 40.71 \\ B\text{'s} & 64.62 \\ C\text{'s} & 27.67 \end{array}\right\}$ The sum 133.

Loss and Gain.

BY this rule we discover what is got or lost *per cent.* in selling and buying goods; and instructs us how to raise or fall the price of goods, to gain or lose so much *per cent.* or otherwise, either with or without time.

This is of excellent use to most traders; and there being a great deal of variety in it, we will endeavour to make all plain in the following questions or examples.

QUEST. I.

If I buy yarn for 9 *d.* the pound, and sell it again for 13 *d.* $\frac{1}{2}$, what is gained *per cent.* or in laying out 100 *l.* at that price?

Say, If 9 *d.* become 13 *d.* $\frac{1}{2}$, what will 100 *l.* become? *Facit* 150, and 150—100, gives 50 *l.* for the gain.

```
d.   d.
9 : 13.5 : : 100
              l.
9) 1350.0 (150
   ...     100
   9       ———
  ——       50 the answer.
   45
   45
  ——
   00
```

QUEST.

QUEST. II.

If I buy broad cloth for 11 *s.* 6 *d.* the yard, how must I sell it to gain 20 *l. per cent.*?

Say, If 100 become 120, what will 11 *s.* 6 *d.* or 11.5 *s.* become? *Facit*, 13.8 or 13 *s.* and 9 *d.* ½; and so much must I sell it for, to gain 20 *l. per cent.*

Here having cut off one figure for the decimal, I cut off 2 more, instead of dividing by 100.

```
100 : 120 :: 11.5
             11.5
           ------
             600
            120
           120
           ------          s.
           13.800   Answer 13.8.
```

QUEST. III.

If I buy a *C.* weight of tobacco for 4 *l.* 13 *s.* 4 *d.* and sell it again for 11 *d.* the pound, whether do I gain or lose, and what *per cent.*?

First find by practice what the *C.* will cost at 11 *d.* the pound, which by the work is found to be 5 *l.* 2 *s.* 8 *d.* Then say,

```
                                    11.2
                                   ------
If 4.666 : 5.133 :: 100 : 110 Facit 2.8
And 110 l. minus 100 l. is 10 l.    1.4
the gain sought.                    0.933
                              ------------ l. s. d.
                                    5.133=5 : 2 : 8
```

QUEST. IV.

If a pack of yarn, weighing 240 *lb.* cost 13 *l.* what must a pound be sold for, to gain 15 *l.* 10 *s. per cent.*?

Find what a pound will cost, which in this case is easy; for a pack weighing 240 *lb.* as many pounds as the pack costeth, so many pence the pound will cost: so here a pound will, by that rule, cost 13 *d.* Then say,

$$\begin{array}{l} \quad l. \quad\; l. \qquad\quad d. \qquad\qquad\quad d. \\ \text{If } 100 : 115.5 :: 13 : \textit{Facit } 15.015 \\ \qquad\quad\;\; 13 \\ \hline \qquad\quad 3465 \\ \qquad\; 1155 \\ \hline \end{array}$$

15.015 the *answer*.

QUEST. V.

A *Manchester* man buyeth yarn for 6 *s.* the bundle; which not proving so good as expected, would put it off again so as but to lose 6 *per cent.* by it. The question is, what will a bundle cost?

$$\begin{array}{l} \qquad\quad l. \quad\; s. \quad s. \\ \text{Say, If } 100 : 94 :: 6 \textit{ Ans. } 5\,s.\ 7\,d.\ 2\,q.\ \tfrac{1}{2} \\ \qquad\qquad\quad\;\; 6 \\ \hline \end{array}$$

Facit 5.64=to 5 *s.* 7 *d.* $\frac{1}{2}$ and half a farthing; and so much he must sell it for.

QUEST. VI.

If I buy incle for 8 *s.* the gross, how many yards may I sell for a penny, to gain 20 *l. per cent.*?

$$\begin{array}{l} \qquad\qquad\quad l. \qquad\; l. \qquad\qquad s. \qquad\quad s. \end{array}$$

First, I say, If 100 : 120 :: what 8 : *Facit* .96. Then turn a gross into yards thus; multiply 24, the pieces in a gross, by 36, the yards in a piece, *Facit* 864 yards.

Then say, If $\frac{96}{10}$ of a shilling buy $\frac{864}{1}$ of a yard, what will $\frac{1}{12}$ of a shilling buy? *Facit* $\frac{8640}{1152}$=to $\frac{15}{2}$, or 7 yards and a half; and so many he may sell for a penny, and gain 20 *l. per cent.*

See the work.

$\frac{96}{10} : \frac{864}{1} : \frac{1}{12}$

$\frac{96}{10}$) $\frac{864}{12}$ ($\frac{8640}{1152}=\frac{15}{2}$, or 7 yards $\frac{1}{2}$.

QUEST.

QUEST. VII.

If I sell incle for 12 *s.* a gross, wherein is lost after 5 *per cent.*; what did a yard cost?

Say, if 95 *l.* come from 100 *l.* what doth 12 *l.* come from? *Facit* 12 $\frac{12}{19}$.

$$95 : 100 :: 12$$

```
        12
     ------
95) 1200 (12 60/95, or 12/19
     95
    ----
     250
     190
    ----
      60
```

QUEST. VIII.

A *Manchester* chapman going to a fair, sold fustains for 11 *s.* 6 *d.* the yard, wherein was gained 15 *l. per cent.* and seeing no other chapman had so good, raiseth them at the latter end of the fair to 12 *s.* I demand what he gained *per cent.* by this last sale?

Say, If 11 *s.* 6 *d.* gain 15 *l.* what will 12 *s.* gain? Multiply and divide, and the answer will be found to be 15 *l.* 13 *s.* 00 *d.* 2 *q.*

See the work.

If 11.5 : 15 :: 12
12

30
15

11.5) 180.000 (15.652 gained by
115 ... the last sale.

650
575

750
690

600
575

250
230

20

QUEST. IX.

A *Manchester* man buys 20 tun of cheese, with which he went into *Ireland*; it cost him 400*l.*; the freight and custom came to 50 *l.* his own expences and charges came to 16*l.* 13 *s.* 4*d.* how must he sell it *per* pound to gain 20 *per cent.* by it?

Collect the cost and charges into one sum, and say, If 100*l.* become 120*l.* what will 466*l.* 13*s.* 4*d.*?

Cost, First penny——	400 : 00 : 0
Freight and custom	50 : 00 : 0
His own charges—	16 : 13 : 4
Sum	466 : 13 : 4

See the work.

$$\begin{array}{l} \quad l. \quad\; l. \qquad l. \qquad\qquad l. \\ \text{If } 100 : 120 :: 466.666 : \textit{Facit } 560 \end{array}$$

Say again, If 44800 pound of cheese (and so many is in 20 tun) cost 560 pound, what will one pound cost? *Answ.* 3 *d.* as by the work appeareth.

$$\begin{array}{l} \quad\; lb. \qquad l. \quad lb. \\ \text{If } 44800 : 560 :: 1 \end{array}$$

```
                     1
               -------------
44800) 560.0000 (.0125=to 3 d.
             ...
         44800
         -----
        112000
         89600
        -------
        224000
        224000
        -------
             0
```

QUEST. X.

A merchant selling corn at 8 *s.* the bushel, gained 10 *l. per cent.* but afterwards being, by a falling market, forced to sell it for 7 *s.*; what did he gain or lose *per cent.* by his last sale?

Say, If 1 *s.* make 110. what 7 *s.*? *Facit* 96¼, whereby he lost 3 *l.* 15 *s. per cent.* by this sale.

If

$$\begin{array}{l} \quad s. \quad l. \quad s. \\ \text{If } 8 : 110 :: 7 \\ \qquad\quad 7 \\ \hline 8)\ 770\ (96\tfrac{1}{4} \\ \qquad 72 \\ \hline \qquad 50 \\ \qquad 48 \\ \hline \qquad\ 2 \end{array} \qquad \begin{array}{r} 100 \\ 96\tfrac{1}{4} \\ \hline 3\tfrac{3}{4} \end{array}$$

QUEST. XI.

If I buy yarn for 9*d.* and sell it for 12*d.* and allow 3 months for payment, what do I gain *per cent. per ann.*?

This question admits of a double meaning, and by that means of a double answer.

For first, it is evident, if he on this sale had received ready money, he would have gained 33*l.* $\frac{1}{3}$ *per cent.* but giving 3 months for payment, his gain must needs be less by as much as the rebate of 133*l.* $\frac{1}{3}$ for 3 months amounts to. Which, by proposition the second of simple interest, will be found to be 1*l.* 19*s.* 5*d.* which subtract from 33*l.* 6*s.* 8*d.* leaves 31*l.* 7*s.* 3*d.* the gain in this case sought.

So he makes his 100*l.* to be 131*l.* 7*s.* 3*d.*

But some authors would answer this question thus; *First*, they say,

$$\begin{array}{llll} d. & d. & l. & l. \\ \text{If } 9 :: & 12 :: & 100 : & \textit{Facit } 133\tfrac{1}{3} \text{ as before.} \end{array}$$

But instead of making the gain less, by giving time, they make it vastly more, thus; for they say,

If 3 months gain 33*l.* $\frac{1}{3}$: what will 12 months gain? *Facit* 133*l.* 6*s.* 8*d.* the gain sought.

So they make his 100*l.* to be 133*l.* 6*s.* 8*d.*

But

But you may uſe that which agrees beſt with your own reaſon.

Note, We allow 6 *per cent.* ſimple intereſt for the rebate in the queſtion foregoing, that being the rate allowed by the ſtatute.

The Rule of ALLIGATION.

ALligation teacheth how to mix or unite many ſimples or particulars into one maſs or ſum, according to any price or proportion required.

But, for the eaſe of the learner, we ſhall divide this rule into four varieties, that ſo, when a queſtion is propounded, it is but conſidering what variety it falls under, and the work will ſoon be finiſhed.

VARIETY I.

In this variety we have given the prices and quantities of ſeveral ſimples to be mixed, and the mean rate or price of any part of ſuch mixture is required.

To find which, the proportion is,
As the ſum of ſimples to be mixed :
To the total value thereof : :
So is any part of the compoſition :
To the value thereof.

EXAMPLE.

A tobacconiſt would mix 20 *lb.* of tobacco at 9 *d.* the pound, with 60 *lb.* at 12 *d.* the pound, and with 40 *lb.* at 18 *d.* the pound, and with 12 *lb.* at 2 *s.* the pound. The queſtion is, what a pound of this mixture is worth?

Place

Place the numbers and their values as underneath.

lb.		*s. d.*			*l.*	*s. d.*
20	at	0 9	*per lb.*	will cost	0	15 0
60	at	1 0	*per lb.*	will cost	3	00 0
40	at	1 6	*per lb.*	will cost	3	00 0
12	at	2 0	*per lb.*	will cost	1	04 0
Sum simple = 132				Total value =	7	19 0

Then say, If 132 pound cost 7 *l.* 19 *s.* what will 1? *Facit* 1 *s.* 2 *d.* 2 *q.*

See the work.

lb. *l.* *lb.*

132 : 7.95 : : 1

1

132) 7.95 (.06022=to 1 *s.* 2 *d.* $\frac{1}{2}$.
792

300
264

36

QUEST. II.

A farmer would mix 5 bushels of wheat at 6 *s.* the bushel, with 12 bushels of rye, at 4 *s.* the bushel, with 8 bushels of beans, at 5 *s.* the bushel, and with 18 bushels of barley, at 2 *s.* 6 *d.* the bushel. The price of one bushel of this mixture was demanded.

Place your numbers and their values as under.

B.		*s. d.*			*l.*	*s. d.*
5	at	6 0	the bush.	will cost	1	10 0
12	at	4 0	the bush.	will cost	2	8 0
8	at	5 0	the bush.	will cost	2	0 0
18	at	2 6	the bush.	will cost	2	5 0
Num. bush. 43				Total value =	8	3 0

Then

Then ſay, If 43 buſhels coſt 8 *l.* 3 *s.* or 8.15, what will one buſhel? *Facit* 3 *s.* 9 *d.* ½.

See the work.

```
B.    l.     B.
43 : 8.15 : : 1
        1
     ----
43) 8.15 (.1895=to 3 s. 9 d. ½ fere.
    43
    ----
    385
    344
    ----
     410
     387
     ----
      230
      215
      ----
       15
```

VARIETY II.

In this variety the price of the ſimples is expreſſed, but no quantity given; and it is required how much of each ſimple we muſt take to ſell one quantity or meaſure at a mean rate propounded.

The whole work of this variety is in linking the extremes truly together, and taking the true differences betwixt them and the mean; and theſe differences are the true quantities ſought.

EXAMPLE.

A merchant hath ſpices, ſome at 9 *d.* the pound, ſome at 12 *d.* ſome at 24 *d.* and ſome at 30 *d.* how much of each ſort muſt he take, that he may ſell a pound for 20 *d.*?

Firſt, Set down the ſeveral prices of the ſpices orderly

derly under one another, with a line of connection to the left hand, as in the example may be seen.

Note, That 9, 12, 24, 30, I call the extremes; 9 and 12 being the two lesser extremes, and 24 and 30 the two greater: then, on the other hand, set down the mean price, as you see.

That done, I link or join together two extremes, no matter which, so as one be bigger than the mean, and the other less; and after that other two, till I have finished. So in this example, I link 9 and 30 together, and 12 and 24, as you see; then take the difference betwixt each extreme and the mean price, and place it over against its yoke-fellow: so the difference betwixt 9 and 20 is 11, which place against 30 its yoke-fellow; the difference betwixt 12 and 20 is 8, which place against 24 its yoke-fellow; betwixt 24 and 20 is 4, which place against 12 its yoke-fellow; and lastly, the difference betwixt 30 and 20 is 10, which place over against 9 its yoke-fellow, as you see here done. And the differences here found will be the answer to the question: for as oft as he takes 10 *lb.* of 9 *d.* a pound, he must take 4 of 12 *d.* a pound, and 8 at 24 *d.* the pound; that so a pound may be afforded for 20 *d.* or 1 *s.* 8 *d.*

20	9	10
	12	4
	24	8
	30	11

The proof thereof is easy by the last variety; for the sum of the difference found, multiplied by 20, is equal to the sum of the products of the differences and extremes. But if, at any time, as here it happens, that the extremes may be linked more ways than one, then the question admits of more answers than one, yet all true; for if in this question we link 9 and 24, and 12 and 30, and place the differences respectively, then we shall have 4 at 9 *d.* a pound, 10 at 12 *d.* a pound, 11 at 24 *d.* a pound, and 8 at 30 *d.* a pound, and

20	9	4
	12	10
	24	11
	30	8

and that this is likewise true, may be proved, as in the last, and the mean price will be found to be 20 *d.*

Again, if we link 9 and 30 together, 12 and 30, and 12 and 24, there will arise a new method in placing the differences; for if any extreme have two yoke-fellows, it will likewise have two differences:

20	9	10	10
	12	4 10	14
	24	8	8
	30	11.8	19

so the difference betwixt 9 and 20 is 11, which place over-against 30, its yoke-fellow; the difference betwixt 12 and 20 is 8, which, because it has two yoke-fellows, place against them both, to wit, against 24 and 30, as you see; the difference betwixt 24 and 20 is 4, which place against 12 its yoke-fellow; and lastly, the difference betwixt 30 and 20 is 10, which place against both its yoke fellows, to wit, 12 and 9. Then draw a straight line, and beyond it the differences; so first I set down 10, then 10 and 4 is 14, which set down; then set down 8, then 11 and 8 is 19, which likewise place down; then as oft as he takes 10 at 9 *d.* the pound, he must take 14 at 12 *d.* the pound, and 8 at 24 *d.* the pound, and 19 at 30 *d.* the pound, that so a pound may be worth 20 *d.*

And lastly, If we link 9 both with 24 and 30, and 12 both with 24 and 30, and 24 both with 12 and 9, and 30 both with 12 and 9, then the difference betwixt 9 and 20, which is 11, is to be placed against 24 and 30; and also the difference betwixt 12 and 20, which is 8, against 24 and 30, for the same reason;

20	9	4.10	14
	12	4.10	14
	24	11. 8	19
	30	11. 8	19

and the difference betwixt 24 and 20, which is 4, against 12 and 9, they being thereunto linked; and lastly, the difference betwixt 20 and 30, which is 10, place against 12 and 9, for the same reason; then, on the other side the line, place the sum of the differences against their respective numbers, as you see

in the example: ſo by this *Alligation*, you will find he muſt take 14 *lb.* at 9 *d.* and 12 *d.* the pound, and 19 *lb.* at 24 *d.* and 30 *d.* to make a pound worth 20 *d.*

I ſhall not name any more ways of linking theſe numbers, theſe being ſufficient to underſtand how to link them any way, and leave the reſt to the ſcrutiny of the learner, to exerciſe himſelf with: for I have been the larger in theſe examples, that I may not trouble myſelf any more with the method of linking in the examples following; only note, that if at any time you have but one extreme, either leſſer or bigger than the mean, it will but admit of one way of linking, and the queſtion will have but one anſwer, as in the following example may be ſeen.

EXAMPLE.

A merchant hath wines; Canary at 24 *d.* the quart, Sherry at 16 *d.* the quart, and Malaga at 12 *d.* the quart; how much of each ſort muſt he take to ſell a quart for 18 *d.*? This queſtion, you ſee, admits but of one way of linking, and ſo but one anſwer; and obſerving the directions before given, you will find 8 of Canary, 6 of Sherry, and 6 of Malaga muſt be mixed together, that ſo a quart may be ſold for 18 *d.*

18	24	2.6	8
	16	6	6
	12	6	6

It muſt however be obſerved, that though the *Rule of Alligation* will give but one anſwer, yet all numbers that are in the ſame proportion between themſelves, as the numbers which compoſe the anſwer, will likewiſe ſatisfy the condition of the queſtion. Thus 4 quarts of Canary, 3 of Sherry, and 3 of Malaga, will compoſe a mixture worth 18 *d.* *per* quart: for as 8 : 6 : : 4 : 3. In like manner, 24 quarts of Canary, 18 of Sherry, and 18 of Malaga, will alſo ſatisfy the queſtion. For as 8 : 6 : : 24 : 18. And after the ſame manner an infinite number of anſwers may be found.

Nor is this all; for by the help of a particular method, which we ſhall now explain, an innumerable number of other anſwers may be found, without the numbers which compoſe them being in the ſame proportion.

Let us, for inſtance, ſuppoſe that the merchant is determined to uſe 6 quarts of Malaga conſtantly in the mixture; but to increaſe or diminiſh the Canary or Sherry, in any proportion that will produce a mixture worth 18 *d.* *per* quart.

Then may the quantity of Canary be increaſed or diminiſhed by 2, the difference between the price of the Sherry, and the price of the mixture, provided the quantity of Sherry be, at the ſame time, increaſed or diminiſhed by 6, the difference between the price of the Canary, and the price of the mixture. That is, the Canary may be increaſed to 10 quarts, if, at the ſame time, the Sherry be increaſed to 12.

For 10 quarts of Canary at 24 *d.* are worth	240 *d.*
12 quarts of Sherry, at 16 *d.* are worth	192 *d.*
6 quarts of Malaga, at 12 *d.* are worth	72 *d.*
Therefore 28 quarts of the mixture are worth	504 *d.*
And 1 quart is worth $\frac{504}{28}$ or	18 *d.*

The ſame may be done by diminiſhing both the quantities, inſtead of increaſing them. And if, inſtead of the numbers found, others in the ſame proportion be taken, a prodigious number of anſwers may be found.

If the Sherry be ſuppoſed invariable, then the quantity of Canary may be increaſed by 6, the difference between the prices of the Malaga and the mixture, provided, at the ſame time, the quantity of Malaga be increaſed by 6, the difference between the prices of the mixture and Canary; that is, the quantity of Canary may be increaſed to 14 quarts, provided, at the ſame time, the Malaga be increaſed to 12.

 For

For 14 quarts of Canary, at 24*d*. are worth 336*d*.
6 quarts of Sherry, at 16*d*. are worth 96*d*.
12 quarts of Malaga, at 12*d*. are worth 144*d*.

Therefore 32 quarts of the mixture are worth 576*d*.
And 1 quart is worth $\frac{576}{32}$, or 18*d*.

If the Canary be ſuppoſed invariable, the quantity of Sherry may be increaſed or diminiſhed by 6, the difference between the prices of the Malaga and the mixture, provided, at the ſame time, the Malaga be diminiſhed or increaſed by 2, the difference between the prices of the Sherry and the mixture; that is, the quantity of Sherry may be increaſed to 12 quarts, provided, at the ſame time, the Malaga be diminiſhed to 4.

For 8 quarts of Canary, at 24*d*. are worth 192*d*.
12 quarts of Sherry, at 16*d*. are worth 192*d*.
4 quarts of Malaga, at 12*d*. are worth 48*d*.

Therefore 24 quarts of the mixture are worth 432*d*.
And 1 quart is worth $\frac{432}{24}$, or 18*d*.

By the ſame method, all queſtions of this nature may be ſolved, and an infinite number of ſolutions found to each, out of which may be ſelected thoſe that are moſt proper for the purpoſe, which is an advantage that cannot be had from the common rules of Alligation.

VARIETY III.

In this variety we have the price of all the ſimples, and the quantity of one given to find the quantity of all the reſt, ſo as one meaſure or quantity may bear a mean rate or price propounded; which to do, obſerve the proportion following:

As

As the difference ſtanding againſt the quantity given :
To the reſt of the differences beſides : :
So the quantity given :
To the quantities ſought, each to its reſpective difference.

EXAMPLE I.

A tobacconiſt hath 30 *lb.* of the beſt tobacco at 2 *s.* or 24 *d.* *per* pound, which he would mix with ſome at 12 *d.* ſome at 9 *d.* and ſome at 7 *d.* and he would know how much of each ſort of the ſaid leſs prices muſt be mixed with the 30 *lb.* of the beſt, that he may ſell it for a penny the ounce, or 16 *d.* the pound.

16	24	4.7.9	20.30
	12	8	8
	9	8	8
	7	8	8

Having by the laſt variety ſet down the numbers, linked them, and found the differences, as in the example; then ſay, As 20, the difference againſt the quantity given : to 8, the next difference : : So 30 : to 12, the quantity required at 12 *d.* the pound. And ſeeing the other differences are equal, it will require 12 pound of each; ſo that he muſt take of the leſs prices at 12 *lb.* a-piece, to mix with the 30 *lb.* of the beſt, that ſo a pound may be afforded for 16 pence. The proof of this, and the following, are by the firſt *variety*.

EXAMPLE II.

A *goldſmith* hath 20 ounces of gold at 20 carracts fine, and would mix it with ſome at 22 carracts fine, and ſome at 24 carracts fine; how much of 22 and 24 carracts fine, and how much alloy muſt he mix with the 20 ounces of 20 carracts fine, ſo as an ounce, and conſequently the whole maſs, may be 18 carracts fine?

Note, That alloy is a ſort of coarſe ſilver, or cop-

per, or ſome baſe metal, with which *goldſmiths* mix gold or ſilver, to abate the fineneſs thereof.

An ounce of gold is divided into 24 parts, called *carracts*, and an ounce of ſilver into 20 parts, called *penny-weights*; therefore, to diſtinguiſh fineneſs of metals, ſuch gold as will abide the fire without loſs, is accounted 24 carracts fine; if it loſe 2 carracts in trial, it will then be 22 carracts fine, &c.

Silver is valued in ounces; and a pound of ſilver, which loſeth nothing in trial, is called 12 *ounces fine*; but if it loſe 2 penny-weight, it is then ſaid to be 11 ounces 18 penny-weight fine.

Firſt ſet down the values in order as uſual, with the mean value, and in the place of the alloy, becauſe it is not accounted of any value, place a cipher; then take the differences, which by the linking you may ſee will all be the ſame, except only in the place of the alloy.

Then ſay,

If 18 : 18 : : 20 : 20
If 18 : 18 : : 20 : 20
If 18 : 12 : : 20 : $13\frac{1}{3}$.

18	20	18	18
	22	18	18
	24	18	18
	1	2.4.6	12

Thus you ſee, that with the 20 ounces of 20 carracts fine, there muſt be mixed 20 of 22 carracts fine, and 20 of 24 carracts fine, and 13 ounces and $\frac{1}{3}$ of alloy, that ſo an ounce would bear 18 carracts fine.

By changing the two laſt terms of the proportion in the aforegoing rule, which is the ſame thing in effect, we may work any of theſe queſtions by the contracted way in *Fellowſhip*; and if there be any fractions, bring them up decimally.

If the terms be changed, the proportion ſtands;

As

As the difference against that price whose quantity is given :

Is to the quantity given ::
So any other difference :
To its quantity sought.

EXAMPLE III.

A chapman hath yarn at several rates, and would mix 40 pound, at 24 *d.* the pound, with some at 20 *d.* some at 14 *d.* some at 9 *d.* and some at 7 *d.* How much of each sort must he mix with the 40 pound, at 24 *d.* the pound, that he may sell a pound for 16 pence?

Having placed your numbers, and linked them, and taken the differences as in the margin:

16	24	7.9	16
	20	2	2
	14	4	4
	9	8	8
	7	8	8

Then the proportion is, as 16 to 40, what 2? what 4? what 8? what 8? Instead of these operations, divide 40 by 16, the quotient, multiplied by every difference, gives every particular quantity sought.

See the whole work.

First, Mult. 2.5
By 2

5.0 at 20 *d.* the pound.

16) 40 (2.5
32
80
80
0

2*dly*, Mult. 2.5
By 4

10.0 at 14 *d.* the pound.

3*dly*, Mult. 2.5
By 8

20.0 at 9 *d.* and 7 *d.* the pound.

So you see, with the 40 at 24 *d.* the pound, he must mix

mix 5 at 20 *d.* the pound, 10 at 14 *d.* the pound, and 20 at 9 *d.* and 7 *d.* the pound; and ſo a pound will be worth 16 pence.

VARIETY IV.

In this *variety* the price of every ſimple is expreſſed, and the mean rate or price; and it is required to find how much of each ſort muſt be taken, to make up a certain quantity propounded, agreeable to the mean rate given?

Which to do, obſerve the proportion following:

As the total ſum of the differences :
To the total quantity given : :
So any particular difference :
To its particular quantity ſought.

EXAMPLE.

A *grocer* hath 4 ſorts of currants, one at 4 *d.* the pound, another at 6 *d.* another at 9 *d.* and the beſt at 11 *d.* the pound: the worſt would not ſell, and the beſt were too dear, and he concludes to mix 240 pound, and ſo much of each ſort, as to ſell a pound for 8 pence: how much of each ſort muſt he take?

8	4	3	72
	6	1	24
	9	2	48
	11	4	96
		10	240

Having placed your numbers with the mean price, linked them, and taken the difference, as here, divide 240 by 10; quotes 24, multiplied by every difference, gives 72, for the quantity of 4 *d.* the pound; and 24 for the quantity at 6 *d.* the pound; and 48 for the quantity at 9 *d.* the pound; and 96 for the quantity at 11 *d.* the pound, the ſum making 240, is the proof.

Note, If he hath a deſire to put off more of his worſt ſort, he may alter the quantities by ſome other way of linking, as was ſhewed in the ſecond *variety.*

QUEST.

QUEST. II.

A *goldsmith* hath ſeveral ſorts of gold, ſome of 24 carracts fine, ſome of 22 carracts, ſome of 18 carracts, ſome of 16 carracts fine, and is deſirous to melt of all theſe ſorts, ſo much together, as may make a maſs of 60 ounces of 21 carracts fine. How much of each ſort muſt he take?

The numbers being placed, linked, and differenced, as hath been ſhewed, and is here expreſſed; I ſay,

21	24	5	25
	22	3	15
	18	1	5
	16	3	15
		12	proof 60

1 As 12 : 60 : : 5 : 25
2 As 12 : 60 : : 3 : 15
3 As 12 : 60 : : 1 : 5
4 As 12 : 60 : : 3 : 15

Or if you will uſe the contracted way, divide 60 by 12, quotes 5; by which multiplying each difference, gives the ſame quantities. So I conclude, that 25 ounces of 24 carracts fine, 15 ounces of 22 carracts fine, 5 ounces of 18 carracts fine, and 15 ounces of 16 carracts fine, will produce a maſs of gold of 60 ounces, and 21 carracts fine.

QUEST. III.

How many gallons of water muſt be mixed with wine, at 3 ſhillings the gallon, to fill a veſſel of 100 gallons, ſo as a gallon may be afforded for 2 *s.* 6 *d.*?

Firſt, ſet down the value of a gallon of wine, and the water being of no value, put a cipher: then having ſet down the mean rate, and linked, and taken the difference; ſay, If 3 the ſum of the difference, give 100, what will 2.5? *Facit* 83.333, or 83 and $\frac{1}{3}$

	s.	
2.5	3	2.5
	0	.5
		3

of

of wine? which subtracted from 100, leaves 16⅔ for the quantity of water.

QUEST. IV.

A *vintner* hath two vessels, one will hold 50 gallons, and the other 30; and would know how much water he must mix with wine at 4 shillings the gallon, to fill the bigger vessel, that every gallon drawn may be worth 3 shillings the gallon; and with wine at 2 shillings and 6 pence the gallon, to fill the less vessel, that a gallon may be worth 2 shillings.

The quantity of water is demanded.

PROP. I.

$$3\left\{\begin{matrix}4\\0\end{matrix}\right]\begin{matrix}3\\1\end{matrix}$$

If 4 : 50 : : 3 : 37.5

Facit 37½ gallons of wine; then there must be 12½ of water in the greater vessel.

PROP. II.

$$2\left\{\begin{matrix}2.5\\0\end{matrix}\right]\begin{matrix}2\\.5\end{matrix}$$

If 2.5 : 30 : : 2 : 24

Facit 24 gallons of wine; then there must be 6 of water in the lesser vessel.

The Rule of FALSE.

THis Rule is more for recreation and delight, than for any solid use; but because it is an ingenious rule, and may exercise the wits of youth, we shall here insert it.

The Rule of False is so named, not from the falsity of it; but because we, by supposed numbers, taken at adventure, and by them working the question according to the nature thereof, do, by those false supposed numbers, find the true numbers sought.

This

This rule is divided into two parts, commonly called the *Single Rule* and *Double Rule*.

The Single Rule of FALSE.

IN the *Single Rule* we need but uſe one ſuppoſition, as may be ſeen in the queſtions following.

QUEST. I.

A certain ſum of money put out at 6 *per cent.* ſimple intereſt, at the end of 10 years amounts to 20 *l.* what was the ſtock? *Anſwer,* 12 *l.* 10 *s.*

Here I ſuppoſe any number, as 10 *l.*; then, according to the nature of the queſtion,

What will 10 *l.* amount to, forborne 10 years? Which, by the table of *ſimple intereſt*, or by the *Double Rule of Three*, will be found to be 16 *l.* which ſhould have been 20 *l.* if I had gueſſed right.

Now I ſay, If 16 *l.* come from 10 *l.* my ſuppoſed number, what will 20 *l.* come from? *Anſwer,* 12 *l.* 10 *s.* the ſtock ſought.

See the work.

```
     l.   l.    l.
If  16 : 10 :: 20 :
           20
          ———
     16) 200 (12.5
         16
         ———
          40
          32
          ———
           80
           80
           ——
            0
```

QUEST.

QUEST. II.

A *ſchoolmaſter* being aſked, how many ſcholars he had? anſwered, If I had as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$, or 1 quarter as many, I ſhould have 99: How many had he? *Anſwer* 36.

Suppoſe he had any number, as 40, then as many, $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, would make 110, which ſhould have been 99. Then ſay,

If 110 come from 40, what will 99 come from? *Anſ.* 36, the number of ſcholars ſought.

See the work.

	ſc.	*ſc.*	*ſc.*
If	110 :	40 : :	99 :
			40

110) 3960 (36
330
660
660
0

QUEST. III.

There is a ciſtern with 3 unequal cocks, containing 60 gallons of water; and if the greateſt cock be opened, the ciſtern will be empty in one hour; if the ſecond cock be opened, it will be empty in two hours; if the third be opened, it will be empty in three hours. Now I demand in what time it will be empty, if all run together?

Suppoſe in $\frac{1}{2}$ an hour, or 30; then muſt there empty at the greateſt cock 30 gallons, or $\frac{1}{2}$, and by the ſecond cock 15 gallons, or $\frac{1}{4}$, and by the leaſt cock

10

10 gallons, or $\frac{1}{8}$, which added together make 55, which should have been 60. Now say,

If 55 gallons run in 30 minutes, what will 60 gallons run in? *Answ.* 32.727, the time sought.

QUEST. IV.

Three merchants, *A*, *B*, *C*, put in money together, and gained 100 *l.* of which *A* took up a certain sum; *B* took up twice as much as *A*, or double to *A*; and *C* took up thrice as much as *B*, or trible to *B*: What did each take up a-part?

Suppose *A* took up 3 pound, then *B* must have 6 pound, and *C* 18 pound, which makes 27 pound, which should have been 100 pound.

Then say, if 27 *l.* should be 100 *l.* what	3	*Facit,*	11.111111 = *A*
	6		22.222222 = *B*
	18		66.666666 = *C*
		Proof,	99.999999

See the work.

27) 100 (3.703703 or 370 repeated.
81

190
189

100
81

190
189

100
81

19

This quotient multiplied by 3 for *A*, by 6 for *B*, and by 18 for *C*, produceth the former numbers. See the contracted way in *fellowship*.

Thus may any question of these natures be wrought;

so I shall forbear mentioning any more of *single position;* only note, that if there be no partition in numbers to make a proportion, you must use the *double rule,* which now we shall begin with.

Double Rule of FALSE.

IN the *double Rule* we use two suppositions; and if with either we find the numbers that satisfy the question, there is no more to be done; but if, as commonly it happens, we err in both suppositions, see whether they be greater or lesser than the solution requires, which mark with + *plus,* or — *minus;* and over-against either supposition its respective errour; then observe this general rule:

As the difference of errours if alike, or sum if unlike:
Is to the difference of suppositions : :
So is either errour: to a fourth number; which, added to, or subtracted from, the supposition over-against it, gives the number sought. *See the examples.*

QUEST. I.

Good-morrow, good fellow, with your 20 geese. Nay, says he, I have not 20; but if I had as many, $\frac{1}{2}$ as many, 2 geese and $\frac{1}{2}$, then had I 20. I demand how many he had?

First, Suppose 6, then as many, $\frac{1}{2}$ as many, two geese and $\frac{1}{2}$, would make 17 and $\frac{1}{2}$, which should be 20. The errour therefore is—$2\frac{1}{2}$, which mark as in the margin.

6—2.5
9+5

Secondly, Suppose he had 9, then as many, $\frac{1}{2}$ as many, $2\frac{1}{2}$, would make 25, which should be 20. The errour therefore is + 5, which put down under the other, as you see done. Then, because the errours are unlike, that is, one *plus,* the other *minus,* I

say,

ſay, As the ſum of the errours 7.5 to the difference of the ſuppoſitions 3 : : So either errour, ſuppoſe the firſt 2.5 : to 1; which, becauſe the firſt ſuppoſition was *minus*, according to the rule added to 6, makes 7, the number of geeſe ſought.

QUEST. II.

A gentleman had two horſes of good value, and a ſaddle worth 50 *l.* which ſet on the back of the firſt horſe, makes his value double the ſecond; but if ſet on the back of the ſecond horſe, makes his worth treble the firſt horſe. The price of each horſe is demanded?

Suppoſe the price of the firſt horſe be 20 pound, which, with the ſaddle, makes 70 *l.* then ſeeing this is double the price of the ſecond horſe, the ſecond will be worth 35 *l.* which, with the ſaddle, would be 85, which ſhould be 60, 3 times 20, the price of the firſt horſe. The errour therefore is—25, which put down as you ſee.

Suppoſe again the firſt horſe worth 25 *l.* which, with the ſaddle, would be 75 *l.* then the ſecond would be worth 37 *l.* 10 *s.* or 37.5 (*l.*), which, with the ſaddle, would amount to 87 *l.* 10 *s.* or 87.5 (*l.*), which would be 75=3 times 25, the price of the firſt horſe. The errour therefore is—12.5, which place under the other errour, and ſay,

20	—25
25	—12.5

As the difference of errours, becauſe alike 12.5 :
To the difference of ſuppoſitions 5 : :
So either errour, ſuppoſe the firſt 25 : to 10, which added to the ſuppoſition over againſt it, becauſe—, makes 30, the price of the firſt horſe; and by conſequence the price of the ſecond will be 40. And if you had taken the ſecond errour 12.5, the fourth number would have been 5, which added to 25, makes 30, as before.

PROOF.

First horse 30
Sadle=50
———
Sum=80
———
$\frac{1}{2}$=40=Second horse.
Saddle=50
———
Sum=90
$\frac{1}{3}$=30=First horse.

QUEST. III.

A stealing apples, was taken by *B*, and to appease him, gives him half he had, and *B* gives him back 10; and going further met with *C*, and was forced to give him half of what he had left, and he returns him back 4; and going further, meets *D*, and gives him half he had, and he returns him back 1; and getting safe away, finds he had 13 left? What had he at first?

Suppose first 80, and working according to the nature of the question, he had $15\frac{1}{2}$ left, which is +2.5.

Supposing again he had 40, and working as before, he will have $10\frac{1}{2}$ left, which is—2.5.

Then working by the general rule, you will find he had 60 apples at the first.

80+2.5
40—2.5

But the number sought in this question may more quickly be found: for, *note*, That if at any time, as here it happens, that the errours are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought; and the sum of the suppositions is 120, half of which is 60 as before.

See

See the work both ways.

As 5 : 40 : : 2.5 | 80
40 | 40

5) 100.0 (20+40=60. | Sum 120
10 or 80—20=60. | 60.

00

QUEST. IV.

Three men, as *A*, *B* and *C*, bought a ship for 200*l*. *A* says to *B*, Give me half your money, and I will pay for the ship; *B* says to *C*, Give me $\frac{1}{3}$ of your money, and I will pay for the ship; *C* says to *A*, Give me $\frac{1}{4}$ of your money, and I will pay for the ship; what sum of money had each? *Answer*, *A* 128, *B* 144, *C* 168.

1. Sup. 120—50
2 Sup. 130+12.5

$$\text{Proof}\begin{cases}128+\frac{1}{2}144=200\\144+\frac{1}{3}168=200\\168+\frac{1}{4}128=200\end{cases}$$

QUEST. V.

Three men, as *A*, *B*, and *C*, thus discoursed of their money: *A* saith to *B* and *C*, Give me half your money, and I shall have 100*l*. *B* saith to *C* and *A*, Give me one third of your money, and I shall have 100*l*. *C* saith to *A* and *B*, Give me one fourth of your money, and I shall have 100*l*. What had each?

This question will require more suppositions than two, before it can well be wrought by this rule; which may convince some who affirm, if a question require more suppositions than two, it will not be wrought by the rule of *False*: but the contrary may be seen in the following work.

Let the first general supposition for *A* be 20 *l.* then he wanted 80 *l.* which is the half of *B* and *C*'s money; then *C* must have 120 *l.* Now *B* will have of *C* and *A* $\frac{1}{3}$ of their money, which is 46 *l.* $\frac{2}{3}$, which added to *B*'s money 40 *l.* makes 86$\frac{2}{3}$, which should be 100 *l.* So we have supposed too little for *B* by 13$\frac{1}{3}$, for 100 less 86$\frac{2}{3}$=13$\frac{1}{3}$.

Suppose again *B* had 70 *l.* then *C* must have 90 *l.* Now *B* will have of *C* and *A* $\frac{1}{3}$ of their money, which is 36$\frac{2}{3}$; which added to *B*'s money 70 *l.* makes 106$\frac{2}{3}$, which should be 100 *l.* Here we have supposed too much for *B* by 6$\frac{2}{3}$.

Now say, according to the rule, As 20 : 30 : : 6$\frac{2}{3}$: 10; and 10 subtracted from 70, resteth 60 for *B*; then if *A* had 20, *B* had 60, *C* 100.

S. 1=40—13$\frac{1}{3}$ *B.*
S. 2=70+ 6$\frac{2}{3}$ *B.*

But the question saith, *C* will have of *A* and *B* $\frac{1}{4}$ of their money, which is 20 *l.* which added to 100 *l.* of *C*'s money, makes 120, which should be 100 *l.* Therefore our supposition for *A* is too little by 20 *l.*— S. 1=20—20 *A.*

Let the second general supposition for *A* be 30 *l.* then he wanted 70 *l.* therefore 70 *l.* is the half of *B* and *C*'s money; then they must have 140 *l.* whereof we suppose *B* had 30 *l.* then *C* must have 110 *l.* Now *B* will have of *C* and *A* one third of their money, which is 46$\frac{2}{3}$, which added to 30 *l.* *B*'s money, makes 76$\frac{2}{3}$. which should be 100 *l.* Here we have supposed too little for *B* by 23$\frac{1}{3}$.

S. 1=30—23$\frac{1}{3}$ *B.*

Suppose again, *B* had 50 *l.* then must *C* have 90 *l.* Now *B* will have of *C* and *A* one third of their money, which is 40 *l.* which added to *B*'s money 50 *l.* makes 90 *l.* which should be 100. Here we have supposed too little for *B* by 10 *l.*

S. 1=30

$$S.\ 1 = 30 - 23\tfrac{1}{3}\ B.$$
$$S.\ 2 = 50 - 10\quad B.$$

Then ſay, If $13\frac{1}{3} : 20 :: 10 : 15$; and 15 added to 50, makes 65 for *B*. Then if *A* had 30, *B* would have 65, and *C* 75.

But the queſtion ſaith, *C* will have of *A* and *B* $\frac{1}{4}$ of their money, which is $23\frac{3}{4}$, which added to 75 of *C*'s money, makes 98 *l.* $\frac{3}{4}$. which ſhould be 100 *l.* therefore our ſecond ſuppoſition for *A* is too much by $1\frac{1}{4}$. S. $2 = 30 + 1\frac{1}{4}$ *A.*

$$S.\ 1\ A = 20 - 20$$
$$S.\ 2\ A = 30 + 1\tfrac{1}{4}$$

Now I ſay, If $21\frac{1}{4} : 10 :: 20 : 9\frac{7}{17}$, and $9\frac{7}{17}$ added to 20, give 29 *l.* $\frac{7}{17}$, for the true ſhare of *A*.

Seeing then that *A* hath 29 *l.* $\frac{7}{17}$; he wanteth 70 *l.* $\frac{10}{17}$, which is half *B*'s and *C*'s money; then they muſt have 141 $\frac{3}{17}$. whereof ſuppoſe *B* had 30 *l.* then *C* had 111 *l.* $\frac{3}{17}$. Now *B* will have of *C* and *A* $\frac{1}{3}$ of their money, which is $46\frac{44}{51}$, which added to his own money 30 *l.* makes $76\frac{44}{51}$, which ſhould be 100 *l.* Here we have ſuppoſed too little for *B* by $23\frac{7}{51}$.

$$S.\ 1 = 30 - 23\tfrac{7}{51}\ B.$$

Suppoſe again, *B* had 65, then *C* had $76\frac{3}{14}$; now *B* will have of *C* and *A* one third of their money, which is $35\frac{10}{51}$, which added to his money 65 *l.* makes $100\frac{10}{51}$, which is too much by $\frac{10}{51}$. S. $2 = 650 + \frac{10}{51}$ *B.*

$$S.\ 1 = 30 - 23\tfrac{7}{51}\ B.$$
$$S.\ 2 = 65 + 0\tfrac{10}{51}\ B.$$

Then ſay, As $23\frac{10}{51}$: to 35 :: ſo is $\frac{26}{51}$: to $\frac{5}{17}$; which ſubtracted from 65, leaves $64\frac{12}{17}$, for *B*'s true ſhare; then muſt *C* have $76\frac{8}{17}$.

Their parts $\left\{\begin{array}{l} \text{l.} \\ A = 29\frac{7}{17} \\ B = 64\frac{12}{12} \\ C = 76\frac{8}{17} \end{array}\right.$ In decimals, $\left\{\begin{array}{l} \text{l.} \\ A = 29.4117647 \\ B = 64.7058824 \\ C = 76.4705882 \end{array}\right.$

Proof

$$\text{Proof}\begin{cases} 29\frac{7}{17}+\frac{1}{2}\begin{Bmatrix} B\ 64\frac{12}{17} \\ C\ 76\frac{8}{17}\end{Bmatrix}=\text{to } 70\frac{10}{17},=100 \text{ for } A. \\ 64\frac{12}{17}+\frac{1}{3}\begin{Bmatrix} C\ 76\frac{8}{17} \\ A\ 29\frac{7}{17}\end{Bmatrix}=\text{to } 35\frac{5}{17},=100 \text{ for } B. \\ 76\frac{8}{17}+\frac{1}{4}\begin{Bmatrix} A\ 29\frac{7}{17} \\ B\ 64\frac{12}{17}\end{Bmatrix}=\text{to } 23\frac{9}{17},=100 \text{ for } C.\end{cases}$$

This queſtion is not capable of an exact anſwer in *Engliſh* coin, as you may ſee: but if you would have an anſwer in integers, you muſt make the common ſum in this queſtion 100 *l.* ſome multiple or 17; or if you reduce their ſhares into improper fractions, then

A will have $\frac{500}{17}$
B will have $\frac{1100}{17}$
C will have $\frac{1300}{17}$

And ſeeing the denominators are equal, neglect them, and the numerators will be proportional numbers for *A*, *B*, *C*, which you abbreviate into leſſer, by cutting two ciphers from each; then will *A* have 5 *l.* *B* 11 *l.* and *C* 13 *l.* Then if *A* have half of *B* and *C*'s money, he will have 17 *l.* If *B* have one third of *C*'s and *A*'s money, he will have 17 *l.* And if *C* have one fourth of *A*'s and *B*'s money, he will have 17 *l.* So this queſtion conſiſteth all of integers; and 17 *l.* falls into the place of 100 *l.*

I have been the longer upon this queſtion, that the learner may obſerve the variety of work that may proceed from ſuch like queſtions.

Here I have annexed two or three more, with their anſwers, which may ſerve for the learner's exerciſe, and ſo conclude this rule.

Queſtions in the rule of Falſe.

QUEST. I.

What number is that, which multiplied by 20, and divided by 6, gives 140 in the quotient? *Facit* 42.

QUEST.

QUEST. II.

What number is that, which added to its half, and its one fourth, and to 3 more, makes 108? *Facit* 60.

QUEST. III.

A veſſel that holdeth 60 gallons, hath 4 cocks; and being filled with water, or any other liquor, if they all be ſet open at once, the liquor will run out in 24 hours. Now the ſecond cock will empty twice as much as the firſt during the ſame time; and the third will empty three times as much as the firſt in the ſame time; and the fourth will empty 5 times as much as the firſt. What number of gallons doth each cock empty?

Facit, the firſt $5\frac{5}{11}$ gallons, the ſecond, $10\frac{10}{11}$ gallons, the third $16\frac{4}{11}$ gallons, the fourth, $27\frac{3}{11}=60$ gallons.

QUEST. IV.

A gentleman hired a workman for 40 days, and agreed for every day he worked he ſhould have 8 pence, and every day he played, he ſhould return back 4 pence. At the end of 40 days the labourer received 10*s.* 5*d.* How many days did he play? *Facit* 16 days, and one quarter.

QUEST. V.

A young man coming into a garden, ſaith, Bleſs you all, you 10 fair maids! Sir, you miſtake yourſelf, ſaith one, for we are not 10; but if we were thrice as many as we are, we ſhould be as many above 10 as now we are leſs. How many were there? *Facit* 5 maids, or rather maidens.

QUEST. VI.

What numbers are they whoſe $\frac{5}{6}$ of the one, = ſuppoſe *A*, is equal to $\frac{4}{7}$ of the other *B*? *Facit* *A* 72, *B* 105.

QUEST.

QUEST. VII.

Suppose there are two numbers, *A* and *B*, the lesser of which, to wit *A*, hath such proportion to the greater, to wit *B*, as $2\frac{4}{5}$ to 6, and the sum of the said numbers, hath such proportion to the sum of the square of the same numbers, as $5\frac{1}{2}$ to $68\frac{1}{2}$; I demand each number? *Facit* 7 for *A*, and 15 for *B*.

QUEST. VIII.

Divide 45 into two such parts, that the greater may be in triple proportion to the less; what are those parts? *Facit* $11\frac{1}{4}$ and $33\frac{3}{4}$.

QUEST. IX.

Divide 10 into two such parts, as if the greater be divided by the lesser, the quotient may be 20; what are both parts? *Facit* $\frac{10}{21}$, and $9\frac{11}{21}$.

QUEST. X.

A vessel of 63 gallons was filled with *French* wine of two sorts; the one at 2 *s.* the gallon, and the other at 2 *s.* 6 *d.* The wine in the hogshead thus filled did cost 7 *l.* 4 *s.* How much was there of each sort? *Facit* 27 gallons of 2 *s.* and 36 of 2 *s.* 6 *d.*

QUEST. XI.

A gentleman bought a house with a garden, and a good horse in the stable, for 500 *l.* Now he paid 4 times the price of the horse for the garden, and 5 times the price of the garden for the house: What did the house, garden, and horse cost?

Answer, The house cost 400 *l.* the garden 80 *l.* the horse 20 *l.*

Note, Some of the foregoing questions may be wrought by the *Single Rule of False*; though, notwithstanding, they

they may be wrought by two ſuppoſitions, as you may try at your leiſure. And, to conclude, any queſtion whatſoever, if not impoſſible, may be reſolved; if by comparing, adding, ſubtracting, or proportion, you could prove your queſtion, if the true reſolution was given; for otherwiſe the queſtion cannot be reſolved, becauſe you cannot come to know what the errours were at the poſitions; but then they muſt be wrought by *Algebra*.

In the next place, we ſhall proceed to *Logarithmical Arithmetic*; and in that ſhall be very conciſe: their chiefeſt uſe in Arithmetic being in reſolving queſtions concerning *Compound Intereſt*, and *Annuities*.

Logarithmical ARITHMETIC.

Logarithms are artificial numbers which differ equally, fitted to the natural, for eaſe in calculation.

The firſt figure, called the *Index*, or *Characteriſtic*, ſhews how many figures the anſwering number conſiſts of, which are always more by one than the index, if the ſame be whole.

So the index of any number under 10, is (0); betwixt 10 and 100 (1); betwixt 100 and 1000, is (2), &c.

The logarithm of a fraction, or decimal number, is all one as an integer, only with this difference for the index, that if the firſt figure of the decimal to the left hand be ſignificant, the index is (.9; if there be one cipher before it, the index is (.7), &c.

EXAM-

E X A M P L E.

W. Numb.	*Logarithms.*	*Defect. Numbers.*	*Logarithms.*
2345	3.3701428	.2345	.9.3701428
234.5	2.3701428	.02345	.8.3701428
23.45	1.0701428	.002345	.7.3701428
2.345	0.3701428	.0002345	.6.3701428

Thus may you see the logarithms are the same, but the index thereof differeth, according as the first figure thereof is removed from unity.

Construction of Logarithms.

Their construction according to the common rules, given by many extractions of roots, is tedious; the best way yet known, is this, which follows.

To make a table of Logarithms.

First, Put for the logarithm of 1, a cipher for the index, and a competent number of ciphers for the logarithm, according to the number of places you would have your logarithms consist of; for 10 an unit, with the same number of ciphers; for 100, 2, with as many ciphers; for 1000, 3, with as many ciphers, &c.

Secondly, Find the difference between some two logarithms above 1000, or rather above 10000, that differ by unity; thus, multiply the two numbers together, and that product you must multiply again by 43429448190325183896; which last product divided by the arithmetical mean between both numbers, the quotient is the difference sought.

Suppose we would find the difference between the log. 10000 and 10001, the product of these two numbers is 1.00010000, which multiplied by 4343 produceth 43434343; this divided by 10000.5, quotes 4343 Now, if to the logarithm of 10000 which is 4.0000000 you add the difference before found, to wit,

wit, 4343, the ſum 40000434 is the true logarithm of 10001 to 7 places.

Thirdly, Having thus found the difference of any two logarithms difference by unity, and conſequently ſome of the logarithms, by dividing the difference found by the arithmetical mean, between any two numbers difference by unity, you ſhall have the difference of the logarithm of thoſe two numbers.

Thus to find the difference between the logarithm of 274 and 275; divide 4343, the difference of the logarithm of 10000, and 10001 by 274.5, the quotient 15821, is the difference ſought.

Fourthly, Having by this means found a few of the prime logarithms, the reſt are made by addition and ſubtraction; and having made the cannon upward, above 1000 to 10000, by conſequence it is made for all inferiour numbers.

The prime numbers to which logarithms muſt be found in the firſt place, are theſe, 2 . 3 . 7 . 11 . 13 . 17 . 19 . 23 . 29 . 31 . 37 . 41 . 43 . 47 . 53 . 59 . 61 . 67 . 71 . 73 . 79 . 89 . 97 . *&c.* or the ſame numbers with ciphers. There being ſeveral tables of logarithms, we ſhall only explain thoſe, which in this place we have made uſe of, which are Mr *Oughtred*'s in his *Trigonometry*, they being of as good if not of a better character than any extant; and the logarithms extended to 7 places after the index. Of the ſame ſort are Mr *Gunter*'s, Mr *Norwood*'s, Mr *Leyburn*'s, &c. and above all *Sherwin*'s tables, *&c.*

The logarithm of any number under 10000, is found by inſpection; ſo the logarithm of 1234 is 3.0913151; but if your number given conſiſts of 5 or 6 places, then you muſt uſe proportion. Thus ſeek for the firſt 4, as before, noting the difference betwixt that logarithm and the next greater; then ſay, As 10, if your number conſiſts of 5 places, or as 100, if of 6 places: To the ſaid difference: : So the figure, or figures wanting to the part proportional, which added

to the logarithm before found, the ſum (when a true characteriſtic or index is fitted): To the logarithm of the number ſought.

So the logarithm of 12345 is 4.0914910 }
And of 12356 is 4.0918778 } &c.

To find the number anſwering a logarithm given, is but the converſe; the firſt four figures are found by inſpection. But if you want for 5 or 6 places, do thus: Seek the logarithm next leſs, and againſt it are the four firſt figures; then ſeek the difference betwixt that and the next greater, as likewiſe betwixt the given logarithm and the next leſs, and ſay, As the firſt difference: To 10, if for 5 figures, or to 100, if for 6 places :: So the other difference: To the remaining figure or figures ſought.

So the number anſwering 4.0918778 is=12356, as above.

Addition of Logarithms.

In *Addition* take this general rule.

A general Rule.

If your indices be affirmative, add them as uſual, and you have the true ſum.

But if they be negative, add them as before; only note, That if the ſum of the indices be under 10, add 10; if juſt 10, add unity; if above 10, caſt 10 away, the ſum, or remainder, will be negative.

But if the indices be of different kinds, that is, one affirmative and the other negative, add them alſo.

If the ſum be 10, or above, caſt away 10, the remainder is affirmative; if under 10, negative.

EXAM-

EXAMPLES.

I. Unto 2.2671717
Add 3.1414498

Sum=5.4086215

II. Unto .3.2671715
Add .5.1414498

Sum=18.4086213

II. Add { .2.2671717 / .9.1414498 / .8.8750613 }

Sum=0.2836828.

IV. Add { .9.1414498 / .7.2671717 / .8.8750913 }

Sum=5.2837128

More EXAMPLES.

V. Unto 2.2671717
Add .8.1414498

Sum=0.4086215

VI. Unto .9.2671717
Add 3.1414498

Sum=2.4086215

In the adding of theſe logarithms there is no difficulty, excepting in the ſecond, which may appear abſtruſe, where the ſum of the indices is .18; which ſhews there is 11 ciphers before the firſt ſignificant figure, the 10 of which ſignifies 10 ciphers, and the 8 being defective, always is the ſign of one cipher before the firſt ſignificant figure, as was noted before.

And now we ſhall proceed to *Subtraction*.

Subtraction of Logarithms.

A general Rule.

If your indices be affirmative, and the higher the greater, then as uſual.

If one or both be negative, obſerve if the index of the higher be ſmaller than the lower; if it be, add 10 to it; and if the higher be of greater value, the remains are affirmative; if not, they are negative.

 EXAM-

EXAMPLES.

I. From	3.1414498	II. From	2.2671717
Subtract	2.2671717	Subtract	3.1414498
Rest	0.8742781	Rest	.9.1257219
III. From	.9.2971717	IV. From	3.1414498
Subtract	3.1414498	Subtract	.9.2671717
Rest	.6.1257219	Rest	3.8742781
V. From	.8.8750613	VI. From	.9.1414498
Subtract	.9.1414498	Subtract	.8.8750613
Rest	.9.7336115	Rest	.0.2663885

In these there is nothing obscure.

Multiplication of Logarithms.

To multiply one number by another, is nothing but to add their logarithms together, their sum is the logarithm of their product.

EXAMPLES.

I. Multiply	144	Log. 2.1583625	Add
By	12	Log. 1.0791812	
Product	1728	Log. 3.2375437	
II. Mult.	1385	Log. 3.1414498	Add
By	185	Log. 2.3671717	
Prod.	256225	Log. 5.4086215	
III. Mult.	.1385	Log. 0.1414498	Add
By	.0185	Log. .8.2671717	
Prod.	.0256225	Log. .8.4086215	

IV. Mult.

IV. Mult.	138.5	Log. 2.1414498	} Add
By	18.5	Log. 1.2671717	
Prod.	2562.25	Log. 3.4086215	

And so of any other.

Division in Logarithms.

To divide one number by another, is nothing but to subtract the logarithm of the divisor from the logarithm of the dividend, the remainder is the logarithm of the quotient.

EXAMPLES.

I. Divide	1728	Log. 3.2375437	} Subtract.
By	12	Log. 1.0791812	
Quotes	144	Log. 2.1583625	
II. Divide	256225	Log. 5.4086215	} Subtract.
By	185	Log. 2.2671717	
Quotes	1385	Log. 3.1414498	
III. Divide	.0256225	Log. .8.4086215	} Subtract.
By	.1385	Log. 4.1415498	
Quotes	.0185	Log. .4.2671717	
IV. Divide	256.225	Log. 2.4086215	} Subtract.
By	138.5	Log. 2.1414498	
Quotes	1.85	Log. 0.2671717	

And thus of any other.

Golden Rule in Logarithms.

In this rule we have 3 numbers given, to find a fourth; wherefore, if your question be direct, work thus: Add the logarithms of the second and third,

and from that ſum ſubtract the logarithm of the firſt, the remainder is the logarithm of the 4th proportional ſought.

E X A M P L E.

If 13 groſs of incle coſt 7 *l.* 12 *s.* what will $66\frac{1}{2}$ coſt?

See the work.

If 13 groſs, Log.	— —	1.1139434
l. Coſt. 7.6, Log.	— —	0.8808136
What 66.5 groſs, Log.	— —	1.8228216
		2.7036352
Anſ. 38.8769		Log.=1.5896918

This may be performed by addition, thus: Add the arithmetical complement of the logarithm of the firſt unto the logarithm of the ſecond and third, the ſum is the logarithm of the fourth.

Arith. comp.

The arithmetical complement is only the remainder of every figure to 9, and the laſt to 10. So the arithmetical comp. of 0.8808136 is 9.1191864, of 2.0000000 is 8.0000000.

13.	8.8860566
7.6.	0.8808136
66.5.	1.8228216
38.8769.	1.5896918

Here you may ſee the anſwer is the ſame as it was before.

But if your queſtion be inverſe, work thus: Add the logarithm of the firſt and ſecond together, and from that ſum ſubtract the logarithm of the third, the remainder is the logarithm of the fourth proportional ſought.

E X A M P L E.

If 12 men do a piece of work in 20 days, in how many days will 60 men do the ſame work?

The

The operation.

If 12 men, Log.	1.0791812
Require 20 days, Log.	1.3010300
The ſum	2.3802112
What will 60 men require? Log. Sub.	1.7781512
Anſ. 4 days. Log.	0.6020600

This may likewiſe be performed by Addition, by adding the arithmetical complement of the logarithm of the third, to the logarithm of the firſt and ſecond; the ſum is the logarithm of the fourth.

Thus, If 12 men, Log.		1.0791812
Require 20 days, Log.	Add	1.3010300
What will 60 men require, *arith com.*		8.2218488
Anſ. 4 days, as before, Log.		0.6020600

And ſo in any other.

Whence you may obſerve, that in multiplication, inſtead of adding the two logarithms together, you may ſubtract the arithmetical complement of the logarithm of the one, from the logarithm of the other, the remainder is the logarithm of the product.

Likewiſe in diviſion, inſtead of ſubtracting one from the other, you may add the arithmetical complement of the logarithm of the diviſor to the logarithm of the dividend, the ſum is the logarithm of the quotient.

Extraction of the Square Root.

Though extraction of roots by natural numbers, be one of the difficulteſt parts of arithmetic, yet by artificial numbers or logarithms, nothing is more eaſy and plain, as may be ſeen in the practice thereof.

To extract the ſquare root of any number is performed by parting or halving its logarithm; the ſaid half is the logarithm of the root ſought.

EXAM-

EXAMPLE I.

What is the ſquare root of 144? Log. 2.1583625
Half is 1.0791812
Which is the logarithm of 12, the root ſought.

EXAMPLE II.

Let it be required to find the ſquare root of 160.
Log. of 160, is 2.2041200
Half is the logarithm of 12.6491 1.1020600
Which is the ſide of a ſquare acre, and true to 4 places of decimals; which is exact enough for common uſe, 160 being a ſurd number, its true root is inexpreſſible.

Note, If the number whoſe root is ſought be a decimal, add 10 to the index, and halve it, as in this.

EXAMPLE III.

What is the ſquare root of .225? Log. 19.3521825
Half is 9.6760912
Which is the logarithm of .4743, the root ſought.

And ſeeing halving the logarithm of any number gives the logarithm of its root, then it follows, that multiplying the logarithm of any number by 2, gives the ſquare root thereof; as may be ſeen in this example.

EXAMPLE IV.

What is the ſquare of 11826? Its Log. 4.0728378
Multiply by 2

Which is the logarithm of 139854276=8.1456756
Proof is its root, *viz.* 11826, $\frac{1}{2}$=4.0728378

This number 139854276 is a very remarkable number. *Firſt*, It is a ſquare number; *Secondly*, It contains 9 places, and they are 9 digits, and I think there is not another that does.

Extraction of the Cube-Root.

As the ſquare root was found by bipartition, or halving its logarithm, ſo the cube root is found by tripartition or taking one third part of its logarithm, which will be the logarithm of the cube root ſought.

EXAMPLE I.

What is the cube root of 1728?
The logarithm of 1728, is 3.2375437
One third of this is 1.0791812
Which is the logarithm of 12, the root ſought.

EXAMPLE II.

What is the cube root of 123456?
Its logarithm is 5.0915121
One third thereof is 1.6971707
Which is the logarithm of 49.7932, the cube root ſought.

If your number be a decimal, add 20 to its index, and take $\frac{1}{3}$, as before.

So the cube root of .256, its Log. 29.4082400
will be .6350 9.8027466

Hence if you have a mind to cube any number, you muſt multiply its log. by 3, and you have the logarithm of its cube: ſo the cube of 9 will be found to be 9 Log. 0.9542425
3

which is the Log. of 729=2.8627275

Proportions in Logarithms.

To find a mean proportional between 2 numbers.

RULE.

Add the logarithm of the two numbers into one ſum

ſum, the ½ of which is the logarithm of the mean proportional ſought.

EXAMPLE.

Let the numbers be 16 and 144, and let a mean proportional be required.

Log. of 16 is	1.2041200
Log. of 144 is	2.1583625
Their ſum	3.3624825
Half of which is	1.6812412

Which is the logarithm of 48, the mean proportional ſought.

Note, If one be a decimal, if the ſum of the indices be 10, or above, caſt away 10, and then halve it; if it be not 10, add 10 to it, and then halve it.

So a mean proportional betwixt 12 Log.	1.0791812
And 25 Log.	9.3979400
The ſum	0.4771212
Will be 1.732	0.2385606

Between two numbers given, to find any number of mean proportionals deſired.

RULE.

Subtract the logarithm of the leſs number out of the logarithm of the greater; the remainder divided by a number greater by one than the number of means ſought; this quotient added to the logarithm of the leſs number, the ſum is the logarithm of the firſt mean; to which adding again the ſaid quotient, the ſum is the logarithm of the ſecond mean, and ſo forward, as far as you have occaſion.

EXAM-

EXAMPLE.

Betwixt 16 and 64 find five mean proportionals:

Log. of 64 is	1.8061800
Log. of 16 is	1.2041200
The difference is	0.6020600
$\frac{1}{6}$ Part for 5 means is	0.1003433
To which add the log. of 16	1.2041200
The ſum is the log. of the 1ſt mean 20.158	1.3044633
To which add again	1003433
The ſum of the log. of the 2d mean 25.398	1.4048066
To which add again	·1003433
The log. of the 3d mean 32	1.5051499
To which add again	1003433
The log. of the 4th mean 40.317	1.6054932
To which add again	1003433
The log. of the laſt mean 50.796	1.7058365

This propoſition is of excellent uſe in the calculation of tables belonging to compound intereſt, as ſhall be ſhewn in due place.

Having three numbers given, to find a fourth in a duplicated proportion.

RULE.

Double the difference of the logarithm of thoſe two numbers, which have the ſame denomination; then according as the firſt term is leſs or greater than the ſecond, add or ſubtract the double difference to, or from, the logarithm of the other number: this done, the ſum or remainder is the logarithm of the 4th number ſought.

EXAM-

EXAMPLE.

If the content of a circle, whose diameter is 7 inches, be 38.484; what is the content of that circle, whose diameter is 21? *Answer*, 346.3561.

See the work.

Diameter 7 inches,	Log.	0.8450980
Diameter 21 inches,	Log.	1.3222193
Difference is		0.4771213
Difference doubled		0.9542426 } Add
Content given, 38.484	Log.	1.5852802
Content required, 346.3561	Log.	2.5395228

By this proposition we find the proportion betwixt like superfices, which, by *Euclid* the 6th and 19th and 20th, is proved to be in duplicate proportion of their homologous sides.

So if a field, measured by a statute perch, contain 36 acres, it would, if measured by a *Cheshire* perch of 24 feet to the pole, be found to contain but 17 acres, and $\frac{1}{8}$ part.

Having these numbers given, to find a fourth in a triplicated proportion.

RULE.

Triple the difference of the logarithm of those two numbers, which have the same denomination; then, according as the first term is lesser or greater than the 2d, add or subtract the triple difference to or from the logarithm of the other number; this done, the sum or remainder is the logarithm of the 4th number sought.

EXAMPLE.

If a bullet, whose diameter is 9 inches, do weigh 72

72 pound, what will a bullet of the same metal weigh, whose diameter is 6 inches?

Diameter 9 inches,	Log.	0.9542425
Diameter 6 inches,	Log.	0.7781512
Difference is		0.1760913
Difference tripled		0.5282739 } Sub.
Weight given 72 *lb.*	Log.	1.8573325 }
Weight required	$21\frac{1}{3}$	1.3290586

By this proposition we find the proportion betwixt like solids; for as like superfices do hold in a duplicated proportion; so like solids do hold in a triplicated proportion of their homologous sides, diameters, *&c.*

In the next place, we shall give you a few propositions in *military orders*, and then proceed to our chief design, which is *compound interest.*

Military Orders by the Logarithms.

PROP. I.

Any number of soldiers being given, to place them in a square battalia of men.

RULE.

One half the logarithm of the number of soldiers given, will be the logarithm of the number of men, both in rank and file, to make a square battalia of men.

EXAMPLE.

Let 1764 men be given, and let it be required to frame them in a square battalia.

Log. 1764 is	3.2464986
$\frac{1}{2}$ is	1.6232493

which is the logarithm of 42, which is the number of

men

men that must be placed both in rank and file, to make a square battalia of men.

P R O P. II.

Any number of men being given, to order them into a double battalia; that is, that shall have twice as many in rank as file.

R U L E.

Half the logarithm of $\frac{1}{2}$ the number of men, is the logarithm of the number of men to be placed in file; and that number doubled is the number to be placed in rank.

E X A M P L E.

Let the number of men be 8450, and let it be required to make a double battalia of them.

Half the given number of men is 4225, Log. 3.6258267
The half of which is 1.8129133
which is the logarithm of 65; and so many must be placed in file; which doubled makes 130, which is the number of men to be placed in rank; for 65 times 130 is equal to 8450, the number of men given.

P R O P. III.

Any number of soldiers given, to order them into a quadruple battalia of men; that is, that shall have 4 times as many in rank as file.

R U L E.

Half the logarithm of one quarter of the number of soldiers given, is the logarithm of the number of men to be placed in file; which number multiplied by 4, is the number to be placed in rank.

E X A M-

EXAMPLE.

Let the given number of men be 4900 to be ordered into a quadruple battalia.

One quarter of which number is 1225, Log. 3.0881361
The half of which is 1.5440680
which is the logarithm of 35, the number to be placed in file, which multiplied by 4, gives 140, which must be the number to be placed in rank.

PROP. IV.

Any number of soldiers given, to order them into three equal square battalions.

RULE.

Half the logarithm of $\frac{1}{3}$ part of the number of soldiers given, is the logarithm of the number to be placed both in rank and file in every battalion.

EXAMPLE.

Let the given number of soldiers be 6075, and let it be required to form 3 square battalions of them, that shall have an equal number of men, both in rank and file.

One third of 6075 is 2025, its Log. 3.3064250
The half of which is 1.6532125
which is the logarithm of 45, the number of men in each battalion that must be placed both in rank and file.

PROP. V.

Any number of soldiers given, to place them in rank and file, according to the proportion of any two numbers given.

RULE.

From the sum of the logarithms of the number of soldiers given, and the proportional number for the rank, subtract the logarithm of the proportional number of files, half the remainder is the logarithm of the men to be placed in rank; and the same logarithm subtracted from the logarithm of the whole number of soldiers, leaves the logarithm of the number to be placed in file.

EXAMPLE.

Let 360 soldiers be so placed, that the number in file may be to those in rank as 5 to 9.

The log. of 3600 is	3.5563025
The log. of 9, the prop. of rank is	0.9542425
The sum is	4.5105450
Log. of 5, prop. for file is	0.6989700
Difference is	3.8115750
Half of which is	1.9057875

which is the logarithm of 80, nearest the number in rank; and the last log. subtracted from the first, leaves the log. of the file, *viz.* 1.6505150, the log. of 44, nearest the number in file.

PROP. VI.

Any number of soldiers given, with their distance in rank and file, to order them into a square battalia of ground.

RULE.

From the sum of the logarithm of the number of soldiers, and of the distance in rank, subtract the logarithm of their distance in file, half of the remainder is the

the number in file; which logarithm subtract from the whole number of soldiers, the remainder is the logarithm of the number of soldiers to be placed in rank.

EXAMPLE.

Let 3600 soldiers be ordered into a square battalia of ground, so that the distance in file may be 7 foot, and in rank 3 foot; so that the ground they stand upon may be a true square.

The logarithm of 3600 is	3.5563025
Log. of 3, the distance in rank, is	0.4771212
The sum is	4.0334237
Log. of 7, the distance in file, is	0.8450980
Difference is	3.1883257
Half of which is	1.5941628

which is the logarithm of 39, nearest for the number of men in file; and the last logarithm subtracted from the first logarithm leaves 1.9621397, which is the logarithm of 92, nearest the number of men in rank.

More might be added, but these are sufficient.

Compound INTEREST.

WHen a sum of money is lent, and the interest, when due, is not paid, but kept in the borrower's hands, and by that means becomes a part of the principal, then it is called *Compound Interest*.

As if *A* lend to *B* 100*l.* at the rate of 6 *per cent.* for a year, then it is evident, that at the end of one year, *B* is got into *A*'s debt 106*l.* and if this be continued in *B*'s hand till the end of the second year, there will then be due to *A* the increase of 106, *viz.* 112*l.* 7*s.* 2*d.* $\frac{1}{4}$, which will be a new stock for the third year, if not paid at the second year's end.

Whereby it is plain, that if it be lawful to take interest at all, it is lawful to take compound interest: for if *A* had received this interest annually as it became due, he had the advantage of putting out those annual payments at the same rate, and none would have stiled it *compound interest*. This will yet appear more plain, by supposing *A* laid out his 100 *l.* in purchasing an annualrent of 6 *l.* clear value, which annualrent may be made use of to his best advantage, and none can call him an extortioner.

And lastly, it will appear, that for any time under a year, compound interest is more easy than simple; for he that takes 3 *l.* for the use of an 100 *l.* for one half-year, takes too much; which may be proved thus: for as simple interest was performed by a rank of numbers arithmetically proportional; so compound interest is performed by a rank of numbers geometrically proportional.

And it is to be known, that if three numbers be in geometrical proportion, the product of the two extremes is equal to the square of the mean, by the 20th of the 7th of *Euclid.* So, on the contrary, if the rectangle contained under the extremes of any three numbers, be equal to the square of the mean, then those three numbers are in geometrical proportion.

Now if 3 *l.* be the interest of 100 *l.* for half a year, or six months, then these three numbers, 100, 103, 106, should be in geometrical proportion; but it may be proved, by the aforesaid proposition they are not: for the rectangle of 100 and 106 is but 10600, and the square of the mean 103, is 10609. But if the square root of 10600 be sought, it will be found to be 102.956: so that the true proportional interest of 100 *l.* for six months, or half a year, is but 2 *l.* 19 *s* 1 *d.* $\frac{1}{2}$ *ferè*.

In the solution of questions of compound interest, four things are to be considered.

First,

First, The principal, or money lent.

Secondly, The time of forbearance in years, or parts of a year.

Thirdly, The rate of interest *per cent.* by the year, half-year, or quarter, *&c.* equal to 1.06, 1.08, 1.10, *&c.*

Fourthly, The amount of the said principal for the said rate and time.

Any three of these being given, to find the fourth, as in the four propositions following.

PROP. I.

Principal, rate, and time given, to find the amount.

RULE.

Unto the logarithm of the rate multiplied by the time, add the logarithm of the principal, the sum is the logarithm of the amount.

EXAMPLE.

What will 20 *l.* amount, forborne 7 years, at 6 *per cent.* compound interest?

Principal 20 *l.* rate 1.06, time 7 years.

See the work.

Log. of 1.06, the rate is		0.0253059
Multiply by the time		7
Product of the rate and time	Log.	0.1771413
Add the log. of 20 the principal		1.3010299
The sum is		1.4781712

Which is the log. of 30.7, or 30 *l.* 14 *s.* the amount sought.

EXAMPLE II.

What will 365 *l.* 15 *s.* 6 *d.* amount to, forborne 11 years and a quarter, at 5 *per cent.* compound interest?

Principal 365.775 *l.*, rate 1.05 *l.*, time 11.25 *Y.*

The

The work.

Log. of 1.05, the rate is	0.0211893
Multiply by the time	11.25
Product of the rate and time, Log.	0.2383796
Add the log. of 365.775, the principal,	2.5632140
The sum is	2.8015936

Which is the log. of 663 *l.* 6 *s.* the amount.

PROP. II.

Amount, rate, and time given, to find the principal.

RULE.

From the logarithm of the amount, subtract the logarithm of the rate multiplied by the time, the remainder is the logarithm of the principal, or present worth.

EXAMPLE I.

What present money will pay a debt of 20 *l.* due 7 years hence, at 5 *per cent. per ann.* compound interest?

l. 20 the amount, *l.* 1.05 the rate, and *l.* 7 the time.

Log. of 20 *l.* the amount	1.3010300
Prod. of the log. of the rate and time sub.	0.1483251
The remainder is	1.1527049

Which is the log. of 14 *l.* 4 *s.* 4 *d.* $\frac{3}{4}$, the ready money sought.

EXAMPLE II.

A gentleman left his son 150 *l.* to be paid at the age of 21 years, of which 7 years were spent at the said time.

The

The executors deſire to pay ready money, ſo they may have rebate allowed after the rate of 6 *per cent. per ann.* compound intereſt. The queſtion is, What ready money will pay this debt?

Amount 150, rate 1.06, time 14 years.

Log. of the rate 1.06 is	0.0253059
Multiply by the time	14
	1012236
	253059
Product of the rate and time	3542826
Which ſub. from the log. of 150, *viz.*	2.1760913
Reſt	1.8218087

Which is the log. of 66.345, or 66 *l.* 6 *s.* 11 *d.* which is the anſwer to the queſtion.

PROP. III.

Principal, amount, and rate given, to find the time.

RULE.

From the logarithm of the amount ſubtract the logarithm of the principal, that divided by the logarithm of the rates, gives the time.

EXAMPLE I.

In what time will 20 *l.* amount to 40 *l.* at 6 *per cent. per ann.* compound intereſt?

Principal 20 *l.* amount 40 *l.* rate 1.06.

The log. of 40 *l.* the amount is	1.6020600
The log. of 20 *l.* the principal is	1.3010300
Diff.	.3010300
	.0253059)

.0253059) 3010300 (11.895

253059

479710   *Ans.* 11 years, 11 months, 2 weeks, and 5 days; dividing a year as in simple interest.

253059

2266510

2024472

2420380

2277531

1428490

EXAMPLE II.

In what time will 15*s.* amount to 15*l.* at 10 *per cent. per ann.* compound interest?

Principal 75, amount 15, rate 1.10.

Log. of 15 the amount is 1.1760913

Log. of 75 the principal is 9.8750613

Diff. 1.3010300

.0413927) 1.3010300 (31.4313

1241781

592490

413927   *Answer*, In 31 years, 5 months, 2 weeks, and 3 days.

1785630

1655708

1299220

1241781

PROP. IV.

Principal, time, and amount given, to find the rate.

RULE.

RULE.

From the logarithm of the amount, subtract the logarithm of the principal, the remainder divided by the time, quotes the logarithm of the rate.

EXAMPLE I.

At what rate of compound interest will 20*l.* amount to 30.072, or 30*l.* 1*s.* 5*d.* 1*q.* in 7 years?

Principal 20*l.* amount 30.072, time 7 years.	
Amount 30.072,	Log. 1.4781712
Principal 20*l.*	Log. 1.3010300
	$\frac{1}{7}$ 0.1771412
	=0.0253058

Equal to the logarithm of 1.06 the rate sought.

EXAMPLE II.

At what rate of compound interest will 51*l.* 15*s.* amount to 70*l.* 18*s.* in 5 years?

Principal 51.75, amount 70.9, time 5 years.

Amount 70.9,	Log. 1.8506462
Principal 51.75,	Log. 1.7139103
	0.1367359
	$\frac{1}{5}$=0.0273471

Equal to the logarithm of 1.065, which is 6*l.* 10*s.* *per cent. per ann.* the rate sought.

The two first propositions being often used, we have, as in *simple interest*, annexed tables fitted thereto, at the rates of 5 and 6 *per cent.* and to continue for 31 years.

TABLE

Years.	TABLE I. Shewing the amount of one pound for 31 years, at 5 and 6 per cent. Compound Interest.		TABLE II. Shewing the rebate of one pound for 31 years, at 5 and 6 per cent. Compound Inter.	
	5.	6.	5.	6.
1	1.050000	1.060000	.925381	.943396
2	1.102500	1.123600	.907030	.889996
3	1.157625	1.191016	.863838	.839619
4	1.215506	1.262477	.822703	.792093
5	1.276281	1.338225	.783526	.747258
6	1.340096	1.418519	.746215	.704960
7	1.407100	1.503630	.710683	.665057
8	1.477455	1.593848	.676839	.627412
9	1.551328	1.689479	.644609	.591898
10	1.628895	1.790848	.613913	.558394
11	1.710339	1.898298	.584679	.526787
12	1.795856	2.012196	.556837	.496969
13	1.885649	2.132928	.530321	.468839
14	1.979932	2.260904	.505068	.442301
15	2.078928	2.396558	.481017	.417265
16	2.182874	2.540352	.458111	.393647
17	2.292018	2.692773	.436296	.371364
18	2.406619	2.854339	.415520	.350343
19	2.526950	3.025599	.395734	.330513
20	2.653298	3.207135	.376889	.311804
21	2.785962	3.399564	.358942	.294155
22	2.925261	3.603537	.341849	.277505
23	3.071524	3.819750	.325571	.261797
24	3.225100	4.048935	.310067	.246978
25	3.386355	4.291871	.295302	.232998
26	3.555673	4.549383	.281240	.219810
27	3.733456	4.822346	.267848	.207368
28	3.920129	5.111687	.255093	.195630
29	4.116135	5.418388	.242946	.184556
30	4.321942	5.743491	.231377	.174110
31	4.538039	6.088101	.220359	.164254

The

The construction and uses of the foregoing tables.

For the construction of these two tables are several methods used: we shall only mention that which is most easy and expeditious, which is by the logarithms.

For the first table thus: Seek by the first proportion aforegoing, the amount of one pound for 31 years, and betwixt that log. and the log. of the rate, find 30 geometrical mean proportionals, as before taught, which shall be the log. of the number in the first table; which is nothing else but the continual addition of the log. of the rate to itself, and to its last sum. As if we add the log. of the rate to itself, the sum is the log. of the number belonging to the second year, and to that sum add again the log. of the rate, gives the log. of the number belonging to the third year; and thus you may do till you have finished: or if you multiply the log. of the rate by 1, 2, 3, 4, 5, 6, *&c.* gives the log. of the numbers answering those respective years.

And for the numbers in the second table, take the arithmetical complements of the log. of the numbers in the first table; and you will have the log. of the numbers in the second.

Now for their uses.

These tables are to be used in the same manner as those in *simple interest*, and so need but few examples.

Take an example for the use of the first table.

What will 20 *l.* amount to, forborne 7 years, at 6 *per cent.* compound interest?

In the first table under 6 *per cent.* and over-against 7 years, is	1.50363
Which multiply by	20
The product is	30.07260

which is equal to 30 *l.* 1 *s.* 5 *d.* 1 *q.*

 Take

Take another example for the use of the second table.

What ready money will pay a debt of 36 *l.* 10 *s.* due 21 years hence, at 5 *per cent.* compound interest?

The operation.

In the second table, and under 5 *per cent.* and over-against 21 years, is	.358942
Which multiplied by	36.5
	1794710
	2153652
	1076826
Produceth	13.1013830

which is equal to 13 *l.* 2 *s.* and $\frac{1}{4}$ *d.* the answer.

SECTION II.

In the solution of the questions of *compound interest*, concerning annuities in arrear, we may consider it under these four particulars, *viz.*

First, The annuity or pension.

Secondly, The time of forbearance in years, or parts of a year.

Thirdly, The rate of interest. And,

Fourthly, The amount of the said annuity, for the said rate and time.

Any three of these being given, to find the fourth, as in these four *propositions* following.

PROP. I.

Annuity, rate, and time given, to find the amount.

RULE.

First, you find a correspondent principal in this manner; As the interest : To its principal : : So the given annuity :

annuity : To its correſpondent principal. Next, multiply the logarithm of the rate by the time, to which add the logarithm of the correſpondent principal, the ſum is the logarithm of a number, from which ſubtract the correſpondent principal, leaves the amount.

EXAMPLE I.

An annuity of 20 *l. per annum* is forborne 7 years; what will then be due at 6 *per cent.* compound intereſt?

	l. l. l.	
Firſt ſay,	If 6 : 100 :: 20	
	20	
	2000	
$\frac{1}{6}$	$333\frac{1}{3}$=the correſp. principal.	
Log. of the rate		0.0253059
Multiplied by the time		7
Log. of the rate × by time		0.1771413
Log. of $333\frac{1}{3}$ the correſp. principal add		2.5228788
Equal to the log. of 501.210		2.7000201
Cor. princ. ſubtract. 333.333		
	l. s. d.	
Reſt	167.877=to 167 : 17 : $6\frac{1}{2}$ *ferè*.	

the anſwer.

EXAMPLE II.

There is an annuity of 50 *l. per annum*, payable by quarterly payments, *viz.* 12.5 *l. per* quarter; this annuity is forborne to the end of 11 years and a half; the queſtion is, What will then be due at 6 *per cent.* compound intereſt?

Note, If the interval betwixt any payments be leſs

than a year, as suppose half-yearly, quarterly, monthly, weekly, daily, &c. then you must divide the logarithm of the rate by such parts; as by 2 for half-yearly payments; 4 for quarterly payments, by 52 for weekly payments, and by 365 for daily payments; and your quotient will be a proportional rate, whereby to find your correspondent principal: for if the absolute number answering that quotient, be made less by an unit, it will be a new divisor; by which dividing your half-yearly, quarterly, &c. payments, your quotient will be a correspondent principal; then may you work as before.

See underneath, the logarithm of 1.05 and 1.06, so divided with their natural numbers placed over-against them.

	Logarithms.	*Natural numbers.*
Log. of 1.05	0.0211893 =	1.05
$\frac{1}{2}$	0.0105946 =	1.0246738
$\frac{1}{4}$	0.0052973 =	1.0122722
$\frac{1}{12}$	0.0017658 =	1.0040741
$\frac{1}{52}$	0.0004075 =	1.0009387
$\frac{1}{365}$	0.0000581 =	1.0001336
Log. of 1.06	0.0253058 =	1.06
$\frac{1}{2}$	0.0126529 =	1.0295630
$\frac{1}{4}$	0.0063264 =	1.0146738
$\frac{1}{12}$	0.0021088 =	1.0048675
$\frac{1}{52}$	0.0004866 =	1.0011019
$\frac{1}{365}$	0.0000693 =	1.0001596

So in the last question, the payments being quarterly, I take the natural number answering $\frac{1}{4}$ part of the logarithm of the rate 1.06, which made less by unity, is .014673; by which dividing the quarterly payment 12 *l.* 10 *s.* quotes 851.9048, the correspondent principal.

See

See the work.

```
.014673) 12.5000000 (851.9048
         117384
         ------
          76160
          73365
          -----
          27950
          14673
          ------
          132770
          132057
          ------
            71300
            58692
            -----
            12608
```

Log. of the rate 1.06 is	0.0253058
Multiply by the term	11.5
	1265290
	253058
	253058
Log. of the rate and time	.29101670
Log. of the corresp. principal, add	2.93039110
The sum is the log. of 1665.2053 =	3.22140780
Sub. the cor. principal, 851.9048	
Rest 813.3005 = to 813 *l.* 6*s.*	

which is the answer.

P R O P. II.

Amount, rate, and time given, to find the annuity.

R U L E.

Suppoſe an annuity at pleaſure, and by the laſt *propoſition* find the amount or arrearages; then you may ſay, As the amount to the ſuppoſed annuity :: So the amount given : To the annuity required.

E X A M P L E.

What annuity at 6 *per cent.* compound intereſt, will raiſe a ſtock of 167.877 *l.*, in ſeven years?

Suppoſe 3 *l.*

Then if 6 *l.* : 100 *l.* : : 3 *l.* : *Facit* 50 *l.* a correſp. principal.

Log. of the rate	0.0253059
Multiply by the time	7
	0.1771413
Log. of the correſpond. principal, add	1.6989700
Equal to the log. of 75.181	1.8761113
Cor. princip. ſubtr. 50.000	
25.181	

Then ſay, If 25.181 : 3 : : 167.877

3

25.181) 503.631 (20 *l.*

50362

11

P R O P. III.

Annuity, rate, and amount given, to find the time.

R U L E.

Find a correſpondent principal, add to it the given amount,

amount, and from the logarithm of that ſum, ſubtract the logarithm of the correſpondent principal, the remainder, divided by the logarithm of the rate, quotes the time.

EXAMPLE.

In what time will 20 *l.* *per annum* raiſe a ſtock of 167.877, compound intereſt being computed at 6 *per cent. per annum?*

Firſt, If 6 : 100 : : 20 : *Facit* 333⅓ correſp. principal.

Given ſtock	167.877
Correſp. princ.	333.333
The ſum	501.210
Log. of 501.210 is	2.7000197
Log. of the correſp. principal is	2.5228783
.0253059)	1771414
	1771413

Anſwer, In 7 years. 1

PROP. IV.

The annuity, amount, and time given, to find the rate of intereſt.

RULE.

To anſwer this, we will uſe approximation, it being the moſt conciſe and quickeſt method we can uſe. Wherefore make two or three trials, till you get the anſwer bounded betwixt two of the neareſt reſults; then the work may be performed by proportion, as may be ſeen in the work of the following example.

EXAMPLE.

An annuity of 20 *l.* *per annum* is offered to be let for

180.5

l.
180.5, or 180 *l.* 10 *s.* to be paid at the end of the said term; what interest is allowed in this bargain?

Interest of money being seldom above 10 *l.* and under 5 *per cent.*; wherefore I make a supposition at 8 *per cent.* and by the first *proposition* I find the amount at that rate, to be 178.456 *l.* which is too little by 2.044.

Wherefore, because I see I am near, I make my second trial at 8 *l.* 10 *s.* *per cent.* and working as before, I find the amount to be 181.21, whereby I see I have overshot the truth by 71, and I see the answer is bounded betwixt 8 *l.* *per cent.* and 8 *l.* 10 *s.* *per cent.*

Wherefore, as in the *Rule of False*, by those two suppositions, add their respective errours, I find the rate as under.

First supposition	8	the errour	2.044	—
Second supposit.	8.5	the errour	0.71	+
Supp. difference	.5	sum	2.754	

Then say, As 2.754 : .5 : : 2.044 : .371, which added to the first *supposition* 8, gives 8.371, or 8 *l.* 17 *s.* 5 *d.* the rate of interest sought.

The first proposition being of good and frequent use, we have adjoined a table fitted thereto, and calculated at the rate of 5 or 6 *per cent.* compound interest, and to continue for 31 years.

The

The construction and use thereof, here follow.

Its construction.

The logarithmical differences of .05 or .06 (being the rates here used *minus* unity) and the numbers in the table shewing the amount of one pound at 5 and 6 *per cent.* for 31 years, *minus* unity, are the logarithms of the numbers in this table.

Take an example or two for the use.

What will an annuity of 3 *l.* 15 *s.* 6 *d.* amount to, forborne 21 years, compound interest being computed at 6 *per cent.*?

Tabularnumberanswering 21 years, and under 6 *per cent.* is 39.992727
Multiply by 3.775

199963635
279949089
279949089
119978181

150.972544425

Facit, 150 *l.* 19 *s.* 5 *d.* ¼

A TABLE, shewing the amount of one pound annuity forborne for 31 years, or under, at 5 and 6 per cent. compound interest.

Years.	5	6
1	1.000000	1.000000
2	2.050000	2.060000
3	3.152500	3.183600
4	4.310125	4.374616
5	5.525631	5.637093
6	6.801913	6.975318
7	8.142008	8.393837
8	9.549108	9.897467
9	11.026564	11.491316
10	12.577892	13.180795
11	14.206787	14.971943
12	15.917126	16.869940
13	17.712982	18.882137
14	19.598631	21.015065
15	21.578563	23.275969
16	23.657491	25.672527
17	25.840366	28.212879
18	28.132384	30.905651
19	30.539003	33.759992
20	33.065954	36.785590
21	35.719251	39.992727
22	38.505214	43.392291
23	41.430475	46.995826
24	44.501999	50.815575
25	47.727099	54.864510
26	51.113452	59.156381
27	54.669126	63.705763
28	58.402583	68.528112
29	62.322712	74.639799
30	66.438847	79.058184
31	70.760790	84.801677

EXAMPLE II.

What will an annuity of 50 *l. per annum* amount to, forborne 7 years, at 5 *per cent.* compound interest?

Tabular number under 5 *per cent.* and against 7 years, is 8.142008

Multiplied by 50

Answer, 407 *l.* 2 *s.* 407.100400

In the solutions of questions of compound interest, relating to many equal payments, at many equal times, as in the buying or purchasing annuities, pensions, or leases in reversion, we may consider it under these four particulars:

First, The annuity or pension to be sold.

Secondly, The time of continuance either considered as yearly, half-yearly, or quarterly payments.

Thirdly, The rate of interest. And,

Fourthly, The present worth of the whole, paid at one entire payment, or equally reduced to such.

Any three of these being given, to find the fourth, as in the four *propositions* following.

PROP. I.

Annuity, rate, and time given, to find the present worth.

RULE.

Find a correspondent principal, as before taught; then multiply the logarithm of the rate by the time, which subtract from the corresponding principal, the remainder is the logarithmical difference of the principal and worth, and so by consequence the worth is given.

EXAMPLE I.

There is an annuity of 20 *l. per annum*, payable by yearly payments, and to continue 7 years, to be sold for

for ready money: What is it worth, compound interest being allowed the purchaser at 5 *per cent.*?

See the work.

If 5 *l.* : 100 *l.* : : 20 *l.* : *Facit* 400 *l.* corresp. princip.	
Log. of 1.05 the rate is	0.0211893
Multiply by	7
Log. of the rate and time	0.1483251
Log. of 400 the corresp. princip.	2.6020600
The difference is	2.4537349

l.

equal to the logarithm of 284.2725; which subtracted from 400 *l.* leaves 115 *l.* 14 *s.* 6 *d.* ½, the present worth sought.

EXAMPLE II.

An annualrent of 365 *l.* paid yearly, and to continue 12 years, is to be sold for present money; what is it worth at 5 *l. per cent.* compound interest?

See the work.

If 5 *l.* : 100 *l.* : : 365 *l.* : *Facit* 7300 corresp. princip.	
Log. of 1.05 the rate is	0.0211893
Multiply by the time	12
	0423786
	211893
Logarithm of the rate and time	2542716
Log. of 7300 the corresp. princip.	3.8633228
The difference is	3.6090512

which is the logarithm of 4064.913, or 4064 *l.* 18 *s.* 3 *d.* which subtracted from 7300 *l.* leaves 3235 *l.* 1 *s.* 9 *d.* the worth sought.

EXAM-

EXAMPLE III.

But if the aforenamed annuity were to be paid by quarterly payments, *viz.* 91 *l.* 5 *s. per* quarter, what would be the present worth, allowing the same rate of interest as before?

Proportional interest for quarterly payments at 5 *per cent.* is = to 1.0122722—1=0122722; by which dividing the quarterly rent, quotes the correspondent principal.

.0122722) 91.2500 (7435.5046 corresp. principal.

Log. of the rate 1.05, is	0.0211893
Multiply by the time	12
	423786
	211893
Log. of the rate and time	.2524716
Log. of 7435.5046, the corresp. principal	3.8713104
The difference is	3.6170388

which is the logarithm of 4140.4671; which subtracted from 7435.5046, leaves 3295.0375 equal to 3295*l.* and 9*d.* the present worth; by which you may perceive that quarterly payments in this annuity raiseth the value 59*l.* 19*s.*

Notwithstanding, in the purchasing of annuities, very few persons will value a lease the more for being paid quarterly.

EXAMPLE IV.

But if the said annuity were to be paid by daily payments, *viz.* 20 *s. per* day, what would be the present worth, keeping the same rate of interest still?

Proportional interest for daily payments is .0001336, when unity is subtracted; by which dividing the daily rent, quotes the correspondent principal.

.0001

.0001336) 1.000 (7485.03 cor. principal.

Log. of 7485.03, is	3.8741935
Log. of the rate and time, is	0.2542716
The difference is	3.6199219

which is the logarithm of 4167.9446; which subtracted from 7485.03, will leave 3317.0854, equal to 3317 *l.* 1 *s.* 8 *d.* $\frac{1}{2}$. So the difference of the present worth of this annuity, according to yearly and daily payments, is 82 *l. fere*.

EXAMPLE V.

An annuity of 24 *l. per ann.* to begin after the end of 6 months, whereby the first rent will not be received till after the expiration of 7 years, and to continue 21 years, is to be sold for present money; what is it worth on this condition, allowing the purchaser 6 *per cent.* compound interest?

If this annuity were to begin presently, the worth, by this proposition, would be found to be 382.3378, or 382 *l.* 6 *s.* 9 *d.*

But seeing it begins not till the end of 6 years, you must, by the second proposition of the first section of this chapter, find what ready money will pay a debt of 382.3378, due 6 years hence; which will appear to be 269.5328, or 269 *l.* 10 *s.* 8 *d.* which is the present worth of this annuity according to the condition aforesaid.

PROP. II.

Present worth, rate, and time given, to find the annuity.

RULE.

Suppose an annuity at pleasure, and find the worth by the last; then the proposition runs,

As the worth found : To the supposed annuity : :
So the worth given : To the annuity required.

EXAMPLE.

What annuity, to continue 7 years, will be purchased for 120*l.* at 6 *per cent.* compound interest?

Suppose 15*l.* and by the last proposition the present worth will be found to be 83.7357.

Then say, If 83.7357 *l.* : 15 *l.* :: 120 *l.* : *Facit* 21.4962, or 21*l.* 9*s.* 11*d.* 1*q.* the *answer*. And thus of any other.

PROP. III.

Annuity, present worth, and rate of interest given, to find the time of continuance.

RULE.

Find a correspondent principal, subtract the debt out of the correspondent principal; and the logarithm of their difference out of the logarithm of the correspondent principal; this last difference, divided by the logarithm of the rate, shews the time.

EXAMPLE I.

In what time will 20*l. per ann.* pay a debt of 115.7275; or 115*l.* 14*s.* 6*d.* $\frac{1}{2}$, at 5 *per cent.* compound interest?

First, if 5 : 100 :: 20 : *Facit* 400 cor. prin.

From the cor. prin.	400
Subtract the debt	115.7275
The difference is	284.2725

Log. of the cor. prin. 400*l.* is	2.6020600
Log. of 284.2725, is	2.4537349

.0211893) 1483251 (7
1483251

0.

Answer, In seven years.

EXAM-

EXAMPLE II.

A borrows of *B* 1728 *l.* and at the same time delivers up to *B* an annuity of the clear value of 240 *l.* *per ann.* which he is to enjoy till he be fully satisfied for his 1728 *l.* The question is, How long must *B* enjoy the premises, compound interest being computed at 6 *per cent. per ann.?*

First, I say, If 6 : 100 : : 240 : *Facit* 4000 cor. prin.

From the cor. principal	4000
Subtract the debt	1728
The difference is	2272
Log. of 4000, the cor. principal, is	3.6020600
Log. of the said difference 2272, is	3.3564083
The difference is	2456517

which divide by the log. of 1.06

```
=0253059) 2456517 (9.70729
          2277531
          -------
           1789860
           1771413
           -------
             1844700
             1771413
             -------
               732870
               506118
               ------
               226752
```

Answ. In 9 years, 9 months, 0 weeks, and 5 days, and so long *B* must enjoy the premises.

EXAMPLE III.

A lends *B* 600 *l.* and *B* is willing to pay a quarterly rent of 15 *l.* *per* quarter, till *A* be satisfied for his 600 *l.* How many quarters rent must *A* receive, compound in-

 terest

tereſt being computed at 5 *per cent.* and what will the laſt payment be? Divide 15 by the natural number to $\frac{1}{4}$ of the log. of the rate leſs 1.

.0122722) 15.0000000 (1222.2747 cor. prin.

From the cor. principal	1222.2747
Subtract the debt	600
The difference is	622.2747

Log. of the cor. prin.	1222.2747, is	3.0871688
Log. of the difference	622.2747, is	2.7939821
Their difference is		.2931867

$\frac{1}{4}$ Log. rate.

=0052973) .2931867 (55.3464=to 55 whole quarters, and ſomething above $\frac{1}{3}$ of a quarter. *Anſwer*, He muſt receive 55 quarters rent; and the laſt payment will be 5 *l.* 3 *s.* 11 *d.* 1 *q.*

PROP. IV.

The annuity, preſent worth, and time of continuance given, to find the rate of intereſt.

This propoſition is beſt performed by approximation; for by two or three trials (but they muſt be near the truth) you will have the anſwer bounded betwixt two numbers; as in the laſt propoſition of the laſt ſection.

An annuity of 20 *l. per ann.* to continue for 7 years, is ſold for 100 *l.* ready money; what rate of compound intereſt hath the purchaſer for the money?

Intereſt of money being ſeldom above 10, or under 5 *per cent.* I make my firſt ſuppoſition at 9 *per cent.* and, by the firſt propoſition of this ſection, the preſent worth of 20 *l. per ann.* to continue 7 years, will be
l.
found to be 100.659056, which ſhould have been 100 *l.* wherefore the errour is .659056. And ſeeing the ſuppoſition was ſhort, I place it and the errour as here,

9+.659056
9.25—.18

But

But seeing I am pretty near, I make my next supposition at 9*l.* 5*s.* and by the same proposition the said annuity for the same time will be worth 99.82, which should have been 100; by which I see I have proposed too much, and the errour is .18; which supposition and errour I place under the other, and say,

As .839, the sum of the errours: To .25, the difference of suppositions :: So 18, the latter errour: To .536; which subtract from the latter supposition because it was too great, leaves 9.1964, or 9*l.* 3*s.* 11*d.* the rate sought.

And though this be mathematically true and demonstrable; and that by delivering up of an annuity of 20*l.* *per ann.* to continue for 7 years, for 100*l.* paid in hand, he allows 9*l.* 3*s.* and 11*d.* *per cent.* *per ann.* yet he will never be able to make that interest by his annuity, unless he can find such a fool as will take his annual payments as they become due, and give him 9*l.* 3*s.* 11*d.* *per cent.* compound interest; which will be hard to do, when any responsable man may be fitted for 6, nay, in most places for 5 *per cent.*

Wherefore Mr *Martindale* was in the right, according to the intent and import of his propositions; and that he can but make 7*l.* 13*s.* 7*d.* $\frac{1}{2}$, supposing every payment be taken off his hand at 6 *per cent.* compound interest; and this will be something difficult to do. And if some of his annualrents, or all of them, should not be improved, which is no impossible thing, he will not be able to make 6 *per cent.* by his annuity; so that I had rather put forth my hundred pound at 6 *per cent.* compound interest for 7 years, than stand to the venture of the improvement of the annuity.

The two first propositions being of good use, we have annexed two tables fitted thereto for 31 years, at 5 and 6 *per cent.* compound interest.

Years.	TABLE I. Shewing the present worth of one pound annuity, to continue for 31 years, at 5 and 6 per cent. compound interest.		TABLE II. Shewing what annuity, continued for 31 years, one pound will purchase at 5 and 6 per cent. compound inter.	
	5.	6.	5.	6.
1	0.952381	0.943396	1.050000	1.060000
2	1.859410	1.833392	.537805	.545437
3	2.723248	2.673012	.367208	.374110
4	3.545950	3.465105	.282012	.288591
5	4.329477	4.212363	.230952	.237396
6	5.075692	4.917324	.197017	.203363
7	5.786373	5.582381	.172820	.179135
8	6.463212	6.209792	.154722	.161036
9	7.107821	6.801691	.140690	.147022
10	7.721734	7.360086	.129505	.135868
11	8.306414	7.688873	.120389	.126793
12	8.863251	8.383843	.112825	.119272
13	9.393572	8.852682	.106456	.112960
14	9.898640	9.294983	.101023	.107585
15	10.379658	9.712248	.096342	.102963
16	10.837769	10.105894	.092270	.098952
17	11.274065	10.477258	.088699	.095445
18	11.689586	10.827602	.085546	.092356
19	12.085320	11.158115	.082745	.089621
20	12.462209	11.469920	.080242	.087184
21	12.821152	11.764075	.077996	.085004
22	13.163002	12.041580	.075970	.083045
23	13.488573	12.303377	.074137	.081278
24	13.798641	12.550356	.072441	.079679
25	14.093944	12.783354	.070952	.078227
26	14.375184	13.003164	.069564	.076904
27	14.643033	13.210431	.068292	.075697
28	14.898127	13.406162	.067122	.074592
29	15.141073	13.590721	.066045	.073579
30	15.372450	13.764829	.065051	.072649
31	15.592810	13.929084	.064132	.071792

The

The Construction of the foregoing TABLES.

If from the logarithms of the numbers in that table, under section the second, you subtract the logarithms of the numbers in table the first, section the first, the remainders are the logarithms of the numbers in the first table here.

And their complements arithmetical are the logarithms of the numbers in the second table.

Their USE.

There is no difference betwixt the use of these tables and those going before, as may be seen in the following examples.

Examples in the use of the first TABLE.

I. An annuity of 20 *l. per ann.* clear value, is to be sold for 7 years; what ready money is it worth, at 5 *per cent.* compound interest?

Multiply the tabular number under 5 *per cent.* and over-against 7 years, *viz.*	5.786373
By 20 the given annuity	20
	115.727460

Gives the *answer, viz.* 115 *l.* 14 *s.* 6 *d.* $\frac{1}{2}$

EXAMPLE II.

There is a lease of lands worth 32 *l. per ann.* more than the rent paid to the lord; of which land there is yet a lease in being for 7 years; and the lessee is desirous to take a lease in reversion for 21 years, to begin when his old lease is expired; what sum of money is to be paid for his lease, allowing interest at the rate of 6 *per cent. per annum?*

First, See what this rent of 32 *l.* is worth for 7 years, which will be 178 *l.* 12 *s.* 9 *d. fere.*

Secondly,

Secondly, Add 7 years to 21 years, which makes 28 years, then see what 32 *l.* to continue 28 years, is worth, which will be 428 *l.* 19 *s.* 11 *d.* $\frac{1}{4}$.

Lastly, Subtract the present worth for 7 years from the present worth for 28 years, the difference is the answer to the question, to wit, 250 *l.* 7 *s.* 2 *d.* $\frac{1}{2}$.

The work for 7 years.	*The work for 28 years.*
5.582381	13.406162
32	32
11164762	26812324
16747143	40218486
178.636192	428.997184

From 428.997184
Sub. 178.636192

Sub. 250.360992 the *answers*

Examples in the use of the second TABLE.

I. What annuity, to continue 9 years, will 34 *l.* purchase, compound interest being computed at 5 *per cent.*?

Tabular number in table the second, under 5 *per cent.* and over-against 9 years, is .140690
Multiply by 34

562760
422070

Ans. 4 *l.* 15 *s.* 8 *d.* *Facit*, 4.783460

EXAMPLE II.

What annuity, to continue 21 years, will 365 *l.* purchase, compound interest being computed at 6 *per cent. per annum*?

Tabular number in table the second, under 6 *per cent.* and over against 21 years, is	.085004
Multiply by	365
	425020
	510024
Anſ. 31 *l.* 0 *s.* 6*d.*	255012
	31.026460

SECT. IV.

Now, in the laſt place, we ſhall treat of compound intereſt, as it relates to the purchaſing of freehold eſtates, to be bought or ſold for ever.

This by ſeveral is called *compound intereſt infinite*, becauſe it relates to diverſe equal parts at diverſe equal times; but the number of thoſe equal times are infinite; as in purchaſing an eſtate in fee-ſimple for ever.

And this may be conſidered under theſe three particulars:

Firſt, The annuity paid by yearly or quarterly payments.

Secondly, The price, or preſent worth.

Thirdly, The rate of intereſt.

Any two of theſe being given, to find the third; as in the three propoſitions following.

PROP. I.

The annuity and rate of intereſt given, to find the preſent worth.

RULE.

The annual (half-yearly, or quarterly) payment divided by the rate of intereſt, *minus* unity, quotes the preſent worth.

EXAM-

E X A M P L E.

There is an estate to be sold of the clear value of 20*l.* *per ann.* what sum of ready money is this estate worth, compound interest being allowed the purchaser at 6 *per cent.*?

.06) 20.000 (333.333=333 *l.* 6 *s.* 8 *d.*

18

20
18

20
Ans. 333 *l.* 6 *s.* 8 *d.* 18

20
18

2

E X A M P L E.

But if the said annuity were paid by quarterly payments, *viz.* 5 *l.* *per* quarter; what would be the present worth, holding still the same rate of interest?

Quarterly rate,
minus unity, is .014674) 5.000000 (340.7387

44022

Ans. 340 *l.* 14 *s.* 9 *d.* 59780
58696

Quarterly payment raiseth 108400
the worth 7 *l.* 8 *s.* 1 *d.* 102718

5682

P R O P.

PROP. II.

Present worth, or purchase-money, together with the rate of interest, being given, to find the annuity.

RULE.

Multiply the purchase-money by the rate of interest, *minus* unity, the product shall be the annual rent.

EXAMPLE.

A gentleman hath a desire to lay out 333 *l.* $\frac{1}{3}$, on a freehold estate, provided he meets with such a bargain as shall bring him in 6 *per cent.* compound interest for his money; what annualrent must this be?

$$\begin{array}{r} 333\cdot333 \\ .06 \\ \hline 19\cdot99999 \end{array} \quad \textit{Facit}, \ 20\,l.$$

PROP. III.

The annuity and present worth given, to find the rate of interest.

RULE.

The annualrent divided by the present worth, or sum demanded, quotes the rate, *minus* unity.

EXAMPLE I.

There is an estate to be sold of the yearly value of 20 *l.* for 333 *l.* $\frac{1}{3}$, what rate of compound interest will the purchaser have for his money?

$$\begin{array}{r} 333\cdot333)\ 20.000000\ (.06 \\ 19999999 \\ \hline 0 \end{array}$$

Ans. 6 *per cent.*

EXAMPLE II.

There is a freehold estate to be sold for 1600 *l.* the yearly

yearly rent being 128 *l.* what rate of compound interest shall the purchaser have for his money?

$$\begin{array}{r@{\,}l} 1600) & 128.000\ (.08 \\ & \underline{128000} \end{array}$$

Anſ. 8 *per cent.*

Laſtly, If it be required how many years purchaſe any annuity is worth, work thus; Divide unity by the rate, *minus* unity, the quote exhibits the number of years.

EXAMPLE III.

There is a freehold eſtate to be ſold; how many years purchaſe is it worth at 5 *per cent. per ann.* compound intereſt?

$$\begin{array}{r@{\,}l} .05) & 1.00\ (20 \\ & \underline{10} \\ & 00 \end{array}$$

Anſ. 20 years purchaſe.

What is it worth at 6 *per cent.*?

$$\begin{array}{r@{\,}l} .06) & 1.0\ (16.666 \\ & \underline{6} \\ & 40 \\ & \underline{36} \\ & 4,\ \&c. \end{array}$$

Anſ. 16 years and $\frac{2}{3}$.

Likewiſe, if an eſtate be offered at any number of years purchaſe, and the rate of intereſt be demanded, do thus; Divide unity by the number of years propoſed, and the quote gives the rate, *minus* unity.

EXAMPLE IV.

An eſtate is offered at 20 years purchaſe; what is the rate of intereſt?

$$\begin{array}{r@{\,}l} 20) & 1.00\ (.05 \\ & \underline{1.00} \\ & 0 \end{array}$$

Anſ. 5 *per cent.*

Here

Here follow diverse questions of interest to exercise the learner, both simple and compound; and so we will conclude *Logarithmical Arithmetic.*

QUEST. I.

A owes *B* 800 *l.* to be paid in 4 years, that is, at the end of every two years 400 *l.* *B* owes *A* 900 *l.* to be paid in six years, that is, at the end of every two years 300 *l.* They agree to clear their debts, and allow each other 8 *per cent.* compound interest: which must pay money, and how much?

Ans. B must pay unto *A* 29 *l.* 16 *s.* 3 *d.* 1 *q.*

QUEST. II.

A owes *B* 455 *l.* to be paid in 14 years; that is, at the end of every two years 65 *l.* He would agree with his creditor to pay him in 7 years, *viz.* each year one equal payment, which *B* agrees to; and they conclude compound interest shall be allowed at 6 *per cent.*: what will this equal payment be? *Ans.* 52 *l.* 10 *s.* 8 *d.*

Found by seeking the present worth of the 7 payments, paid each two years, which will be 293 *l.* 5 *s.* 2 *d.* Then seek what annuity, to continue 7 years, 293 *l.* 5 *s.* 2 *d.* will purchase; which will be found to be 52 *l.* 10 *s.* 8 *d.* the *answer* sought.

QUEST. III.

A merchant hath owing to him 10000 *l.* to be paid in five years, *viz.* at the end of every year 2000 *l.* and agrees with his debtor, that if he will pay him 5000 *l.* ready money, he will take the remainder in 21 years by an equal annual payment, compound interest being computed at 6 *per cent.* to which his debtor assents: the question is, what will this equal annual payment be?

l.
Ans. 291.11725, or 291 *l.* 2 *s.* 4 *d.*

 Found

Found by ſeeking the preſent worth of 2000 *l. per ann.* to continue 5 years, which will be 8424 *l.* 14 *s.* 6 *d.* ½. from which ſubtracting 5000 *l.* reſts 3424 *l.* 14 *s.* 6 *d.* ½. Then find what annuity, to continue 21 years, 3424 *l.* 14 *s.* 6 *d.* ½, will purchaſe, *viz.* 291 *l.* 2 *s.* 4 *d.* and that is the *anſwer.*

QUEST. IV.

There is annuity of 64 *l.* 10 *s.* to continue 120 years, to be ſold for ready money; whether is it better to purchaſe this annuity at 6 *per cent.* ſimple intereſt, or at 6 *per cent.* compound intereſt? likewiſe, what is the difference? and laſtly, what is its value taken as a freehold eſtate?

The work according to ſimple intereſt.

120	120		120
·12	·06		120
240	7.20		240
120			120
14.40	16.4) 70743.6 (4313.6341		14400
2. Add	656		.06
16.40	514		864.00
	492		240 Add
	223		1104
	164	Deduct	7.2
	596		1096.8
	492		64 5
	1040		54840
	984		43872
			65808
	560		70743.60
	492		
Anſ. 4513 *l.* 12 *s.* 8 *d.* according to ſimple intereſt	680		
	656		
	24		

The

The work, according to compound interest.

$$\text{First, If } 6 : 100 :: 64.5 : \textit{Facit}, 1075 \text{ cor. prin.}$$

(l. l. l.)

```
6) 64500 (1075
   6...
   ----
   045
    42
   ----
     30
     30
     --
      0
```

{ *Anf.* 1074 *l.* 00 *s.* 3 *d.* according to compound interest.

Log. of the rate	.0253059
Multiply by the time	120
	5061180
	253059
Log. of the rate and time	3.0367080
Log. of the cor. principal 1075, is	3.0314084
Log. of the rate and time, is	3.0367080
Difference is	9.9947004

Which is the logarithm or .98788; which subtracted from the cor. principal, leaves the present worth, *viz.* *l.* 1074.01212, which is equal to 1074 *l.* 0 *s.* 3 *d.*

The work, as a freehold estate at 6 *per cent.* compound interest.

```
.06) 64.50 (1075
     6..
     ---
     045
      42
     ---
      30
      30
      --
       0
```

Anf. 1075 *l.* as a freehold estate.

 The

	l.	*s.*	*d.*
The present worth at 6 *per cent.* simp. int. is	4313	12	8
The present worth at 6 *per cent.* comp. int. is	1074	0	3
Difference of the worth is	3239	12	5

The present worth, as a freehold estate, at 6 *per cent.* compound interest, is 1075 *l.*

By which you may see it is better to purchase it at compound interest, by 3239 *l.* 12 *s.* 5 *d.* which is a very great difference, being more than the estate is worth for ever.

And though the present worth of this estate, to continue 120 years at 6 *per cent.* compound interest, comes so near the worth of the same estate, to continue for ever, the same rate of interest being computed; yet if this estate were to continue 200 years, nay double that time, yet it would not reach 1075 *l.*; which shews the agreement of the rules: for if it were otherwise, it would not be found better to purchase an annuity for ever, than for a certain number of years; which would be a paradox.

QUEST. V.

A gentleman pays 350 *l.* for a lease in reversion, to commence at the end of 13 years and a quarter, and to continue for 21 years and 3 quarters: what quarterly rent may he let the premisses for, after he comes to be in possession thereof, so as to gain 8 *per cent.* compound interest for his money?

The log. of 350 *l.*	2.5440680
Worth of 1 *l.* forborne 53 quarters =	0.4428627
Log. of the increase of 350 *l.* *i. e.* of 970.4 =	2.9869307
The log. of the annuity that 1 *l.* will purchase for 87 quarters.	= 2.3689282
Sum of the log. of 22.69 the *Ans.*	= 1.3558589

PRO-

PROBLEMS OR QUESTIONS, IN ALGEBRA.

1. TO find a number, which being multiplied by 3, subtracting 5 from the product, and the remainder divided by 2, if the number sought be added to the quotient, that the sum may be 40.

2. To find a number, which being multiplied by 12, and 48 added to the product, as much may be produced as if the same number sought were multiplied by 18.

3. To find a number, to which if 11 be added, and 7 subtracted from the same number, (*viz.* the first), the sum of the addition may be double the remainder.

4. To find a number, to which if its double, treble, quadruple, &c. be added, the square of the same number may be produced.

5. To find a number, which if added to itself, and the sum multiplied by the same, and the same number still subtracted from the product; and lastly, the remainder divided by the same, that it may produce 13.

6. To divide the number 16 into two parts, so that the square of the greater part may exceed the square of the less by 32.

7. To divide the number 36 into two parts, so that if 12 be added to the first, and 6 to the second, the former may be double of the latter.

8. Let the line AB (of 70 parts) be divided any how in C, (so that AC may be 24, BC 48), it is required to divide the same line again in another point; *for example*, in D, so that the rectangle ADC may be equal to the square DB: let the segment CD be enquired; which being obtained, AD, DB, will be known.

9. Let the line EF be divided any how in G, (so that FG may be 6, GF 4); it is required to produce this right line EF, (for example unto H), so that the rectangle EHF may be equal to the square GH; the length of FH is required.

10. A general disposing his army into a square battle, finds he has 284 soldiers over and above; but increasing each side with one soldier, he wants 25 soldiers to fill up the square: how many soldiers had he?

11. A certain captain sends out $\frac{1}{3}$ of his soldiers $+ 10$, there remain $\frac{1}{2} + 15$: how many soldiers had he?

12. There is an army, to which if you add $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ of itself, and take away 5000, the sum-total will be 100000: what was the number of the army?

13. In the rectangle ABCD, the difference of the greater side AB, and of the lesser side BC, is 12; but the difference of the squares of the sides 1680: what are the sides of the rectangle ABCD?

14. The

14. The length DE of the rectangle DEFG, is twice the breadth EF; and the sum of the squares of the length and breadth is ten times the sum of the two sides DE, EF: what are the sides of the rectangle DEFG?

15. To find two numbers in the proportion of 2 to 3; whose product, if they be multiplied by one another, shall be 54.

16. To find two numbers whose *ratio* is to one another as 4 to 5; and the sum of the squares of both is 262.

17. To find the side of a square, whose *area* is to the sum of the sides, in a given *ratio*, as 45 to 12.

18. To find the side of a cube, whose *superficies* is to the solidity in a given *ratio*, as 6 to 11.

19. A certain man hires a labourer, on this condition, that for every day he wrought, he should receive 12 pence; but for every day he was idle, he should be mulcted 8 pence: when 390 days were passed, neither of them were indebted to one another: how many days did he work, and how many was he idle?

20. A certain gentleman hires a servant, and promises him 24 pounds yearly wages, together with a cloak: at 8 months end the servant obtains leave to go away, and instead of his wages receives a cloak + 13 pounds: how much did the cloak cost?

21. A person being asked how old he was? answered, If I quadruple $\frac{2}{3}$ of my years, and add $\frac{1}{2}$ of them + 50 to the product, the sum will be so much above 100, as the number of my years is now below 100.

22. One

22. One being asked, what hour of the day it was? answered, The day at this time is 16 hours long; if now $\frac{1}{2}$ of the hours past be added to $\frac{2}{3}$ of the remainder, you will have the hour desired, reckoning from sun-rising.

23. From *Noremberg* to *Rome* are 140 miles: a traveller sets out at the same time from each of the two cities, one goes 8 miles a day, the other 6: in how many days from their first setting out will they meet one another, and how many miles did each of them go?

24. A certain messenger goes 6 miles every day: eight days after, another follows him, and he goes 10 miles a-day: in what number of days will he come up to the first?

25. A certain messenger goes 6 miles a-day: and after he has gone 56 miles, another follows him, who goes 8 miles a-day: in how many days will he come up to him?

26. One bought three books, whose prices were in proportion as 12, 5, 1: if the price of the first be doubled, of the second trebled, of the third quadrupled, the sum of these products will as much exceed 10 crowns, as the sum of the prices of the greatest and middle is below 5: how much did the said books cost?

27. Suppose the number 50 were to be divided into two parts, so that the greater part being divided by 7, and the less multiplied by 3, the sum of this product, and the former quotient, may make the same number proposed, which was 50.

28. Let the number 20 be divided into two parts, ſo that the ſquare of the leſſer part, being taken out of the ſquare of the greater, may leave the very number propoſed, which was 20, or may leave the double, treble, *&c.* of the number propoſed.

29. If a man gain 30 crowns a-week; how much muſt he ſpend a-week to have 500 crowns, together with the expence of four weeks, remaining at the year's end?

30. A labourer after 40 weeks, in which he had been at work, lays up 28 crowns—the pay of three weeks; and finds that he had expended 36 crowns + the pay of eleven weeks: what pay did he receive a-week?

31. In the rectancle ABC, is given the baſis AB = 9, and the difference of the other ſides, that is the ſegment BD = 3: required the ſides AC, BC?

32. In the rectangle triangle ABC, is given the baſis AB = 5, and the ſum of the other ſides AC + BC = 25: required the ſides AC, BC ſeverally?

33. Suppoſe two towers, AB 180 feet high, and CD 240, at the diſtance AC 360 feet; a ladder is to be ſet upon the line AC, at ſome point, ſuppoſe in E, of ſuch a length, as from thence it may reach the top of both the towers: we require the point E in the line of diſtance, as alſo the length of the ladder EB, ED?

34. In the triangle ABC, the ſeveral ſides AB = 13, AC = 14, BC = 15 are given; and the perpendicular BD being drawn: required the ſegments of the baſis AD, DC?

35. In

35. In the obtuse angled triangle DEF, the several sides are given, *viz.* DE 11, EF 13, DF 20; and the perpendicular FG, being let fall upon the basis produced: required the prolongation of the basis EG?

36. In the rectangle ABCD, is given the difference between the length AB and the diagonal BD, that is DE=2; and likewise the difference between the breadth AD, and the diagonal BD, that is FB=9: required the sides of the rectangle AB, AD?

37. In a rectangle DEFG, the right line DK is drawn from the angle D to the opposite side, cutting the diagonal EG at right angles in H: and there is given the segment HK=2, and HE=26: required the sides of the rectangle?

38. Let there be a circle whose diameter is AB, with another less circle, whose diameter AC touches within A; and from the centre of the greater circle D, draw the radius DE at right angles to AB, cutting the periphery of the lesser circle in F. Now there is given BC (the difference of the diameters) =9, with the segment EF=5: required the diameters AB, AC, of the said circles?

39. Two companions have got a parcel of guineas: says *A* to *B*, If you will give me one of your guineas, I shall have as many as you will have left. Nay, replies *B*, If you will give me one of your guineas, I shall have twice as many as you will have left: how many guineas had each of them?

40. A certain person bought two horses, with the trappings, which cost 100 pounds; which trappings, if laid on the first horse A, both the horses would be of equal value: but if the trappings be laid on the other horse, he will be double the value of the first: how much did the said horses cost?

41. A

41. A vintner has two sorts of wine, *viz.* A and B, which if mixed in equal parts, a flaggon of mixed will cost 15 pence; but if they be mixed in a *sesqui-alter* proportion, as if you should take two flaggons of A as often as you take three of B, a flaggon will cost 14 pence: required the price of each wine singly?

42. A son asked his father how old he was? His father answered him thus: If you take away 5 from my years, and divide the remainder by 8, the quotient will be $\frac{1}{3}$ of your age: but if you add 2 to your age, and multiply the whole by 3, and then subtract 7 from the product, you will have the number of the years of my age: what was the age of the father and the son?

43. To find out two numbers, to the sum whereof if you add 6, the whole shall double the greater; and if you subtract 2 from their difference, the remainder will be half of the least.

44. To find two numbers, the product whereof is 240, and the triple of the greater divided by the less is 5.

45. Two men have a mind to purchase a house rated at 1200 pounds; says *A* to *B*, If you give me $\frac{2}{3}$ of your money, I can purchase the house alone; but says *B* to *A*, If you will give me $\frac{3}{4}$ of yours, I shall be able to purchase the house: how much money had each of them?

46. Some young men and maids had a reckoning of 37 crowns to pay for a treat, and this was their conditions, that every young man should pay 3 crowns, and every maid 2. Now, if there had been as many young men as there were maids, observing the same conditions, the reckoning would have come to 4 crowns less than it did: how many young men and maids were there?

47. A general who had fought a battle, upon reviewing his army, whose foot was thrice the number of his horse, finds that before the battle $\frac{1}{12}-120$ of his foot had deserted, and of his horse $\frac{1}{20}+120$, besides $\frac{1}{4}$ of his whole army were sent into garrison, (reckoning the sick and wounded), and $\frac{3}{8}$ of his army remained; the rest, who were wanting, being either slain or taken prisoners: now, if you add 3000 to the number of the slain, the sum will be equal to half the foot he had at the beginning: what was the number of each?

48. To divide 100 twice into two parts, so that the *major* part of the first division may be triple the *minor* part of the second division; and the *major* part of the second may be double the *minor* part of the first.

49. To divide 30 twice into two parts, so that the *major* part of the first division, with the minor of the second, may be 33: and the sum of the minor parts subtracted from the sum of the *major*, may leave 14 remaining.

50. A man, his wife, and his son's age, make up 96 years, so that the husband's and son's years together make the wife's $+15$; but the wife's and the son's make the husband's $+2$: what was the age of each?

51. Three merchants, from three different fairs, meet together at an inn, where they reckon up their gains, and find them the sum of 780 crowns. Moreover, if you add the gain of the first and second, and subtract the gain of the third from the sum, there remains the gain of the first $+82$ crowns; but if you add the gain of the second and third, and from the sum subtract the gain of the first, there remains the gain of the third -43 crowns: what was the gain of each?

52. There

52. Three perſons, *A*, *B*, *C*, owe a certain ſum of money, ſo that *A* and *B* together owe 210 crowns; *B* and *C* 290, and *C* and *A* 400; what did each of them owe?

53. To find three numbers, ſo that the firſt and half of the remainder, the ſecond and $\frac{1}{3}$ of the remainder, and the third and $\frac{1}{4}$ of the remainder, may always make 34.

54. Let a ſquare be divided into 9 ſmall ſquares: we are to find and diſpoſe the numbers through the ſeveral *areas*, ſo that the ſum of every three, taken either literally or diagonally, may always be 15.

55. [Theorem.] Let any numbers whatſoever be given, if you ſubtract every leſs number from that which is the next greateſt; I ſay, that the ſum of thoſe differences is equal to the difference of the greateſt and leaſt numbers.

56. To find a number, which being multiplied by 6, and the product ſubtracted from the ſquare of the number to be found, the remainder will be 280.

57. To find a number, which being multiplied by 8, and the product added to the ſquare of the number to be found, the ſum will be 660.

58. To divide 140 into two parts, ſo that the product of thoſe parts may = the ſquare of 56, that is 3136.

59. Let 969 ſoldiers be drawn up into an oblong battle, ſo that the difference of the greater and leſs ſides is 40; required the number of the ſoldiers of each rank in length and breadth?

60. Again, let 480 ſoldiers be drawn up into an oblong battle, ſo that the ſum of the greater and leſs ſides is 52: required the number of the ſoldiers of each rank in length and breadth?

61. In the ſquare ABCD is given the difference of the diagonal and the ſide, that is EC=6: required the ſide of the ſquare?

62. The rectangle EK is added to the ſquare DF, (being of the ſame heighth), whoſe breadth EL is given=2, and alſo the *area* of the whole compound rectangle DK=60: required the firſt ſide of the ſquare?

63. A man buys ſome ells of cloth for 70 crowns; and finds that if he had 4 ells more, he had then bought every ell 2 crowns cheaper: how many ells did he buy?

64. A ſet of boon companions dining at an inn, the reckoning in all came to 175 ſhillings: but before the bill was paid off, two of them ſlunk away, and then the club of thoſe that remained came to 10 ſhillings a man more: how many were there in company?

65. To divide the number 21 in two parts, ſo that if the greater be divided by the leſſer, and again the leſſer by the greater, and then the firſt quotient being multiplied by 4, and the latter by 25, the numbers produced may be equal:

66. Let the line AB be divided in C, ſo that AC may be 8, and CD 6: we are to divide the ſame line AB in D, ſo that the rectangle under AB and DC may be equal to the rectangle under AC and CB, or to the product from 8 and 6, which is 48: required the ſegment CD?

67. Let

67. Let there be a rectangle garden ABCD, the length of which AB is thrice the breadth AD; and reckoning 18 perches from B towards A, that is BE, and drawing EF parallel to AD, let the *area* of the remaining rectangle ED be given=120 square perches: what was the length and breadth of the said garden?

68. Let 600 soldiers be disposed into an oblong battle; which the colonel, willing to make broader, finds that if he takes away 10 ranks from the length, he shall augment the breadth with two ranks: what was the number of his soldiers through every rank in length and breadth?

69. A man buys a horse, which he sells again for 56 crowns, and gains as many crowns in 100, as the horse cost him: how much did he give for the horse?

70. A certain linen-draper buys two sorts of linen for 30 crowns, one finer, the other coarser. An ell of the finest cost as many crowns as he had ells: and also 28 ells of the coarsest at such a price, that 8 ells cost as many crowns as one ell of the finest: how many ells of the finest linen did he by, and what price did he give for them both?

71. In a certain rectangular garden, the length of which AD is 22 perches, and the breadth AD is 10, the walk DG is to be made in a situation parallel to the sides of the figure, so that the *area* of the said walk or gnomon DG may be equal to the remaining rectangle FC, or that the gnomon DG may be half of the whole figure ABCD proposed: required the breadth of the said gnomon DE, or BG?

72. Of three proportional numbers there is the middle term given=12, and the difference of the extremes=10: required the extremes?

73. Of three proportional numbers there is given the sum of the first and second=10, and the differences of the second and third=24: required several numbers?

74. Of four proportional numbers there is given the third=12, also the sum of the first and second =8; besides the second number being subtracted from its square, the remainder is to be the fourth: required the said numbers?

75. Of four numbers in continued proportion, there is given the sum of the means=24, and likewise the sum of the extremes=56: required the said numbers, supposing that the first is the least of all?

76. Two country-women, *A* and *B*, carry 100 eggs together to market, and in the sale of them, one took as much money as the other: but *A* (who had the largest, and consequently the best eggs), says to *B*, Had I carried as many eggs as you, I should have had 18 pence for them, *B* replies, If I had brought as many eggs as you, I should have had but 8 pence for them: how many eggs had each?

77. Two country-men, *A* and *B*, sell their corn at different prices: *A* sells 20 bushels; and *B* received for one bushel as many crowns as he sold bushels: *A* perceives, that if he had sold as many bushels as *B* received crowns, he should then have received 252 crowns; but both together received 176 crowns: how many bushels did *B* sell? and what price had *A*?

78. Two

78. Two merchants sell 21 ells of cloth: the first sells 1 ell for as many crowns, as is $\frac{1}{6}$ of the number of ells that the second had; and the second sells 1 ell for as many crowns, as is $\frac{1}{3}$ of the number of the ells that the first had, the sale being over, they had taken 48 crowns in all: how many ells did each sell, and at what price?

79. Two merchants have a parcel of silk: the first 40 ells, the second 90: the first sells for a crown $\frac{1}{3}$ of an ell more than the second: when the sale was over, they had taken between them 42 crowns: how many ells did each of them sell for a crown?

80. To find a number, to the quadruple of which if you add 91, the whole shall be the square of the number sought, as 3 to 4.

81. To find a number, from the double of which if you subtract 12, the square of the remainder less 1, will be 9 times the number sought.

82. To divide the number 19 into 2 parts, so that the sum of the squares of the parts will be 193.

83. To divide 7 into two parts, so that the difference of the squares, which are made from the treble of the lesser part, and the double of the greater, may be 17.

84. A man buys a piece of linen; and by selling it again, he gains 12 crowns—$\frac{1}{10}$ of what he bought it for; and finds by this means that he had gained as much for 100 crowns as the linen cost him: what price was the linen bought and sold at?

85. A man buys 18 ells of cloth of different ſorts and colours, ſuppoſe red and black; what he bought of each, coſt 40 crowns; and he pays for every ell of red cloth 1 crown more than for the black: how many ells of each did he buy?

86. A man buys 123 pounds of pepper, and as many of ginger; and received for a crown one pound of ginger more than of pepper; ſo that the whole price of the pepper came to 6 crowns more than the price of the ginger: how many pounds of each did he buy for a crown?

87. A man buys 80 pounds of pepper, and 36 pounds of ſaffron, ſo that for 8 crowns he had 14 pounds of pepper more than he had of ſaffron for 26 crowns, and what he laid out amounted to 188 crowns: how many pounds of pepper had he for 8 crowns, and how many of ſaffron for 26?

88. *A* and *B* between them owe 174 pounds; *A* pays 8 pounds a-day, and *B* pays the firſt day 1 pound, the ſecond 2, the third 3, and ſo on: in how many days will they clear the debt, and how much did each of them owe?

89. A certain man intends to travel as many days as he has crowns: it happens, that every following day of his journey he had as many crowns as he had the day before, beſides two crowns over and above; and when he came to his journey's end he finds he had in all 45 crowns: how many crowns had he at firſt?

90. A certain traveller goes 9 miles a-day, three days after another follows him, who the firſt day travels 4 miles, the ſecond 5, and the third 6, and ſo on, gaining a mile every day: in what time will he overtake the former?

91. Two